Photonic Quantum Information Processing
OPTI 647: Lecture 14

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Recap of lecture 11

1. Description of a Gaussian state on phase space (covariance matrix, first moments).

2. Gaussian Wigner function.

3. How to transform the covariance matrix and the 1st moments under symplectic transformations.

4. Symplectic eigenvalues.

Outline of Lecture 11

1. Working out transformations.
2. Partial trace.
3. Thermal states.
4. CV teleportation.

$$\hbar = 1: \ \hat{a} = \frac{\hat{q}_a + i\hat{p}_a}{\sqrt{2}}, \ \hat{a}^\dagger = \frac{\hat{q}_a - i\hat{p}_a}{\sqrt{2}}$$

2-mode squeezed vacuum state

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Vacuum state (2-modes) $V_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

No displacements

2-mode squeezer $S_\xi = \begin{pmatrix} \cosh\xi & \sinh\xi & 0 & 0 \\ \sinh\xi & \cosh\xi & 0 & 0 \\ 0 & 0 & \cosh\xi & -\sinh\xi \\ 0 & 0 & -\sinh\xi & \cosh\xi \end{pmatrix}, \; \xi > 0$

1 phase shift for each mode $S_{\theta,\phi} = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & 0 & -\sin\left(\frac{\theta}{2}\right) & 0 \\ 0 & \cos\left(\frac{\phi}{2}\right) & 0 & -\sin\left(\frac{\phi}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & 0 & \cos\left(\frac{\theta}{2}\right) & 0 \\ 0 & \sin\left(\frac{\phi}{2}\right) & 0 & \cos\left(\frac{\phi}{2}\right) \end{pmatrix}$

$$V = S_{\theta,\phi} S_\xi^T V_0 S_\xi^T S_{\theta,\phi}^T = \frac{1}{2}\cosh 2\xi I_{2\times 2} + \frac{1}{2}\sinh 2\xi M$$

$$M = \begin{pmatrix} 0 & \cos\left(\frac{\theta+\phi}{2}\right) & 0 & \sin\left(\frac{\theta+\phi}{2}\right) \\ \cos\left(\frac{\theta+\phi}{2}\right) & 0 & \sin\left(\frac{\theta+\phi}{2}\right) & 0 \\ 0 & \sin\left(\frac{\theta+\phi}{2}\right) & 0 & -\cos\left(\frac{\theta+\phi}{2}\right) \\ \sin\left(\frac{\theta+\phi}{2}\right) & 0 & -\cos\left(\frac{\theta+\phi}{2}\right) & 0 \end{pmatrix}$$

A 3-mode squeezed state

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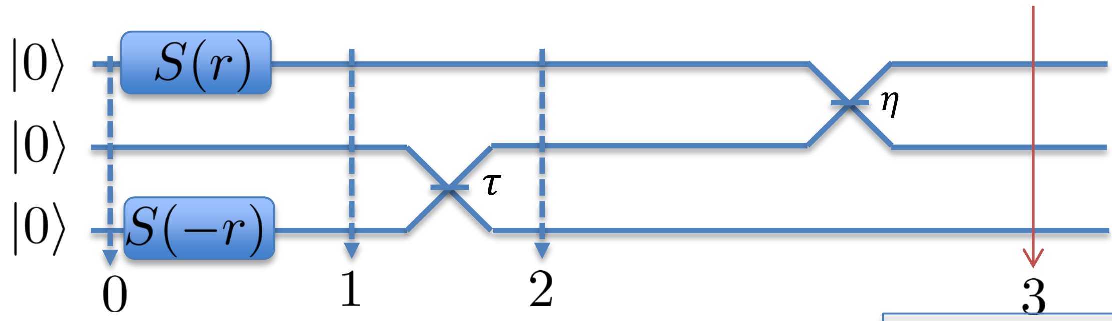

We pay attention on which modes each transformation acts on

Initial CM: $V_0 = \frac{1}{2} I_{6\times 6}$

Transformation between steps 0 and 1:

$$S_{01} = \mathrm{diag}\left(e^{r},\ 1,\ e^{-r},\ e^{-r},\ 1,\ e^{r}\right)$$

Transformation between steps 1 and 2:

$$S_{12} = \begin{pmatrix} 1 & 0 & 0 & & & \\ 0 & \sqrt{\tau} & \sqrt{1-\tau} & & & \\ 0 & -\sqrt{1-\tau} & \sqrt{\tau} & & & \\ & & & 1 & 0 & 0 \\ & & & 0 & \sqrt{\tau} & \sqrt{1-\tau} \\ & & & 0 & -\sqrt{1-\tau} & \sqrt{\tau} \end{pmatrix}$$

Transformation between steps 2 and 3:

$$S_{23} = \begin{pmatrix} \sqrt{\eta} & \sqrt{1-\eta} & 0 & & & \\ -\sqrt{1-\eta} & \sqrt{\eta} & 0 & & & \\ 0 & 0 & 1 & & & \\ & & & \sqrt{\eta} & \sqrt{1-\eta} & 0 \\ & & & -\sqrt{1-\eta} & \sqrt{\eta} & 0 \\ & & & 0 & 0 & 1 \end{pmatrix}$$

$$S = S_{23} S_{12} S_{01}$$

$$V = S V_0 S^T$$

2-mode thermal state from TMSV (partial trace)

$$\theta + \phi = 0$$

When we trace out a part of the system we discard the CM elements that contain momentum and/or position of the system to be traced out.

$$V_{\mathrm{TMSV}} = \frac{1}{2}\begin{pmatrix} \cosh 2\xi & \sinh 2\xi & 0 & 0 \\ \sinh 2\xi & \cosh 2\xi & 0 & 0 \\ 0 & 0 & \cosh 2\xi & -\sinh 2\xi \\ 0 & 0 & -\sinh 2\xi & \cosh 2\xi \end{pmatrix}$$

$$V_{\mathrm{th}} = \frac{1}{2}\begin{pmatrix} \cosh 2\xi & 0 \\ 0 & \cosh 2\xi \end{pmatrix}$$

Which in the form of a thermal state as we saw in previous lecture for $\lambda = \tanh\xi$

$|\lambda|^2 = \frac{\mathrm{M}}{\mathrm{M+1}}$, M: thermal photons

Problem 60: Trace out one of the modes of the following CM:

$$V = \frac{1}{2}\cosh 2\xi I_{2\times 2} + \frac{1}{2}\sinh 2\xi \begin{pmatrix} 0 & \cos\left(\frac{\theta+\phi}{2}\right) & 0 & \sin\left(\frac{\theta+\phi}{2}\right) \\ \cos\left(\frac{\theta+\phi}{2}\right) & 0 & \sin\left(\frac{\theta+\phi}{2}\right) & 0 \\ 0 & \sin\left(\frac{\theta+\phi}{2}\right) & 0 & -\cos\left(\frac{\theta+\phi}{2}\right) \\ \sin\left(\frac{\theta+\phi}{2}\right) & 0 & -\cos\left(\frac{\theta+\phi}{2}\right) & 0 \end{pmatrix}$$

n-mode thermal state

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$$\hat{\rho} = (1 - |\lambda|^2) \sum_{n=0}^{\infty} |\lambda|^{2n} |n\rangle\langle n|, \ 0 \leq |\lambda| < 1$$

Thermal photons

$$V_{\mathrm{th},1} = \frac{1}{2} \begin{pmatrix} \frac{1+|\lambda|^2}{1-|\lambda|^2} & 0 \\ 0 & \frac{1+|\lambda|^2}{1-|\lambda|^2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \cosh 2\xi & 0 \\ 0 & \cosh 2\xi \end{pmatrix} = \begin{pmatrix} M_1 + \frac{1}{2} & 0 \\ 0 & M_1 + \frac{1}{2} \end{pmatrix}$$

Tensor product of n thermal states →

Symplectic eigenvalues

$$V_{\mathrm{th},n} = \begin{pmatrix} M_1 + \frac{1}{2} & & & & & \\ & \ddots & & & & \\ & & M_n + \frac{1}{2} & & & \\ & & & M_1 + \frac{1}{2} & & \\ & & & & \ddots & \\ & & & & & M_n + \frac{1}{2} \end{pmatrix}$$

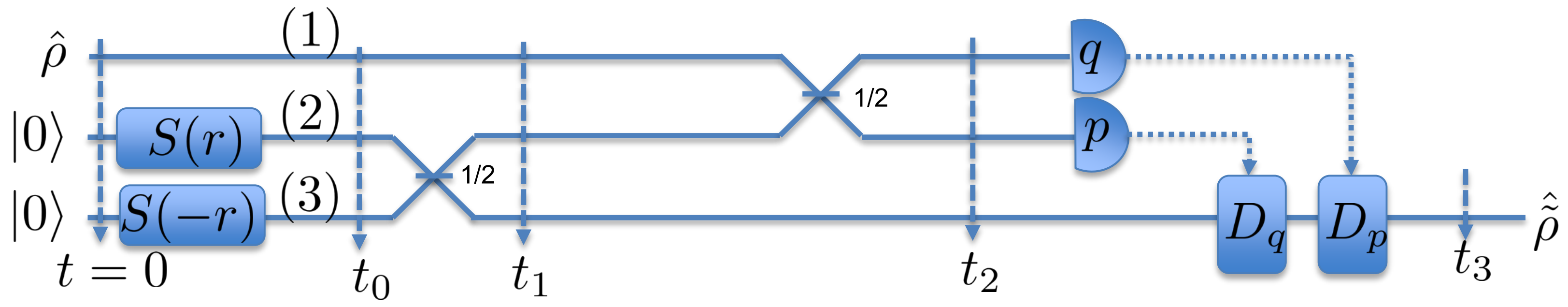
Mode 1

Mode n

Remember: n-mode thermal state = generic Gaussian state, i.e, we can construct any Gaussian state by applying symplectic transformations on a thermal state.

Therefore, the symplectic eigenvalues ν_i of any CM are of the form $M_i + \frac{1}{2}$ and their minimum value is $\frac{1}{2}$ when thermal photons are 0 (pure states). That is, $\nu_i \geq \frac{1}{2}$ where equality holds for pure states

CV teleportation of any state



$$\hat q_1(t_0) = \hat q_1(0)$$
$$\hat q_2(t_0) = \hat q_2(0)e^{r}$$
$$\hat q_3(t_0) = \hat q_2(0)e^{-r}$$

BS1 →

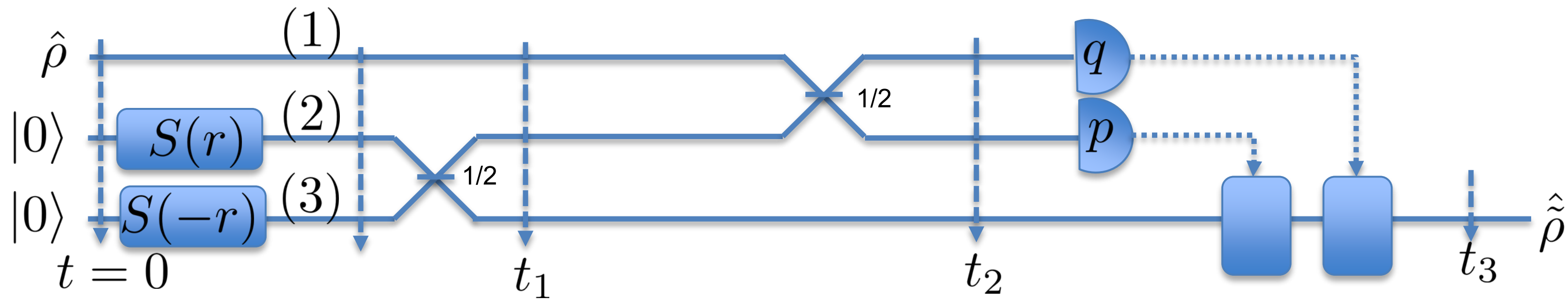
$$\hat q_1(t_1) = \hat q_1(t_0)$$
$$\hat q_2(t_1) = \frac{\hat q_2(t_0)+\hat q_3(t_0)}{\sqrt2}$$
$$\hat q_3(t_1) = \frac{\hat q_2(t_0)-\hat q_3(t_0)}{\sqrt2}$$

BS2 →

$$\hat q_1(t_2) = \frac{\hat q_1(t_1)-\hat q_2(t_1)}{\sqrt2} = \frac{\hat q_1(0)}{\sqrt2} - \frac{\hat q_2(t_0)+\hat q_3(t_0)}{2}$$
$$\hat q_2(t_2) = \frac{\hat q_1(t_1)+\hat q_2(t_1)}{\sqrt2} = \frac{\hat q_1(0)}{\sqrt2} + \frac{\hat q_2(t_0)+\hat q_3(t_0)}{2}$$
$$\hat q_3(t_2) = \hat q_1(0) - \sqrt2\hat q_1(t_2) - \sqrt2\hat q_3(0)e^{-r}$$

Similarly, for momentum ops: $\hat p_3(t_2) = \hat p_1(0) - \sqrt2\hat p_2(t_2) + \sqrt2\hat p_3(0)e^{-r}$

CV teleportation of any state



$$\hat{q}_3(t_2) = \hat{q}_1(0) - \sqrt{2}\hat{q}_1(t_2) - \sqrt{2}\hat{q}_3(0)e^{-r}$$

$$\hat{p}_3(t_2) = \hat{p}_1(0) - \sqrt{2}\hat{p}_2(t_2) + \sqrt{2}\hat{p}_3(0)e^{-r}$$

Hom. Mes.

$$\hat{q}_3(t_2) = \hat{q}_1(0) - q - \sqrt{2}\hat{q}_3(0)e^{-r}$$

$$\hat{p}_3(t_2) = \hat{p}_1(0) - p + \sqrt{2}\hat{p}_3(0)e^{-r}$$

Displacing

$$\hat{q}_3(t_3) = \hat{q}_1(0) - \sqrt{2}\hat{q}_3(0)e^{-r}$$

$$\hat{p}_3(t_3) = \hat{p}_1(0) + \sqrt{2}\hat{p}_3(0)e^{-r}$$

In CV teleportation, the protocol is perfect in the limit of infinite squeezing r.

CV teleportation of a coherent state

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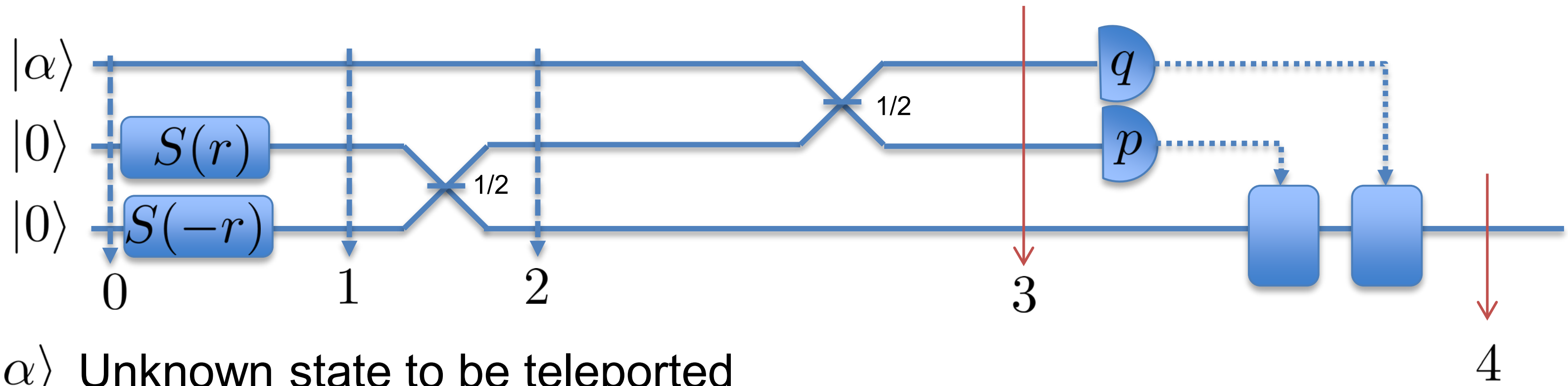

$|\alpha\rangle$ Unknown state to be teleported

Problem 61: Calculate the state in point 3 (covariance matrix and first moments). You already have the result from previous problem. Try to work out up to final point 4. $\alpha \in \mathbb{R}$

Upcoming topics

1. Wigner functions and the rest of quasiprobabilities.
2. More physics into the picture.
3. More topics on Gaussian states: Getting more familiar with the calculations. Fidelity, relative entropy etc → Gaussian metrology.
4. No-go theorems.
5. Non-Gaussian states, photon subtraction/addition.