Photonic Quantum Information Processing
OPTI 647: Lecture 16

Christos Gagatsos
Saikat Guha

October 15, 2019
College of Optical Sciences
Meinel 501A

Announcements

- Apologies for being late on grading HWs!
- The final presentations will be mid-December.
  - Schedule & dates TBD: By alphabetical order of last names.
  - Will invite an external examiner.

Recap

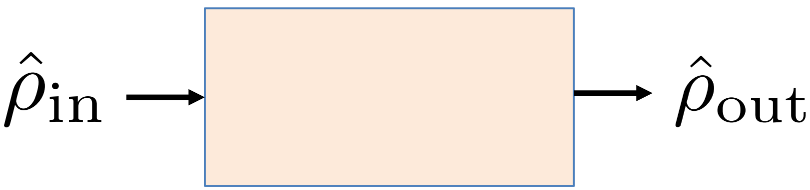
- Multimode Gaussian quantum optics
  - State:
    - Covariance matrix and Wigner function of n-mode Gaussian state
    - Gaussian state is one whose Wigner function is Gaussian
  - Transformation:
    - 1-mode Bogoliubov transformation as a unitary (squeezing)
    - Unitary operators corresponding to displacement and squeezing; they generate all 1-mode Gaussian transformation
    - n-mode linear optical transformation into n(n-1)/2 beamsplitters
    - Decomposition of arbitrary Gaussian transformation (an n-mode Bogoliubov transformation) as: n-mode linear-optical circuit → n squeezers → n-mode linear optical circuit
    - General n-mode Gaussian state = n-mode Bogoliubov transformation applied on n mode vacuum state
  - Measurement:
    - 1-mode homodyne, heterodyne; General development to follow

Recap (continued)

- New tools
  - "Tracing out a mode": quantum state of a sub-system: Two-mode squeezed vacuum, tracing out a mode gives a thermal state
- Applications
  - Continuous-variable teleportation (in progress)
- Notation
  - Saikat: $\hat{a} = \hat{a}_1 + j\hat{a}_2$, $\hat{a}^\dagger = \hat{a}_1 - j\hat{a}_2$
    - Quadrature commutator: $[\hat{a}_1, \hat{a}_2] = j/2$
    - Quadrature variance of vacuum state: $\langle \Delta \hat{a}_1^2 \rangle = \langle \Delta \hat{a}_2^2 \rangle = 1/4$
  - Christos: $\hat{a} = (\hat{q} + i\hat{p})/\sqrt{2}$, $\hat{a}^\dagger = (\hat{q} - i\hat{p})/\sqrt{2}$
    - Quadrature commutator: $[\hat{q}, \hat{p}] = i$
    - Quadrature variance of vacuum state: $\langle \Delta \hat{q}^2 \rangle = \langle \Delta \hat{p}^2 \rangle = 1/2$

Gaussian transformations: Phase

Schroedinger picture: state evolves under a unitary operation



$$U = e^{i\theta \hat{a}^\dagger \hat{a}}$$

$$|\psi_{\text{out}}\rangle = U|\psi_{\text{in}}\rangle$$

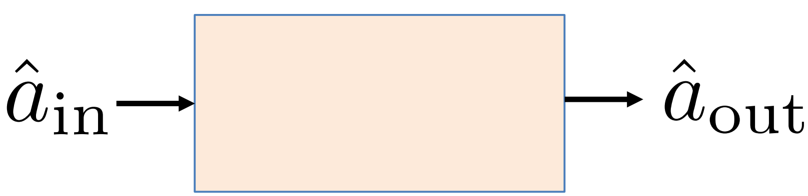
$$\hat{\rho}_{\text{out}} = U\hat{\rho}_{\text{in}}U^\dagger$$

Useful when calculating the output state directly

Example: input coherent state: $U|\alpha\rangle = |\alpha e^{i\theta}\rangle$

Heisenberg picture: operators evolve under a unitary operation; check to see, commutators preserved

$$\hat{a}_{\text{out}} = \hat{a}_{\text{in}} e^{i\theta} = U^\dagger \hat{a}_{\text{in}} U$$



Useful in transforming characteristic functions

Example: input coherent state:

$$\chi_N^{\text{in}}(\zeta) = \langle e^{\zeta \hat{a}_{\text{in}}^\dagger} e^{-\zeta^* \hat{a}_{\text{in}}}\rangle = e^{\zeta\alpha^* - \zeta^*\alpha}$$

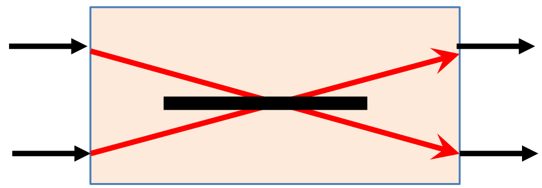
$$\chi_N^{\text{out}}(\zeta) = \langle e^{\zeta \hat{a}_{\text{out}}^\dagger} e^{-\zeta^* \hat{a}_{\text{out}}}\rangle = e^{\zeta e^{-i\theta}\alpha^* - \zeta^* e^{i\theta}\alpha}$$

Gaussian transformations: beamsplitter

$$U = e^{-\left[\arctan\sqrt{\eta^{-1}-1}\right](\hat{a}\hat{b}^\dagger - \hat{a}^\dagger\hat{b})}$$

Schroedinger

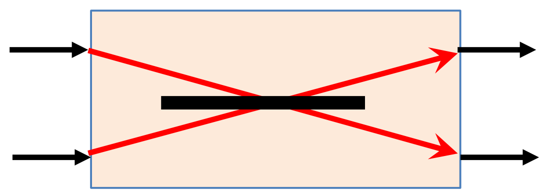
$\hat{\rho}_{\text{in}}$



$\hat{\rho}_{\text{out}}$

Heisenberg

$\hat{a}_{\text{in}}$, $\hat{b}_{\text{in}}$



$\hat{a}_{\text{out}}$, $\hat{b}_{\text{out}}$

$$\hat{a}_{\text{out}} = U^\dagger \hat{a}_{\text{in}} U = \sqrt{\eta}\,\hat{a}_{\text{in}} + \sqrt{1-\eta}\,\hat{b}_{\text{in}}$$

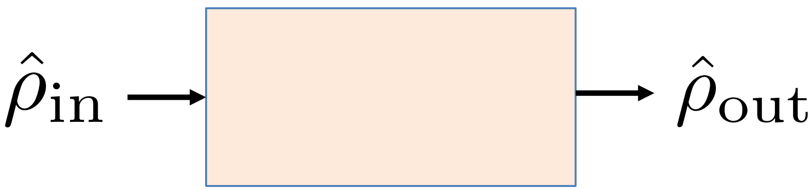
$$\hat{b}_{\text{out}} = U^\dagger \hat{b}_{\text{in}} U = -\sqrt{1-\eta}\,\hat{a}_{\text{in}} + \sqrt{\eta}\,\hat{b}_{\text{in}}$$

$$\chi_A^{a_{\text{out}}}(\zeta) = \langle e^{-\zeta^* \hat{a}_{\text{out}}} e^{\zeta \hat{a}^\dagger_{\text{out}}} \rangle = \chi_A^{a_{\text{in}}}(\sqrt{\eta}\zeta)\,\chi_A^{b_{\text{in}}}(\sqrt{1-\eta}\zeta)$$

Gaussian transformations: Displacement

THE UNIVERSITY OF ARIZONA

Schroedinger



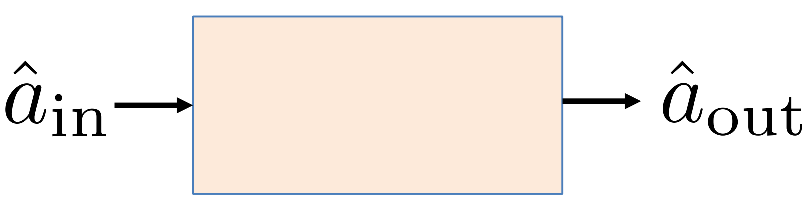
$$U = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}} \equiv \hat{D}(\alpha)$$

$$|\psi_{\mathrm{out}}\rangle = U|\psi_{\mathrm{in}}\rangle$$

$$\hat{\rho}_{\mathrm{out}} = U \hat{\rho}_{\mathrm{in}} U^\dagger$$

Example: input coherent state: $U|\beta\rangle = |\beta + \alpha\rangle$

Heisenberg



$$\hat{a}_{\mathrm{out}} = U^\dagger \hat{a}_{\mathrm{in}} U = \hat{a}_{\mathrm{in}} + \alpha$$

$$\hat{a}^\dagger_{\mathrm{out}} = U^\dagger \hat{a}^\dagger_{\mathrm{in}} U = \hat{a}^\dagger_{\mathrm{in}} + \alpha^*$$

$$\chi_N^{\mathrm{in}}(\zeta) = \langle e^{\zeta \hat{a}^\dagger_{\mathrm{in}}} e^{-\zeta^* \hat{a}_{\mathrm{in}}} \rangle = e^{\zeta \beta^* - \zeta^* \beta}$$

$$\chi_N^{\mathrm{out}}(\zeta) = \langle e^{\zeta \hat{a}^\dagger_{\mathrm{out}}} e^{-\zeta^* \hat{a}_{\mathrm{out}}} \rangle = \langle e^{\zeta (\hat{a}^\dagger_{\mathrm{in}} + \alpha^*)} e^{-\zeta^* (\hat{a}_{\mathrm{in}} + \alpha)} \rangle = e^{\zeta (\beta + \alpha)^* - \zeta^* (\beta + \alpha)}$$

Gaussian transformations: Squeezing

Schroedinger

$$\hat{\rho}_{\mathrm{in}} \longrightarrow \boxed{} \longrightarrow \hat{\rho}_{\mathrm{out}}$$

$$U = \exp\left[\frac{r}{2}\left(e^{-i\theta}\hat{a}^2 - e^{i\theta}\hat{a}^{\dagger 2}\right)\right]$$

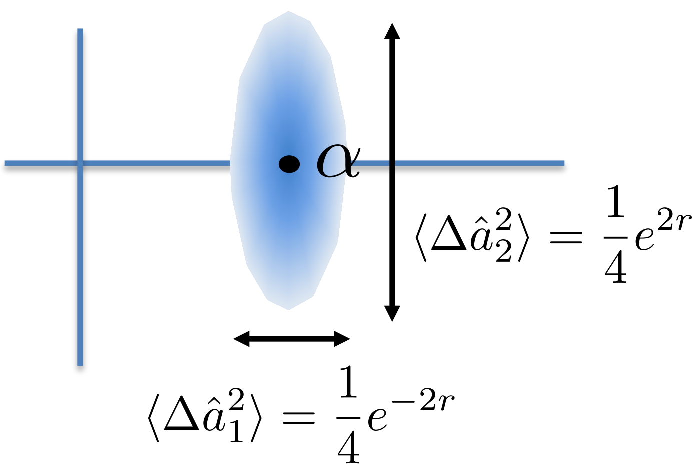
$$\equiv \hat{S}(z),\ z = re^{i\theta}$$

$$|\alpha; r, \theta\rangle = \hat{D}(\alpha)\hat{S}(z)|0\rangle = \hat{S}(z)\hat{D}(\beta)|0\rangle$$

$$\beta = \mu\alpha + \nu\alpha^*$$

$$\mu = \cosh r$$

$$\nu = e^{i\theta}\sinh r.$$



$$\theta = 0, \alpha \in \mathbb{R}$$

$$\beta = \alpha e^r$$

Mean photon number,

$$N = |\alpha|^2 + \sinh^2(r)$$

Heisenberg

$$\hat{a}_{\mathrm{in}} \longrightarrow \boxed{} \longrightarrow \hat{a}_{\mathrm{out}}$$

$$\hat{a}_{\mathrm{out}} = \hat{S}(z)\hat{a}_{\mathrm{in}}\hat{S}(z)^\dagger = \mu\hat{a}_{\mathrm{in}} + \nu\hat{a}_{\mathrm{in}}^\dagger$$

Squeezing is not passive. It adds photons

Gaussian transformations: two-mode squeezing (entangling operation)

THE UNIVERSITY OF ARIZONA

Schroedinger

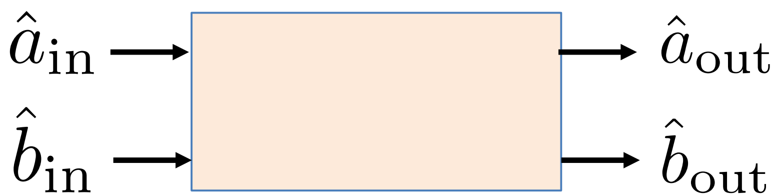
$$\hat{S}(z) = \exp\left(z^* \hat{a}\hat{b} - z\hat{a}^\dagger \hat{b}^\dagger\right),\ z = re^{i\theta}$$

Two-mode squeezed state, $|\alpha, \beta, z\rangle = \hat{D}_A(\alpha)\hat{D}_B(\beta)\hat{S}(z)|0\rangle_A|0\rangle_B$

$$N_A = |\alpha|^2 + \sinh^2(r),\ \ N_B = |\beta|^2 + \sinh^2(r)$$

Two-mode squeezed vacuum, $|0, 0, z\rangle = \sum_{n=0}^{\infty} \frac{N^n}{(1+N)^{n+1}}|n\rangle_A|n\rangle_B$

Heisenberg



$$\beta = \mu\alpha + \nu\alpha^*$$

$$\mu = \cosh r$$

$$\nu = e^{i\theta} \sinh r.$$

$$\hat{a}_{\text{out}} = \hat{S}(z)^\dagger \hat{a}_{\text{in}} \hat{S}(z) = \mu\hat{a}_{\text{in}} - \nu\hat{b}_{\text{in}}^\dagger$$

$$\hat{b}_{\text{out}} = \hat{S}(z)^\dagger \hat{b}_{\text{in}} \hat{S}(z) = \mu\hat{b}_{\text{in}} - \nu\hat{a}_{\text{in}}^\dagger$$

Phase-insensitive Linear Amplifier

$$\hat{\rho}_{\rm in} \qquad \hat{a}_{\rm in} \longrightarrow \boxed{G = \text{gain}} \longrightarrow \hat{a}_{\rm out} = \sqrt{G}\,\hat{a}_{\rm in} + \sqrt{G-1}\,\hat{b}^\dagger_{\rm in} \qquad \hat{\rho}_{\rm out}$$

$$|0\rangle \qquad \hat{b}_{\rm in} \longrightarrow \qquad \longrightarrow \hat{b}_{\rm out} = \sqrt{G}\,\hat{b}_{\rm in} + \sqrt{G-1}\,\hat{a}^\dagger_{\rm in} \quad \text{trace out}$$

Two-mode squeezer: unitary

$$\hat{\rho}_{\rm in} \qquad \hat{a}_{\rm in} \longrightarrow \boxed{\text{PIA, gain G}} \longrightarrow \hat{a}_{\rm out} = \sqrt{G}\,\hat{a}_{\rm in} + \sqrt{G-1}\,\hat{b}^\dagger_{\rm in} \qquad \hat{\rho}_{\rm out}$$

$$|0\rangle$$

Phase insensitive amplifier: not unitary (irreversible)

$$\chi_A^{\rho_{\rm out}}(\zeta) = \langle e^{-\zeta^*\sqrt{G}\hat{a}_{\rm in}} e^{\zeta\sqrt{G}\hat{a}^\dagger_{\rm in}}\rangle \langle e^{-\zeta^*\sqrt{G-1}\hat{b}^\dagger_{\rm in}} e^{\zeta\sqrt{G-1}\hat{b}_{\rm in}}\rangle = \chi_A^{\rho_{\rm in}}(\sqrt{G}\zeta)$$

$$\hat{\rho}_{\rm in} = |0\rangle\langle 0| \Rightarrow \chi_A^{\rho_{\rm out}}(\zeta) = e^{-|\sqrt{G}\zeta|^2} = e^{-(1+(G-1))|\zeta|^2}$$

$$\Rightarrow \hat{\rho}_{\rm out} = \int \frac{e^{-|\alpha|^2/(G-1)}}{\pi(G-1)} |\alpha\rangle\langle\alpha| d^2\alpha$$

Thermal state with mean photon number (G-1)

Phase-insensitive Linear Amplifier

$$\hat{\rho}_{\rm in} \qquad \hat{a}_{\rm in} \longrightarrow \boxed{\text{PIA, gain G}} \longrightarrow \hat{a}_{\rm out} = \sqrt{G}\,\hat{a}_{\rm in} + \sqrt{G-1}\,\hat{b}^\dagger_{\rm in} \qquad \hat{\rho}_{\rm out}$$

$$|0\rangle$$

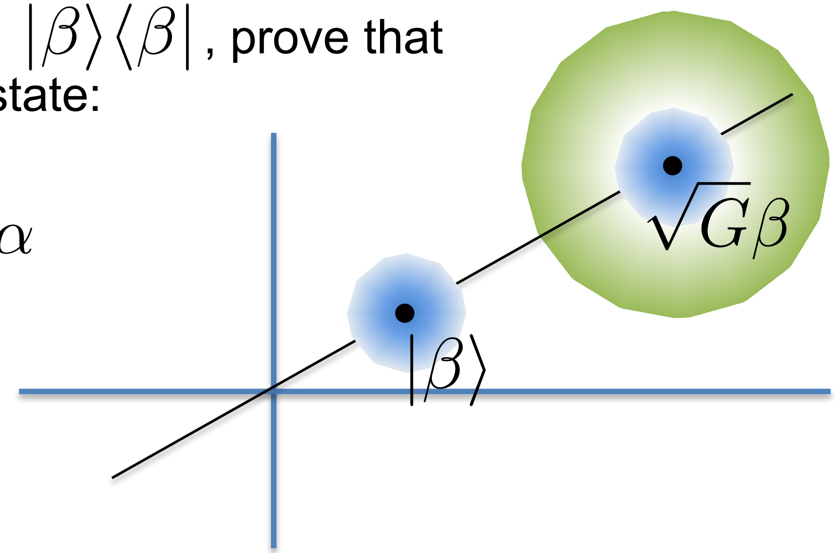
Phase insensitive amplifier: not unitary (irreversible)

$$\chi_A^{\rho_{\rm out}}(\zeta) = \langle e^{-\zeta^*\sqrt{G}\hat{a}_{\rm in}} e^{\zeta\sqrt{G}\hat{a}^\dagger_{\rm in}}\rangle\,\langle e^{-\zeta^*\sqrt{G-1}\hat{b}^\dagger_{\rm in}} e^{\zeta\sqrt{G-1}\hat{b}_{\rm in}}\rangle \;= \chi_A^{\rho_{\rm in}}(\sqrt{G}\zeta)$$

Problem 68

If the input to the PIA is a coherent state, $\hat{\rho}_{\rm in} = |\beta\rangle\langle\beta|$, prove that the output is the following mean-shifted thermal state:

$$\hat{\rho}_{\rm out} = \int \frac{e^{-|\alpha-\sqrt{G}\beta|^2/(G-1)}}{\pi(G-1)}|\alpha\rangle\langle\alpha|d^2\alpha$$



Quadrature variance of $\hat{\rho}_{\rm out}$:

$$\langle\Delta\hat{a}_1^2\rangle = \langle\Delta\hat{a}_2^2\rangle = \frac{2(G-1)+1}{4}$$

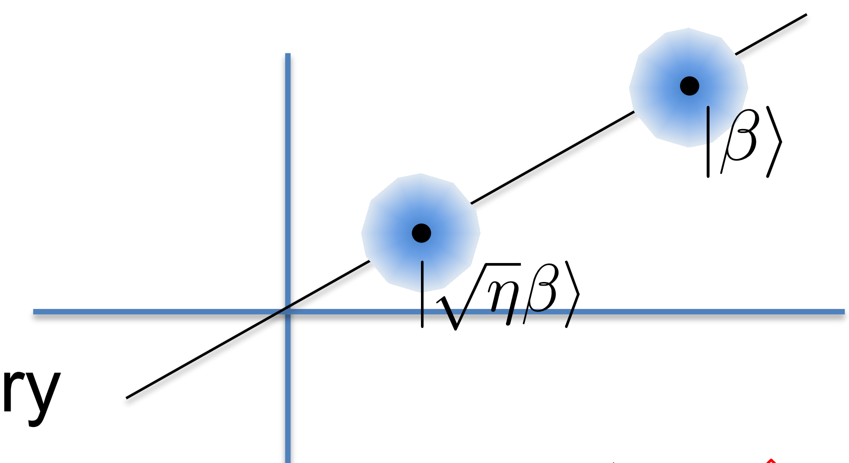
Beamsplitter and loss

- Beam splitter is a unitary

$$\hat{\rho}_{\rm in} \quad \hat{a}_{\rm in} \longrightarrow \boxed{\eta} \longrightarrow \hat{a}_{\rm out} = \sqrt{\eta}\,\hat{a}_{\rm in} + \sqrt{1-\eta}\,\hat{b}_{\rm in} \quad \hat{\rho}_{\rm out}$$

$$|0\rangle \quad \hat{b}_{\rm in} \longrightarrow \boxed{\eta} \longrightarrow \hat{b}_{\rm out} = \sqrt{\eta}\,\hat{b}_{\rm in} - \sqrt{1-\eta}\,\hat{a}_{\rm in} \quad \text{trace out}$$

Beamsplitter: unitary



- Photon loss is not a unitary

$$\hat{\rho}_{\rm in} \quad \hat{a}_{\rm in} \longrightarrow \boxed{\text{Photon loss}} \longrightarrow \hat{a}_{\rm out} = \sqrt{\eta}\,\hat{a}_{\rm in} + \sqrt{1-\eta}\,\underset{|0\rangle}{\hat{b}_{\rm in}} \quad \hat{\rho}_{\rm out}$$

Loss: not unitary (irreversible)

Gaussian transformations

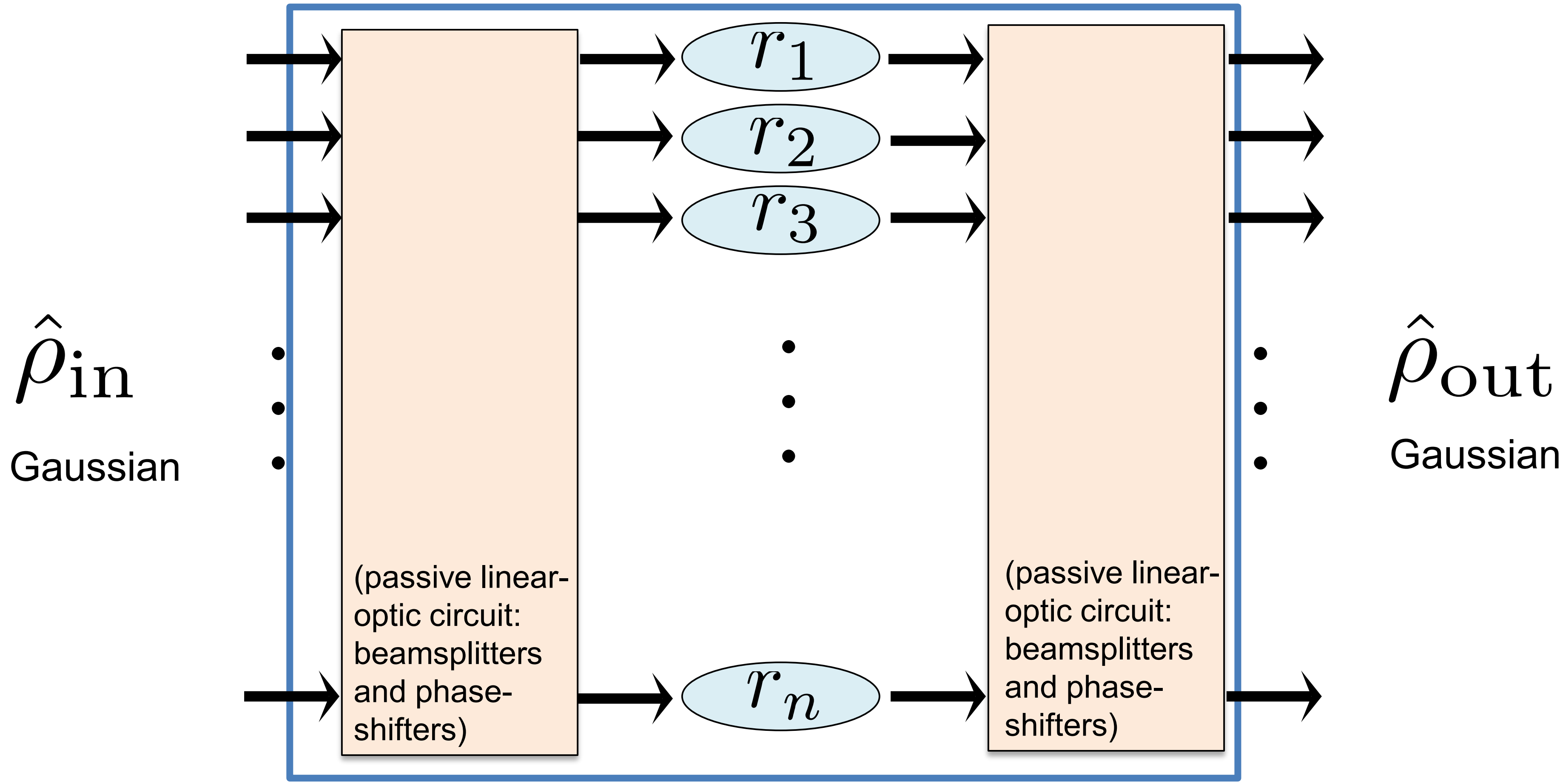
- Phase (1 → 1)
  - $U_{\text{phase}}(\theta), \theta \in [0, 2\pi)$
- Beam splitter (2 → 2)
  - $U_{\text{beamsplitter}}(\eta), \eta \in [0, 1)$
- Squeezing (1 → 1)
  - $U_{\text{squeezing}}(r) = \hat{S}(z), z = r \in [0, \infty)$
- Displacement (1 → 1)
  - $U_{\text{disp}}(\alpha) = \hat{D}(\alpha), \alpha \in \mathbb{C}$

General n-mode passive linear optical transformation

General zero-mean Gaussian unitary (Bogoliubov transformation)

General Gaussian transformation

n-mode zero-mean Gaussian unitary



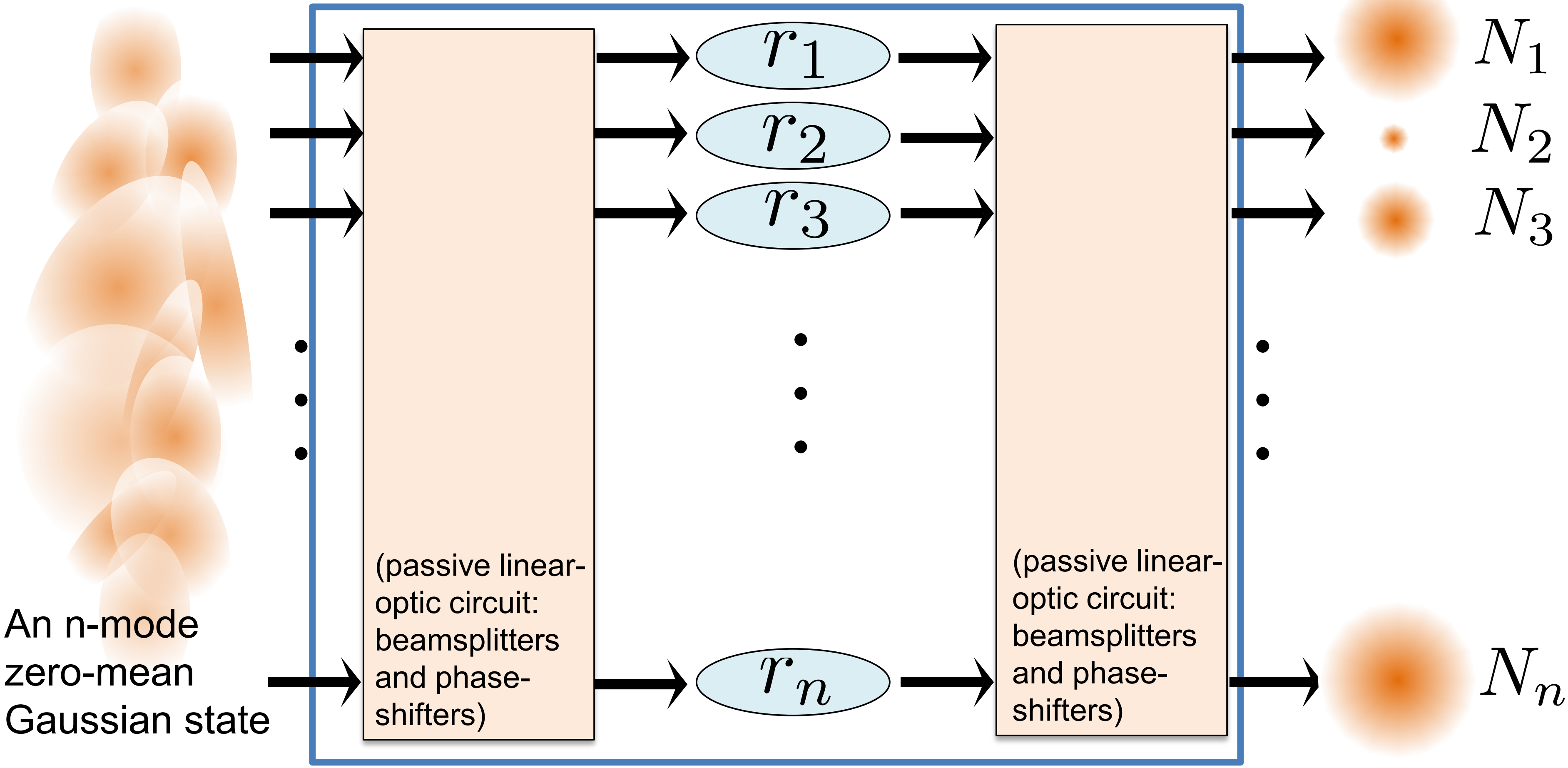
You learnt how to transform the covariance matrix (CM) of input state to that of output

$$V_{\text{in}} \rightarrow V_{\text{out}}$$

Braunstein, ***Squeezing as an irreducible resource,*** **Phys. Rev. A 71, 055801 (2005)**

Williamson's symplectic theorem

Symplectic decomposition



An n-mode symplectic transformation

Lasers, SPDCs, all passive linear optics (beamsplitters, polarizers, wave plates, diffusers), coherent detection, linear amplifiers (EDFA), OPAs

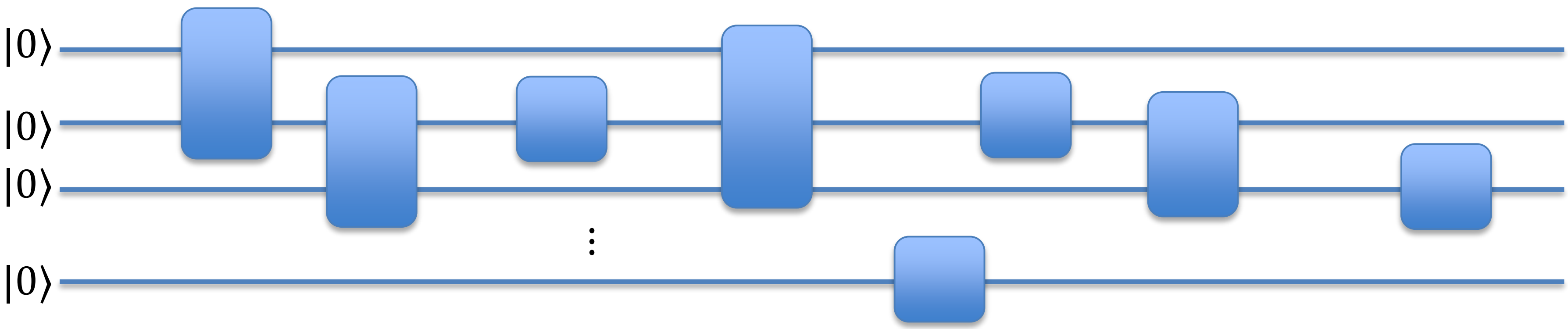
Braunstein, ***Squeezing as an irreducible resource,*** **Phys. Rev. A 71, 055801 (2005)**

Williamson's symplectic theorem

From Gaussian to Non-Gaussian transformations

THE UNIVERSITY OF ARIZONA

It is apparent, that if we want to access **any** transformation we must include non-quadratic Hamiltonians. But how much non-Gausianity is necessary?



When we change position to the operators, we basically have to commute (BCH relation) their generating Hamiltonians.

$\left[\hat{q}_i, \hat{p}_j\right] = i\delta_{ij}$ quadratures of the e/m field

$\hat{q}_i \rightarrow$ Generator of momentum displacement

$\hat{p}_i \rightarrow$ Generator of position displacement

$\hat{\Phi}_i = \hat{q}_i^2 + \hat{p}_i^2 \rightarrow$ Phase generator

$\hat{S}_i = \frac{1}{2}(\hat{q}_i\hat{p}_i + \hat{p}_i\hat{q}_i) \rightarrow$Squeezing generator

Hamiltonians of single mode generators

From Gaussian to Non-Gaussian transformations

By commuting the single mode Gaussian (quadratic) Hamiltonians $\hat{q}_i, \hat{p}_i, \hat{\Phi}_i, \hat{S}_i$, we can produce any other single mode Hamiltonian, *but nothing else.*

To include any number of modes $N > 1$, we just need $\hat{q}_i, \hat{p}_i, \hat{\Phi}_i, \hat{S}_i$ and a beam splitter $\hat{B}_{ij} = \hat{p}_i\hat{q}_j - \hat{q}_i\hat{p}_j$. In that way we can construct any multimode Gaussian Hamiltonian, which will be given by commutating the operators $\{\hat{q}_i, \hat{p}_i, \hat{\Phi}_i, \hat{S}_i, \hat{B}_{ij}\}$. **Recall: Reck decomposition.**

Reck decomposition of U(N)

N-mode passive transformation U(N).
We need **N(N-1)/2** beam splitters and phase shifters.
[Reck et al. 1994]

$$U(N)T_{N,N-1}\dots T_{2,1} = \begin{pmatrix} e^{i\phi_1} & & \\ & \ddots & \\ & & e^{i\phi_N} \end{pmatrix}$$

Where the matrix $T_{n,m}$ is a **unit matrix** with the matrix elements $t_{nn}, t_{mm}, t_{nm}, t_{mn}$ are replaced by the four matrix elements of some 2x2 unitary matrix.

$$U(N) = \begin{pmatrix} e^{i\phi_1} & & \\ & \ddots & \\ & & e^{i\phi_N} \end{pmatrix} T^\dagger_{2,1}\dots T^\dagger_{N,N-2}T^\dagger_{N,N-1}$$

Advanced Problem 5: Study Phys. Rev. Lett. **73**, 58 (1994), present it briefly and apply the Reck decomposition to the Hadamard gate $H = H_1 \otimes \dots \otimes H_1$

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$U(N)T_{N,N-1}T_{N,N-2}\dots T_{N-1,1} = \begin{pmatrix} U(N-1) & \\ & e^{i\phi_N} \end{pmatrix}$$

$$U(N-1)T_{N-1,N-2}T_{N-1,N-3}\dots T_{N-2,1} = \begin{pmatrix} U(N-2) & & \\ & e^{i\phi_{N-1}} & \\ & & e^{i\phi_N} \end{pmatrix}$$

$$\vdots$$

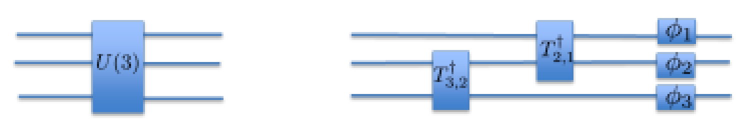
Example on Reck decomposition

$$U(3) = \begin{pmatrix} e^{i\phi 1}\cos(\theta 2) & e^{i\phi 1}\cos(\theta 1)\sin(\theta 2) & e^{i\phi 1}\sin(\theta 1)\sin(\theta 2) \\ -e^{i\phi 2}\sin(\theta 2) & e^{i\phi 2}\cos(\theta 1)\cos(\theta 2) & e^{i\phi 2}\cos(\theta 2)\sin(\theta 1) \\ 0 & -e^{i\phi 3}\sin(\theta 1) & e^{i\phi 3}\cos(\theta 1) \end{pmatrix}$$

$$U(3)\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta 1) & -\sin(\theta 1) \\ 0 & \sin(\theta 1) & \cos(\theta 1) \end{pmatrix} = \begin{pmatrix} e^{i\phi 1}\cos(\theta 2) & e^{i\phi 1}\sin(\theta 2) & 0 \\ -e^{i\phi 2}\sin(\theta 2) & e^{i\phi 2}\cos(\theta 2) & 0 \\ 0 & 0 & e^{i\phi 3} \end{pmatrix}$$

$$U(3)\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta 1) & -\sin(\theta 1) \\ 0 & \sin(\theta 1) & \cos(\theta 1) \end{pmatrix}\begin{pmatrix} \cos(\theta 2) & -\sin(\theta 2) & 0 \\ \sin(\theta 2) & \cos(\theta 2) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} e^{i\phi 1} & 0 & 0 \\ 0 & e^{i\phi 2} & 0 \\ 0 & 0 & e^{i\phi 3} \end{pmatrix}$$

$$U(3)T_{3,2}T_{2,1} = \begin{pmatrix} e^{i\phi 1} & 0 & 0 \\ 0 & e^{i\phi 2} & 0 \\ 0 & 0 & e^{i\phi 3} \end{pmatrix} \Rightarrow U(3) = \begin{pmatrix} e^{i\phi 1} & 0 & 0 \\ 0 & e^{i\phi 2} & 0 \\ 0 & 0 & e^{i\phi 3} \end{pmatrix} T^\dagger_{2,1}T^\dagger_{3,2}$$



From Gaussian to Non-Gaussian transformations

[Lloyd&Braunstein Quantum Computation over Continuous Variables, Vol. 82, Num. 8, p. 1784 (1999)]

If we include just one, single mode, non-quadratic Hamiltonian $\widehat{K}_i$, it is enough to construct any non-quadratic Hamiltonian by commutation relations of $\{\hat{q}_i, \hat{p}_i, \widehat{\Phi}_i, \hat{S}_i, \widehat{B}_{ij}, \widehat{K}_i\}$. For example Kerr non-linearity $\widehat{K}_i = \left(\hat{q}_i^2 + \hat{p}_i^2\right)^2$. Any other non-quadratic Hamiltonian $\widehat{K}_i$ would do the job.

Intuition/proof: for $\widehat{K}_i = \left(\hat{q}_i^2 + \hat{p}_i^2\right)^2$, when trying to commute $\widehat{K}_i$ with the Gaussian set $\{\hat{q}_i, \hat{p}_i, \widehat{\Phi}_i, \hat{S}_i, \widehat{B}_{ij}\}$, you'll need commutations of the form:

$$\left[\hat{q}_i^3, \hat{p}_i^m\, \hat{q}_i^n\right] = i\boxed{\hat{p}_i^{m+2}}\hat{q}_i^{n-1} + \text{lower order terms}$$

The exponent is increasing

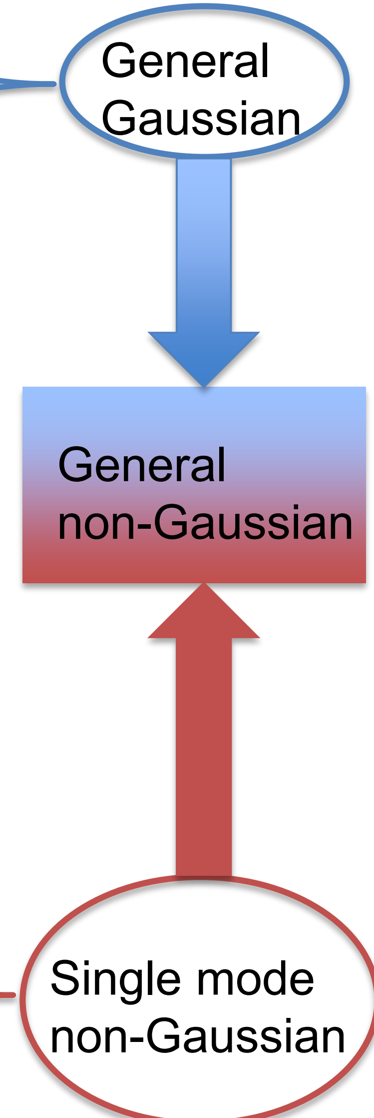
$$\left[\hat{p}_i^3, \hat{p}_i^m\, \hat{q}_i^n\right] = i\hat{p}_i^{m-1}\boxed{\hat{q}_i^{n+2}} + \text{lower order terms}$$

Gaussian transformations not universal. Need **any** one non-Gaussian unitary

THE UNIVERSITY OF ARIZONA

- Phase (1 → 1)
  - $U_{\text{phase}}(\theta), \theta \in [0, 2\pi)$
- Beam splitter (2 → 2)
  - $U_{\text{beamsplitter}}(\eta), \eta \in [0, 1)$
- Squeezing (1 → 1)
  - $U_{\text{squeezing}}(r) = \hat{S}(z), z = r \in [0, \infty)$
- Displacement (1 → 1)
  - $U_{\text{disp}}(\alpha) = \hat{D}(\alpha), \alpha \in \mathbb{C}$

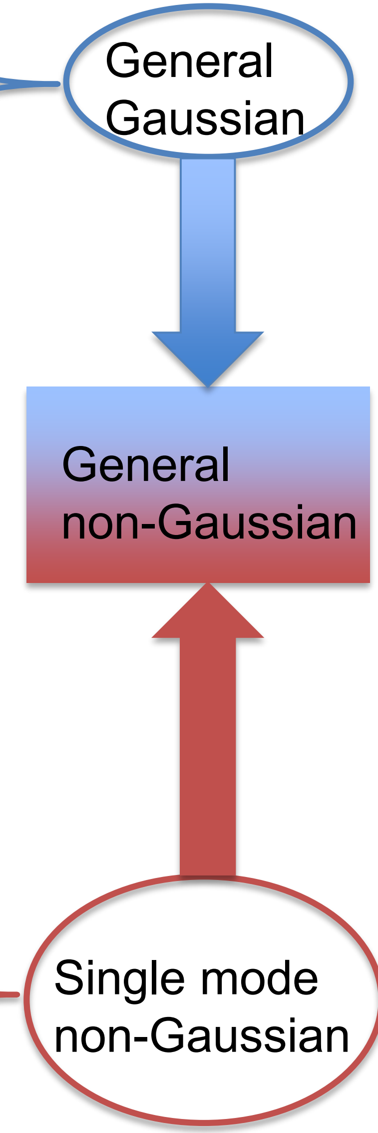
- Self-Kerr (1 → 1)
  - $U(\kappa) = e^{i\kappa(\hat{a}^\dagger \hat{a})^2}$



Gaussian transformations not universal. Need **any** one non-Gaussian unitary

- Phase (1 → 1)
  - $U_{\text{phase}}(\theta), \theta \in [0, 2\pi)$
- Beam splitter (2 → 2)
  - $U_{\text{beamsplitter}}(\eta), \eta \in [0, 1)$
- Squeezing (1 → 1)
  - $U_{\text{squeezing}}(r) = \hat{S}(z), z = r \in [0, \infty)$
- Displacement (1 → 1)
  - $U_{\text{disp}}(\alpha) = \hat{D}(\alpha), \alpha \in \mathbb{C}$

- Cubic phase (1 → 1)
  - $U(\gamma) = e^{i\gamma\hat{q}^3}, \ \hat{q} = \dfrac{\hat{a} + \hat{a}^\dagger}{\sqrt{2}}$



Quantum state of n bosonic modes

THE UNIVERSITY OF ARIZONA

Classical Gaussian e.g. thermal states	Classical Non-Gaussian e.g. a *statistical mixture* of two coherent states
Non-Classical Gaussian e.g. squeezed states	Non-Classical Non-Gaussian e.g. *superposition* of coherent states (cats)

Classical states:
They have P representation.

Gaussian states:
Gaussian (quadratic)Wigner function.

Hudson's theorem:
If a Wigner function of a pure state is positive, then the state is (and its Wigner function of course), is Gaussian. The converse is also valid.

WHEN IS THE WIGNER QUASI-PROBABILITY DENSITY NON-NEGATIVE?

R. L. HUDSON

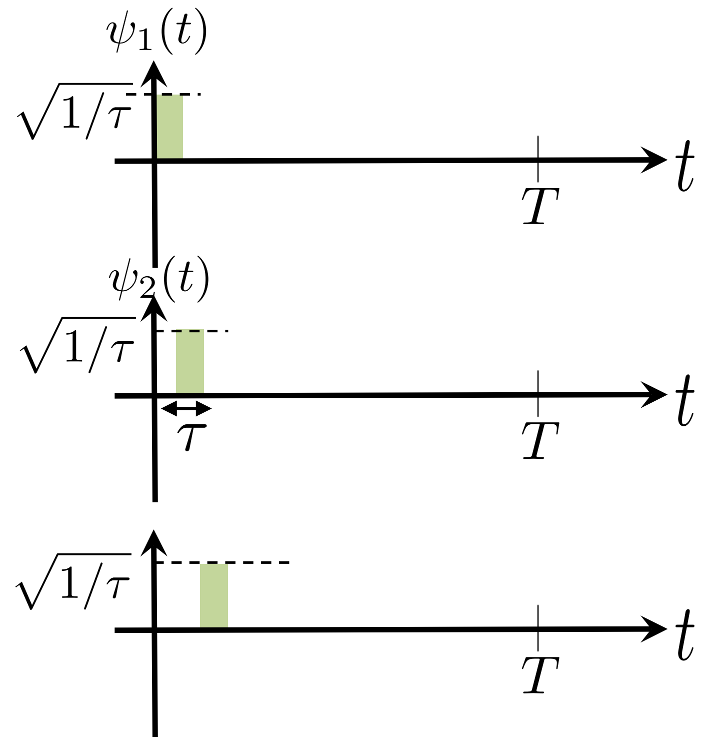
University of Nottingham, Nottingham, Great Britain

Reports on mathematical physics Vol. 6, No. 2, 1976

(*Received June 22, 1973*)

It is shown that a necessary and sufficient condition for the Wigner quasi-probability density to be a true density is that the corresponding Schrödinger state function be the exponential of a quadratic polynomial.

Photodetection on a coherent state



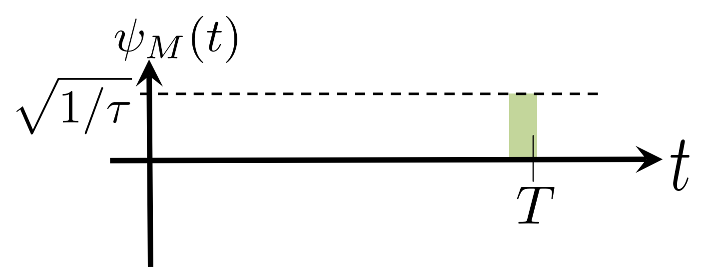
(1) Single-mode coherent state of this mode: $\phi(t)$

$|\alpha\rangle$, $\alpha = \sqrt{N}$, $N = E^2 T$

(2) M-mode coherent state of the modes: $\psi_k(t), k = 1, \ldots, M$

$$|\beta\rangle|\beta\rangle \ldots |\beta\rangle \quad , \beta = \sqrt{\frac{N}{M}} = \frac{\alpha}{\sqrt{M}}$$

$M = \dfrac{T}{\tau}$ Orthogonal temporal modes

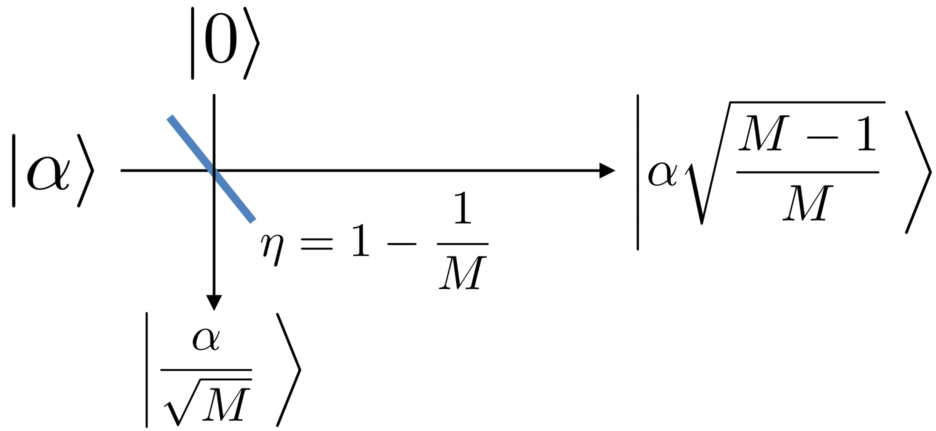


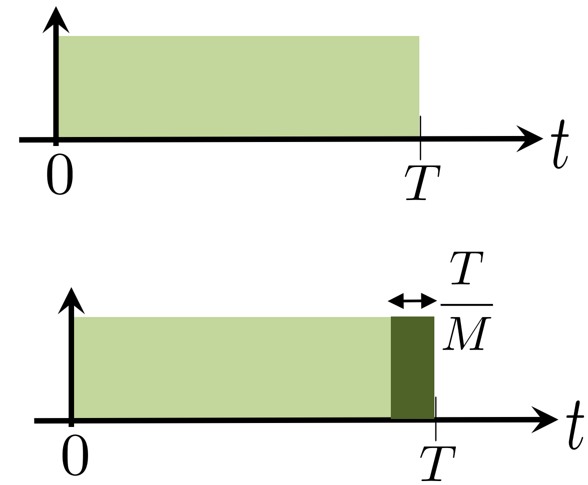
Let us choose #slices, M such that, $|\beta|^2 = N/M \ll 1$

$$|\beta\rangle = \sum_{n=0}^{\infty} \frac{e^{-|\beta|^2/2}\beta^n}{\sqrt{n!}}|n\rangle \approx A(|0\rangle + \beta|1\rangle),\ A = \frac{1}{\sqrt{1+|\beta|^2}}$$

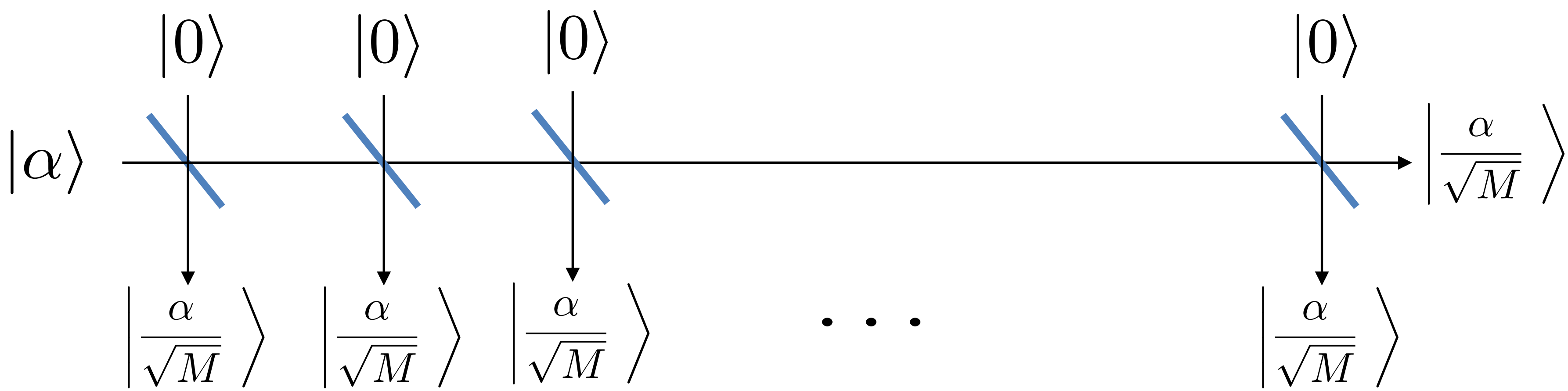
Reinterpret ideal photodetection as detecting slices

- Single time slice seen as a beam-splitter

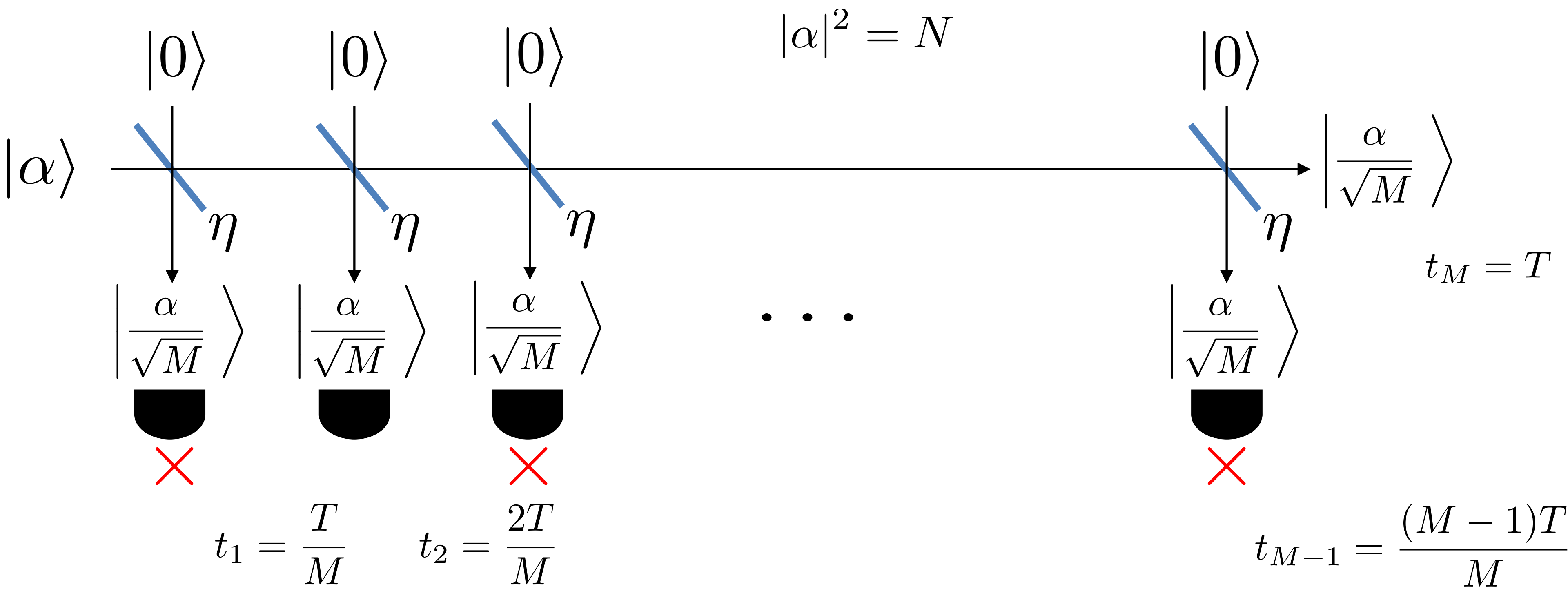




- M-1 BSs in a sequence, $\eta_i = 1 - \frac{1}{M-i}; \ i = 0, 1, \ldots, M-2$



Photon detection statistics of a coherent state pulse



For each detector:

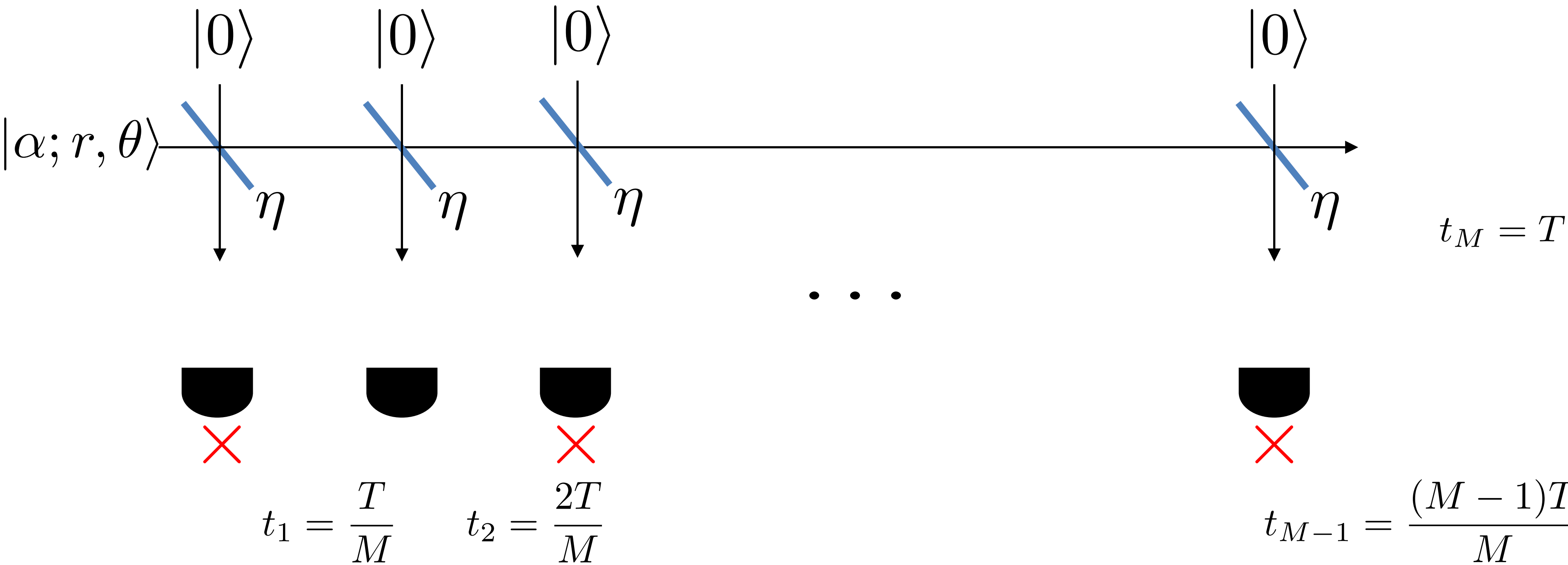
$$P(\text{click}) = 1 - e^{-|\alpha|^2/M} \approx \frac{N}{M}$$

$$P(\text{no click}) = e^{-|\alpha|^2/M} \approx 1 - \frac{N}{M}$$

Each detector's output is a statistically-independent binary-outcome random variable

Continuum limit of this statistics is a Poisson Point Process with arrival rate, $\lambda = N/T$

Photon detection on squeezed state pulse



$$|\alpha|^2 + \sinh^2(r) = N$$

Advanced Problem 7

Work out the photo-detection statistics of detecting a squeezed-state pulse in [0, T] with ideal photon detection. You may take, $\theta = 0$

Upcoming topics

- Non Gaussian states
  - Photon subtraction and cat states
- A few more applications and advanced problems
- **Revision of symplectic Gaussian formalism and introduction to quantum channels.**
- The list of possible forthcoming applications or further theoretical tools is growing. Go back to the slides of previous lectures and tell me if you have a preference (email me or in person).