# Photonic Quantum Information Processing
# OPTI 647: Lecture 6

Saikat Guha

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Associate Professor
College of Optical Sciences
Meinel 523

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# Plan for today

- Recap of important concepts
- Quadrature eigenkets
- Squeezed state

- Announcements:
  - Distance learning
    - Username: opti-647-instructor
    - Password: OSC-Fac-647!1
  - Office hours
    - **OSC 447 on Wednesdays at 11 am**
    - **DL students: https://gotomeet.me/SaikatGuha**

# Recap of a few important concepts

- Coherent states form an over-complete basis, representations in this basis are *not* unique
  $$\frac{1}{\pi}\int_{\mathbb{C}} |\alpha\rangle\langle\alpha| d^2\alpha = \hat{I} \qquad |\psi\rangle = \frac{1}{\pi}\int_{\mathbb{C}} \psi(\alpha)|\alpha\rangle d^2\alpha,\ \psi(\alpha) = \langle\alpha|\psi\rangle$$
- Q-function of a state $\hat{\rho}$ is a proper probability distribution, $Q(\alpha) = \frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle, \alpha \in \mathbb{C}$. We will later see this is a p.d.f. for Heterodyne detection, and can be regarded as the (POVM) measurement of $\hat{a}$
- Non-commuting observables, $[\hat{A}, \hat{B}] = j\hat{C}$; Heisenberg uncertainty principle states: $\langle\Delta\hat{A}^2\rangle\langle\Delta\hat{B}^2\rangle \geq \frac{1}{4}\left|\langle\hat{C}\rangle\right|^2$
  - Equality iff $\Delta\hat{A}|\psi\rangle = j\lambda\Delta\hat{B}|\psi\rangle$ for a real $\lambda$
  - Quadrature operators satisfy, $[\hat{a}_1, \hat{a}_2] = \frac{j}{2}$; $\langle\Delta\hat{a}_1^2\rangle\langle\Delta\hat{a}_2^2\rangle \geq \frac{1}{16}$
  - Coherent states satisfy, $\langle\Delta\hat{a}_1^2\rangle = \langle\Delta\hat{a}_2^2\rangle = 1/4$ (MUP state)
- Mean (field) = $\langle\hat{a}\rangle$ , Mean photon number = $\langle\hat{a}^\dagger\hat{a}\rangle$

# Quantum description of quadrature measurements; quadrature eigenkets

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- Eigenkets of the Hermitian quadrature operators

$$\hat{a}_1|\alpha_1\rangle_1 = \alpha_1|\alpha_1\rangle_1 \quad \text{and} \quad \hat{a}_2|\alpha_2\rangle_2 = \alpha_2|\alpha_2\rangle_2$$

  – Real-valued eigenvalues: $\alpha_1, \alpha_2$
  – Orthonormality, ${}_1\langle\alpha_1|\beta_1\rangle_1 = \delta(\alpha_1 - \beta_1)$
  ${}_2\langle\alpha_2|\beta_2\rangle_2 = \delta(\alpha_2 - \beta_2)$ Infinite energy, unphysical states
  – Resolution of identity, $\hat{I} = \int_{-\infty}^{\infty} d\alpha_k \alpha_k|\alpha_k\rangle_{k\,k}\langle\alpha_k|;\ k = 1, 2$

- We will prove from first principles:
  – Eigenvalues $\alpha_1, \alpha_2$ both live in $(-\infty, \infty)$
  – Inner product, ${}_2\langle\alpha_2|\alpha_1\rangle_1 = \dfrac{e^{-2j\alpha_2\alpha_1}}{\sqrt{\pi}}$, ${}_1\langle\alpha_1|\alpha_2\rangle_2 = \dfrac{e^{2j\alpha_2\alpha_1}}{\sqrt{\pi}}$

# Quadrature eigenkets have infinite energy

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- Suppose a mode $\hat{a}$ is in the $\hat{a}_1$ eigenket $|\alpha_1\rangle_1$
- Mean photon number

$$
\begin{aligned}
{}_1\langle\alpha_1|\hat{a}^\dagger\hat{a}|\alpha_1\rangle_1 &= {}_1\langle\alpha_1|(\hat{a}_1^2+\hat{a}_2^2)|\alpha_1\rangle_1 \\
&\geq {}_1\langle\alpha_1|\Delta\hat{a}_1^2|\alpha_1\rangle_1 + {}_1\langle\alpha_1|\Delta\hat{a}_2^2|\alpha_1\rangle_1 \\
&\geq {}_1\langle\alpha_1|\Delta\hat{a}_1^2|\alpha_1\rangle_1 + \frac{1}{16\,{}_1\langle\alpha_1|\Delta\hat{a}_1^2|\alpha_1\rangle_1} \\
&= \infty
\end{aligned}
$$

– Last equality follows from: $\hat{a}_1^k|\alpha_1\rangle_1 = \alpha_1^k|\alpha_1\rangle_1, k = 1, 2, \ldots$ which implies ${}_1\langle\alpha_1|\Delta\hat{a}_1^2|\alpha_1\rangle_1 = 0$

– Similarly, ${}_2\langle\alpha_2|\hat{a}^\dagger\hat{a}|\alpha_2\rangle_2 = \infty$

# Fourier transform relation of $\psi(\alpha_1)$ and $\Psi(\alpha_2)$

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- Quadrature wavefunctions of a pure state $|\psi\rangle$

$$|\psi\rangle = \int_{-\infty}^{\infty} d\alpha_1 \, \psi(\alpha_1)|\alpha_1\rangle_1; \; \psi(\alpha_1) = {}_1\langle\alpha_1|\psi\rangle, \; p(\alpha_1) = |\psi(\alpha_1)|^2$$

$$|\psi\rangle = \int_{-\infty}^{\infty} d\alpha_2 \, \Psi(\alpha_2)|\alpha_2\rangle_2; \; \Psi(\alpha_2) = {}_2\langle\alpha_2|\psi\rangle, \; p(\alpha_2) = |\Psi(\alpha_2)|^2$$

- We then have…

$$\Psi(\alpha_2) = {}_2\langle\alpha_2|\psi\rangle = \int_{-\infty}^{\infty} d\alpha_1 \, {}_2\langle\alpha_2|\alpha_1\rangle_1 \psi(\alpha_1) = \int_{-\infty}^{\infty} d\alpha_1 \, \psi(\alpha_1) \frac{e^{-2j\alpha_2\alpha_1}}{\sqrt{\pi}} = \frac{1}{\sqrt{\pi}} \mathcal{F}\left[\psi(t)\right]\Big|_{f=\alpha_2/\pi}$$

$$\psi(\alpha_1) = {}_1\langle\alpha_1|\psi\rangle = \int_{-\infty}^{\infty} d\alpha_2 \, {}_1\langle\alpha_1|\alpha_2\rangle_2 \Psi(\alpha_2) = \int_{-\infty}^{\infty} d\alpha_2 \, \Psi(\alpha_2) \frac{e^{2j\alpha_2\alpha_1}}{\sqrt{\pi}} = \frac{1}{\sqrt{\pi}} \mathcal{F}^{-1}\left[\Psi(f)\right]\Big|_{t=\alpha_1/\pi}$$

– where, $\mathcal{F}[x(t)] = X(f) \equiv \int_{-\infty}^{\infty} dt \, x(t) e^{-j2\pi f t}$

$$\mathcal{F}^{-1}[X(f)] = x(t) \equiv \int_{-\infty}^{\infty} df \, X(f) e^{j2\pi f t}$$

# FT uncertainty relation and the HUP

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- Normalized intensities: $p(t) = \frac{|x(t)|^2}{\int_{-\infty}^{\infty} |x(t)|^2 dt}$, $P(f) = \frac{|X(f)|^2}{\int_{-\infty}^{\infty} |X(f)|^2 df}$
- RMS time and bandwidth:

$$T \equiv \sqrt{\int_{-\infty}^{\infty} t^2 p(t) dt} \quad \text{and} \quad W \equiv \sqrt{\int_{-\infty}^{\infty} f^2 P(f) df}$$

- Use Parseval's theorem and Cauchy-Schwarz inequality to show that: $TW \geq 1/4\pi$

**Problem 22**

  - Equality for: $x(t) = \frac{\exp(-t^2/4\sigma^2)}{(2\pi\sigma^2)^{1/4}}$ and $X(f) = (8\pi\sigma^2)^{1/4}\exp(-4\pi^2 f^2\sigma^2)$
  - $T = \sigma$, $W = 1/4\pi\sigma$

- Apply this to the quadrature wavefunctions to show:

$$\langle \hat{a}_1^2 \rangle \langle \hat{a}_2^2 \rangle = \int_{-\infty}^{\infty} |\alpha_1|^2 |\psi(\alpha_1)|^2 d\alpha_1 \int_{-\infty}^{\infty} |\alpha_2|^2 |\Psi(\alpha_2)|^2 d\alpha_2 \geq 1/16$$

  - Equality for: $\psi(\alpha_1) = \frac{\exp(-\alpha_1^2/4\langle \Delta\hat{a}_1^2 \rangle)}{(2\pi\langle \Delta\hat{a}_1^2 \rangle)^{1/4}}$, $\Psi(\alpha_2) = \frac{\exp(-\alpha_2^2/4\langle \Delta\hat{a}_2^2 \rangle)}{(2\pi\langle \Delta\hat{a}_2^2 \rangle)^{1/4}}$

  with $\langle \Delta\hat{a}_2^2 \rangle = 1/16\langle \Delta\hat{a}_1^2 \rangle$

**Problem 23**

# Quadrature wavefunctions of a general MUP state with non-zero means

- Wavefunctions for the case of non-zero means

$$\psi(\alpha_1) = \frac{\exp\left(\left[2j\langle\Delta\hat{a}_2\rangle\alpha_1 - j\langle\Delta\hat{a}_1\rangle\langle\Delta\hat{a}_2\rangle - (\alpha_1 - \langle\Delta\hat{a}_1\rangle)^2\right]/4\langle\Delta\hat{a}_1^2\rangle\right)}{(2\pi\langle\Delta\hat{a}_1^2\rangle)^{1/4}}$$

$$\Psi(\alpha_2) = \frac{\exp\left(\left[-2j\langle\Delta\hat{a}_1\rangle\alpha_2 + j\langle\Delta\hat{a}_1\rangle\langle\Delta\hat{a}_2\rangle - (\alpha_2 - \langle\Delta\hat{a}_2\rangle)^2\right]/4\langle\Delta\hat{a}_2^2\rangle\right)}{(2\pi\langle\Delta\hat{a}_2^2\rangle)^{1/4}}$$

  – with $\langle\Delta\hat{a}_2^2\rangle = 1/16\langle\Delta\hat{a}_1^2\rangle$
  – Derivation follows from shift property of Fourier Transform

- Example: coherent state $|\beta\rangle,\ \beta = \beta_1 + j\beta_2$ is MUP
  – $\langle\Delta\hat{a}_1\rangle = \beta_1, \langle\Delta\hat{a}_2\rangle = \beta_2, \langle\Delta\hat{a}_1^2\rangle = \langle\Delta\hat{a}_2^2\rangle = 1/4$
  – $\psi(\alpha_1) = \dfrac{\exp\left[2j\beta_2\alpha_1 - j\beta_1\beta_2 - (\alpha_1 - \beta_1)^2\right]}{(\pi/2)^{1/4}}$
  – $\Psi(\alpha_2) = \dfrac{\exp\left[-2j\beta_1\alpha_2 + j\beta_1\beta_2 - (\alpha_2 - \beta_2)^2\right]}{(\pi/2)^{1/4}}$

# Quadrature eigenkets from first principles… but first, we need some operator algebra

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- Prove that: $\left[\hat{a}_1, \hat{a}_2^k\right] = \dfrac{jk\hat{a}_2^{k-1}}{2}$, for $k > 2$ **Problem 24(a)**
  - Hint: use $\left[\hat{a}_1, \hat{a}_2\right] = j/2$ to show $\left[\hat{a}_1, \hat{a}_2^2\right] = j\hat{a}_2$; use induction
- Define the following operator derivative: $\dfrac{d\hat{a}_2^k}{d\hat{a}_2} \equiv k\hat{a}_2^{k-1}$
  - So, $\left[\hat{a}_1, \hat{a}_2^k\right] = (j/2)\dfrac{d\hat{a}_2^k}{d\hat{a}_2}, \; k = 1, 2, \ldots$
  - In analogy to above, prove that

$$\left[\hat{a}_2, \hat{a}_1^k\right] = -jk\hat{a}_1^{k-1}/2 = -(j/2)\frac{d\hat{a}_1^k}{d\hat{a}_1}, \; k = 1, 2, \ldots$$

**Problem 24(b)**

- Consider functions with convergent Taylor series

$$F(\alpha_1) = \sum_{n=0}^{\infty} \frac{\alpha_1^n}{n!} \frac{d^n F(\alpha_1)}{d\alpha_1^n}\bigg|_{\alpha_1=0}, \; -\infty < \alpha_1 < \infty \qquad F(\hat{a}_1) = \sum_{n=0}^{\infty} \frac{\hat{a}_1^n}{n!} \frac{d^n F(\alpha_1)}{d\alpha_1^n}\bigg|_{\alpha_1=0}$$

$$G(\alpha_2) = \sum_{n=0}^{\infty} \frac{\alpha_2^n}{n!} \frac{d^n G(\alpha_2)}{d\alpha_2^n}\bigg|_{\alpha_2=0}, \; -\infty < \alpha_2 < \infty \qquad G(\hat{a}_2) = \sum_{n=0}^{\infty} \frac{\hat{a}_2^n}{n!} \frac{d^n G(\alpha_2)}{d\alpha_2^n}\bigg|_{\alpha_2=0}$$

- Prove that: $\left[\hat{a}_1, G(\hat{a}_2)\right] = \left(\dfrac{j}{2}\right)\dfrac{dG(\hat{a}_2)}{d\hat{a}_2}$ and $\left[\hat{a}_2, F(\hat{a}_1)\right] = \left(-\dfrac{j}{2}\right)\dfrac{dF(\hat{a}_1)}{d\hat{a}_1}$

**Problem 25**

# Translation operator that generate the quadrature eigenkets

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- Assume $\hat{a}_1|\alpha_1\rangle_1 = \alpha_1|\alpha_1\rangle_1$ , $\alpha_1$ real
- Define a "translation" operator

$$\hat{A}_1(\xi) \equiv \exp(-2j\xi\hat{a}_2) = \sum_{n=0}^{\infty} \frac{(-2j\xi)^n}{n!}\hat{a}_2^n, \text{ for } -\infty < \xi < \infty$$

  – Use result from Problem 25, and $\hat{a}_1\hat{A}_1(\xi) = \hat{A}_1(\xi)\hat{a}_1 + \left[\hat{a}_1, \hat{A}_1(\xi)\right]$ to show that $\hat{A}_1(\xi)|\alpha_1\rangle_1$ is an eigenket of $\hat{a}_1$ with eigenvalue $\alpha_1 + \xi$ for any real number $\xi$ **Problem 26(a)**
  – Show that $|\alpha_1\rangle_1 = \exp(-2j\alpha_1\hat{a}_2)|0\rangle_1$ is an $\hat{a}_1$ eigenket with eigenvalue $\alpha_1$ and that ${}_1\langle\alpha_1|\alpha_1\rangle_1 = {}_1\langle 0|0\rangle_1$ **Problem 26(b)**
- Similarly, with $\hat{A}_2(\xi) \equiv \exp(2j\xi\hat{a}_1) = \sum_{n=0}^{\infty} \frac{(2j\xi)^n}{n!}\hat{a}_1^n, \text{ for } -\infty < \xi < \infty$
  – $\hat{A}_2(\xi)|\alpha_2\rangle_2 = |\alpha_2 + \xi\rangle_2$

# Inner product of quadrature eigenkets



- Use the results of Problem 26, to prove:

$$ {}_2\langle\alpha_2|\alpha_1\rangle_1 = \exp(-2j\alpha_1\alpha_2)\,{}_2\langle 0|0\rangle_1 $$

**Problem 27**

  - Hint: Show that $|\alpha_2\rangle_2$ is an eigenket of $\hat{A}_1(\xi)$, and $|\alpha_1\rangle_1$ is an eigenket of $\hat{A}_2(\xi)$
- Next, ${}_2\langle\alpha_2'|\alpha_2\rangle_2 = {}_2\langle\alpha_2'|\hat{I}|\alpha_2\rangle_2 = {}_2\langle\alpha_2'|\left(\int_{-\infty}^{\infty} d\alpha_1|\alpha_1\rangle_{11}\langle\alpha_1|\right)|\alpha_2\rangle_2$

$$ = |{}_2\langle 0|0\rangle_1|^2 \int_{-\infty}^{\infty} \exp[-2j(\alpha_2' - \alpha_2)\alpha_1]d\alpha_1 $$

$$ = |{}_2\langle 0|0\rangle_1|^2 \pi\delta(\alpha_2 - \alpha_2') $$

  - Assuming ${}_2\langle 0|0\rangle_1$ is positive real, ${}_2\langle 0|0\rangle_1 = 1/\sqrt{\pi}$
  - Hence, ${}_2\langle\alpha_2|\alpha_1\rangle_1 = \dfrac{e^{-2j\alpha_2\alpha_1}}{\sqrt{\pi}}$

# MUP state and Bogoliubov transformation

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- Heisenberg uncertainty principle: $\langle\Delta\hat{a}_1^2\rangle\langle\Delta\hat{a}_2^2\rangle \geq \frac{1}{16}$
  - Equality for $|\psi\rangle$ iff $\Delta\hat{a}_1|\psi\rangle = -j\lambda\Delta\hat{a}_2|\psi\rangle$ for a real $\lambda$
- Rearrange this condition as an eigenvalue equation

$$(\hat{a}_1 + j\lambda\hat{a}_2)|\psi\rangle = \left(\langle\hat{a}_1\rangle + j\lambda\langle\hat{a}_2\rangle\right)|\psi\rangle$$

  - Some solutions to this *must* exist since we know of the existence of MUP states from our wavefunction analysis
  - Suppose $\lambda$ is non-negative. Re-write the condition as:

$$(\mu\hat{a} + \nu\hat{a}^\dagger)|\psi\rangle = (\mu\langle\hat{a}\rangle + \nu\langle\hat{a}^\dagger\rangle)|\psi\rangle$$

    - with $\mu = (1+\lambda)/2\sqrt{\lambda},\ \nu = (1-\lambda)/2\sqrt{\lambda}$ , note that $\mu^2 - \nu^2 = 1$
  - Define a new operator: $\hat{b} \equiv \mu\hat{a} + \nu\hat{a}^\dagger$
    - Verify that, $[\hat{b}, \hat{b}^\dagger] = 1$ **Problem 28**
    - Bogoliubov transformation ($\hat{a} \longrightarrow \hat{b}$): this will give us new insights on MUP states. We will later see how to realize this

# "coherent states" of the $\hat{b}$ operator

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- From our wavefunction analysis, there must be kets $|\psi\rangle$ that satisfy $(\hat{a}_1 + j\lambda\hat{a}_2)|\psi\rangle = (\langle\hat{a}_1\rangle + j\lambda\langle\hat{a}_2\rangle)\,|\psi\rangle$
  - Let us label them as: $\hat{b}|\beta;\mu,\nu\rangle = \beta|\beta;\mu,\nu\rangle,\ \beta \in \mathbb{C}$
  - By using $\hat{a} = \mu\hat{b} - \nu\hat{b}^\dagger$, we get the mean (field):

    $\langle\hat{a}\rangle = \langle\beta;\mu,\nu|\hat{a}|\beta;\mu,\nu\rangle = \mu\beta - \nu\beta^*$
- As $[\hat{b},\hat{b}^\dagger] = [\hat{a},\hat{a}^\dagger] = 1$, they behave similarly

| | |
|---|---|
| $\hat{N} \equiv \hat{a}^\dagger\hat{a}$ | $\hat{N}_b \equiv \hat{b}^\dagger\hat{b}$ |
| $\hat{N}|n\rangle = n|n\rangle$ | $\hat{N}_b|n;\mu,\nu\rangle = n|n;\mu,\nu\rangle$ |
| $\langle m|n\rangle = \delta_{mn}$ | $\langle m;\mu,\nu|n;\mu,\nu\rangle = \delta_{mn}$ |
| $\hat{I} = \sum_{n=0}^{\infty}|n\rangle\langle n|$ | $\hat{I} = \sum_{n=0}^{\infty}|n;\mu,\nu\rangle\langle n;\mu,\nu|$ |
| $\hat{a}^\dagger = \sum_{n=0}^{\infty}\sqrt{n+1}|n+1\rangle\langle n|$ | $\hat{b}^\dagger = \sum_{n=0}^{\infty}\sqrt{n+1}|n+1;\mu,\nu\rangle\langle n;\mu,\nu|$ |
| $\hat{a} = \sum_{n=1}^{\infty}\sqrt{n}|n-1\rangle\langle n|$ | $\hat{b} = \sum_{n=1}^{\infty}\sqrt{n}|n-1;\mu,\nu\rangle\langle n;\mu,\nu|$ |

# Properties of $|\beta;\mu,\nu\rangle$

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- Define $\hat{b} \equiv \mu\hat{a} + \nu\hat{a}^\dagger,\ \mu,\nu \in \mathbb{C},\ |\mu|^2 - |\nu|^2 = 1$
  - Verify that $[\hat{b},\hat{b}^\dagger] = 1$ still holds
  - $\hat{b}|\beta;\mu,\nu\rangle = \beta|\beta;\mu,\nu\rangle,\ \mu,\nu \in \mathbb{C},\ |\mu|^2 - |\nu|^2 = 1$
  - $\hat{N}_b|n;\mu,\nu\rangle = n|n;\mu,\nu\rangle,\ \mu,\nu \in \mathbb{C},\ |\mu|^2 - |\nu|^2 = 1$ CON basis states
  - Mean, $\langle\hat{a}\rangle = \langle\beta;\mu,\nu|\hat{a}|\beta;\mu,\nu\rangle = \mu^*\beta - \nu\beta^*$
- Prove that:
  - Mean photon number of the state $|\beta;\mu,\nu\rangle$ is given by:

    $$\langle\hat{N}\rangle = \langle\hat{a}^\dagger\hat{a}\rangle = |\langle\hat{a}\rangle|^2 + |\nu|^2$$

    **Problem 29**

    - Hint: $\hat{a} = \mu^*\hat{b} - \nu\hat{b}^\dagger$
    - Even for $\langle\hat{a}\rangle = 0,\ \langle\hat{N}\rangle = |\nu|^2$
  - Second moment, $\langle\hat{a}^2\rangle = \langle\hat{a}^{\dagger 2}\rangle^* = \mu^{*2}\beta^2 + \nu^2\beta^{*2} - 2\mu^*\nu|\beta|^2 - \mu^*\nu$

    **Problem 30**

# MUP states ("squeezed states")

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- Prove that the quadrature variances satisfy:

$$\langle \Delta \hat{a}_1^2 \rangle = \frac{|\mu - \nu|^2}{4}, \ \langle \Delta \hat{a}_2^2 \rangle = \frac{|\mu + \nu|^2}{4}$$

**Problem 31**

  - It is easy to see that $\langle \Delta \hat{a}_1^2 \rangle \langle \Delta \hat{a}_2^2 \rangle = 1/16$ if $\mu^* \nu$ is real
- Intuitive feeling for quadrature "squeezed" states
  - Let us look at the case of $\mu, \nu \in \mathbb{R}$
  - $\Delta \hat{a}_1 = (\mu - \nu) \Delta \hat{b}_1, \ \Delta \hat{a}_2 = (\mu + \nu) \Delta \hat{b}_2$
  - $|\beta; \mu, \nu\rangle$ is a coherent state of $\hat{b}$. Hence, $\langle \Delta \hat{b}_1^2 \rangle = \langle \Delta \hat{b}_2^2 \rangle = 1/4$
  - It follows therefore, $\Delta \hat{a}_1^2 = (\mu - \nu)^2/4, \ \Delta \hat{a}_2^2 = (\mu + \nu)^2/4$
  - If $\mu\nu > 0$, we are attenuating the noise in $\hat{a}_1$ quadrature and amplifying the noise in $\hat{a}_2$ quadrature
  - Squeezed vacuum state: $|0; \mu, \nu\rangle$

# Squeezed states

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- We will learn how to realize the Bogoliubov transformation using chi-2 non-linear optics, and thus how to generate quadrature squeezed from coherent state (laser) light
- Two notations for squeezed states:

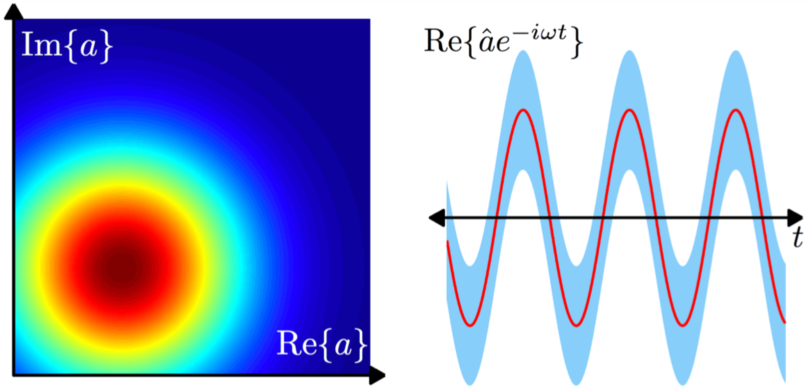
$$|\beta;\mu,\nu\rangle \quad \text{vs,.} \quad |\alpha;r,\theta\rangle$$

Jeff Shapiro and Horace Yuen — Carl Caves

$$\beta = \mu\alpha + \nu\alpha^*$$
$$\mu = \cosh r$$
$$\nu = e^{i\theta}\sinh r.$$

# Physical meaning of squeezed states

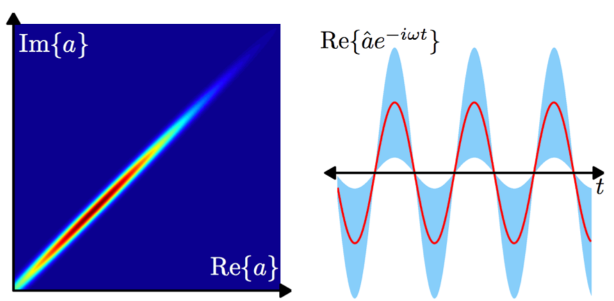
Figure courtesy, Dr. Baris Erkmen, MIT Ph.D. 2008



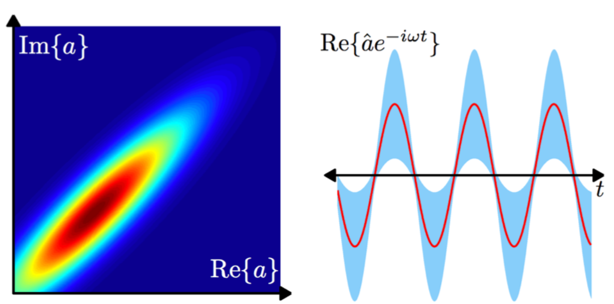

(a) Wigner distribution (b) Real quadrature

Coherent state

## Phase-squeezed state $\theta = \pi/2$

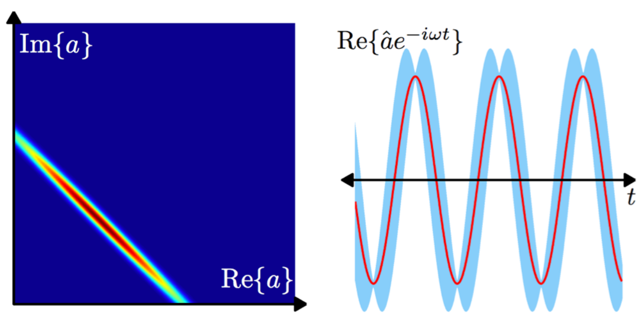



(a) Wigner distribution (b) Real quadrature

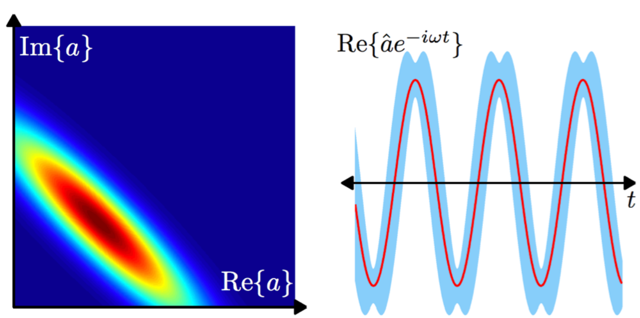



(c) Wigner distribution (d) Real quadrature

## Amplitude-squeezed state $\theta = 3\pi/2$




(a) Wigner distribution (b) Real quadrature




(c) Wigner distribution (d) Real quadrature

# Upcoming topics



- Some more practice and familiarity with quadrature squeezed states
- Phase space picture of quantum optical states
  - Characteristic functions, and Wigner functions