FABRY-PEROT VIEWED MANY WAYS

by

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OUTLINE

Introduction

General

Complex infinite sum
Self consistent field
Cavity Q
Photon lifetime
How FP increases coherence length
Figure perfection and alignment tolerances
FP performs Fourier Transform
Birefringent mirror coatings

Some Applications

FP laser oscillator
Directed energy (diffraction output coupling)
Spectroscopy
Ringdown measurement of pollutants
Pulse shaping (laser induced fusion)
LIGO Michelson FP Interferometer
Laser heterodyne interferometer (dimensional stability measurement)
Rotation rate sensing
Frequency standards

INTRODUCTION

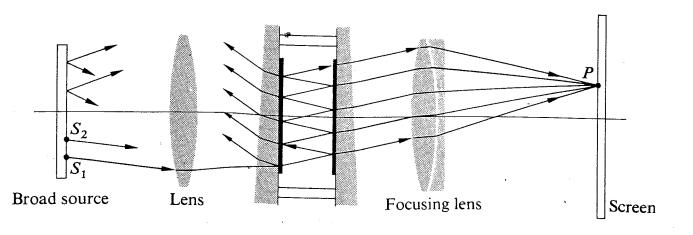
The Fabry-Perot resonator (FPR) is a many-sided and versatile device whose usefulness far exceeds its function as a spectroscopic tool or as a resonator for a laser oscillator. It plays a larger part in our daily lives than most of us realize, being as close to us as our compact disc player (semiconductor laser), optical communication systems for telephone and world-wide web (WDM filters and tunable lasers), and airline navigation systems (ring laser gyros).

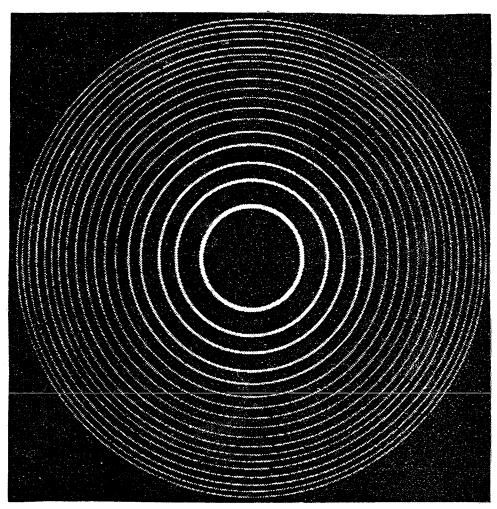
The parallel plate multiple beam interferometer was first constructed by Charles Fabry and Alfred Perot in the late 19th century. If the plate separation can be varied by moving one of the end mirrors it is referred to as an *interferometer*. When the mirrors are fixed, it is said to be an *etalon*, *cavity*, or *resonator*.

It seems worthwhile to understand this remarkable device in as many different ways as possible. In the course of many years of teaching and working with the FPR I have collected a number of ways of looking at it. Some are as simple as back-of-the-envelope estimates. Others are snatches of wisdom gleaned at coffee breaks. Still others are an appreciation of the historically unanticipated role of diffraction in making many curved FPRs resonant. None of these is original with me. And no claim is made for completeness of the collection.

During the race to make the first laser (late 1950's) there was a lot of concern about making FP mirrors flat enough as well as keeping them well enough aligned for a low gain laser. Don Herriott at Bell Labs went to heroic lengths to fabricated mirrors flat to $\lambda/200$ and Gordon Gould at TRG demonstrated automatic alignment using corner reflectors and crossed roof prisms. In fact, the alignment problem proved so difficult that the Bells Labs team used a "random hunter" to search for perfect alignment of the HeNe 1.15 μm laser. The story goes that this failed, but success was achieved due to vibrations from a hammer blow instead. Gould's pre-aligned corner reflector seemed like a good idea, provided it could be fabricated sufficiently accurately and provided polarization wasn't a problem. Gould's group demonstrated resonance with a corner prism as well as a pair of crossed roof prisms. But in both cases, fabrication remained a practical obstacle. It is now a matter of history that none of the above concerns prevented Ted Maiman's ruby laser from oscillating. The gain was so high that alignment losses didn't matter.

What is lost as the years go by is appreciation of the pleasant surprise when it was found that a wide range of *curved mirrors* could make resonant cavities. Thanks to *diffraction*, the ray picture is largely inadequate and most of the worries about mirror fabrication and alignment disappeared.

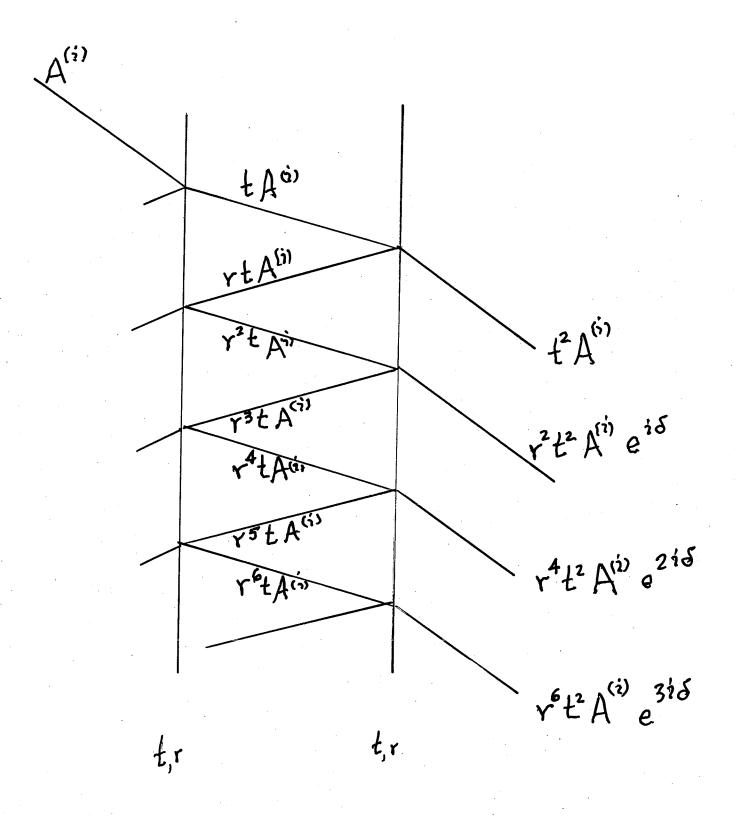




We will define finesse = $\frac{\text{fringe separation}}{\text{fringe width}}$ and show that finesse = $\frac{\pi\sqrt{R}}{1-R}$. For the above fringe pattern estimated finesse = 20 means $1-R \approx \frac{\pi}{20}$ or R = 85%

COMPLEX INFINITE SUM

(FLATS)



$$A^{(t)}_{(p)} = TA^{(i)} \left(1 + Re^{i\delta} + R^2 e^{2i\delta} + \dots + R^{P-1} e^{i(p-i)\delta} \right)$$
R<1

$$T^{(t)} = |A^{(t)}|^2 = |A^{(i)}|^2 \frac{T^2}{(1-Re^{i\delta})(1-Re^{-i\delta})}$$

$$\frac{\text{after many}}{\text{Steps}} \left(\frac{T}{1-R}\right)^2 \frac{1}{1+\frac{4R}{(1-R)^2} \sin^2 \frac{\sigma}{2}}$$

$$+ \frac{1}{2\pi L} = \frac{C(1-R)}{\sqrt{R}}$$

$$C/2L$$

Define |
$$\frac{C/2L}{AV_{FWHI}} = \frac{C/2L}{C/2L \pi R} = \frac{TVR}{1-R}$$

Define resolving power

$$\left(\frac{1}{\text{FWHI}} = \frac{1}{\sqrt{\lambda}} = \frac{1}{\sqrt{\lambda}$$

Since at normal incidence the interference order

m=2L because m halfwaves
span the zavity

Comparing this with Adiffraction = m Negual beams
Suggests that / ~ effective # of equal beams

NOTE

resolving power =
$$\left(\frac{2L}{\lambda}\right)\left(\frac{\pi\sqrt{R}}{1-R}\right)$$

= function * function of R

Thus resolving power
$$\rightarrow \infty$$
 if $L \rightarrow \infty$

Provided there's no problem with

alignment error surface figure diffraction loss

SELF CONSISTENT FIELD

(1)
$$E(L) = E(0) e$$

(2) $E(0) = E(0) r^2 e^{i k 2 L} + E_{in} t$
or $E(0) (1 - Re^{i k 2 L}) = E_{in} t$

To obtain Eour in terms of Ein combine (1) and (3)

same as before (complex infinite sum)

CAVITY Q

A term originally used only by engineers

power crossing surface (oney) = IA wats

Energy in volume Act = (IAt) joules

Thus

$$Q = \frac{(2I/c) \vee \omega}{IA(1-R)^2}$$

$$= \frac{\omega L}{c(1-R)}$$

Or DW. FWHI C(1-R)

PHOTON LIFETIME

Tph

back-of-the-emelope estimate of F-P resolution &

Define photon lifetime as time for I -> I./e



calculate how many hits n

$$R^{n} = \frac{1}{6}$$

$$\log R = -\frac{1}{n} = \log (R-1+1) \stackrel{\text{Liff}}{=} R-1$$

$$-\frac{1}{n} \approx R-1$$

$$N \stackrel{\text{Liff}}{=} \frac{1}{1-R}$$

distance traveled in decaying to Io/e is

nL = (f-R)L = cTph because AWAT ?!

or Tph = cli-R) = AW. or AV. = S(1-R)

FIGURE PERFECTION and ALIGNMENT TOLERANCES

HOW FLAT MUST PFP PLATES BE FOR GOOD PERFORMANCE?

 $n \lambda/2 = L$ Since $\Delta L/L = \Delta v/v$ + LAV shift c/2L If plate separation increases ΔL resonances shift $\Delta v_{\text{shift}} = v \Delta L/L$ Setting $\Delta v_{\text{shift}} = \Delta v_{\text{c}} = \Delta v_{\text{permissible}} = v \Delta L/L$

E.g. for R = 95%, finesse = π / (1-R) so that flatness ~ λ /120

Or $\Delta L_{permissible} = L \Delta v_c/v = \lambda / twice finesse$

F-P PERFORMS FOURIER TRANSFORM

We can show the Fourier Transform action of a Fabry-Perot resonator by examining in the time domain the *relaxation of the filled resonator*.

Previously we considered FP relaxation by detecting an infinite sum of gradually shortening field vectors, each phased identically with respect to the previous one. This was a *space domain analysis* that treated monochromatic light and spatial boundary conditions.

We now consider a high-Q F-P relaxing as an infinite sum of *periodically sampled* fields inside the resonator, which may fluctuate with time, each sample decaying with the same time constant.

$$E(z,t) = \begin{bmatrix} E(z,t) & e \\ \end{bmatrix} = \begin{bmatrix} E(z,t) & e \\ \end{bmatrix}$$
Complex and includes fluctuations
In amplitude and phase

As time evolves, detector sees the sum of periodically sampled fields of the form

$$\begin{bmatrix}
\mathcal{E}(z,t) & e & + & e. & e. \\
+ & \mathcal{E}(z,t-T)e & e & + & e. & e.
\end{bmatrix}$$

$$\begin{bmatrix}
\mathcal{E}(z,t-T)e & e & + & e. & e.
\end{bmatrix}$$

$$\begin{bmatrix}
-i\omega(t-z\tau) & 4i\kappa L & + & e. & e.
\end{bmatrix}$$

$$\begin{bmatrix}
+ & \mathcal{E}(z,t-2\tau)e & e & + & e.
\end{bmatrix}$$

where T = round trip time

The detector reports

The detector reports

$$|\mathcal{E}(z,t)|^2 + \text{other dc (self) terms}|$$

$$+\mathcal{E}(z,t)\mathcal{E}^*(z,t-T)e^{-i\omega T} - 2i\kappa L$$

$$+\mathcal{E}(z,t)\mathcal{E}^*(z,t-T)e^{-i\omega 2T} + \text{other terms}$$

$$+\mathcal{E}(z,t)\mathcal{E}^*(z,t-2T)e^{-i\omega 2T} - 4i\kappa L$$

$$+\mathcal{E}(z,t)\mathcal{E}^*(z,t-3T)e^{-i\omega 3T} - GikL$$

$$|E|^2 \sim \sum_{n=0}^{\infty} \mathcal{E}_{(z,t)} \mathcal{E}_{(z,t+nT)} e^{i\omega nT} = \frac{t}{t_{ph}}$$
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|E| ~ Sat' \(\xi(2,t) \xi(2,t+t') \) e e \\
\[\text{Various periods later} \]

 $|E|^2 \sim \int_0^\infty dt' \langle \mathcal{E}_{(z,t)} \mathcal{E}_{(z,t+t')} \rangle e^{i\omega t'} e^{-t/\tau_{pl}}$ correlation

function ~ FT of [correlation function x e - t/Tpl] ~ FT of correlation function * FT of e - FT of correlation Function X (instrument function)

Wiener-Khinchine theorem states that,

provided field is stationary,

TT of correlation function is spectrum

BIREFRINGENT MIRROR COATINGS

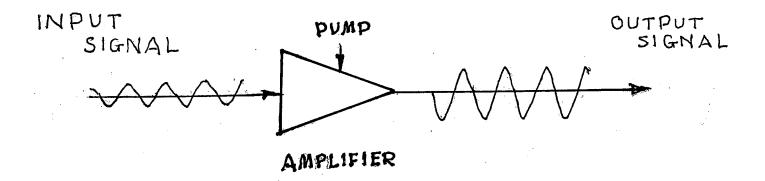
In the course of our work on thermal expansivity we once observed a splitting of what should have been a single laser resonance line. This was found to be due to unwanted birefringence in the dielectric mirror coatings.

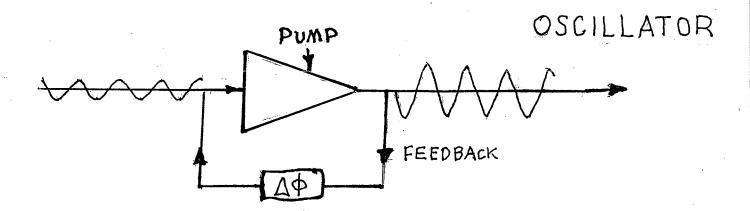
When we frequency tune a laser beam through a Fabry-Perot resonator we expect to see light transmitted whenever the laser frequency coincides with a cavity resonance frequency. We were surprised to find that with some of our mirrors the resonances were split into two components, and we found that by rotating the linear polarization of the incident light either resonance could be individually excited. The magnitude of the splitting was $\sim 20\, MHz$; the Fabry-Perot length was $10\, cm$. $(c/2L=1500\, MHz)$. Since the round trip phases of adjacent F-P modes are spaced by 2π radians we estimate the difference in round trip phase due to mirror birefringence was 20/1500 of 2π , or $0.083\, radians$. (No way to tell whether this birefringence was due to a single mirror or divided between the two mirrors).

S.C. Johnston and S.F. Jacobs, "Some Problems Caused by Birefringence in Dielectric Mirrors," *Appl. Optics*, **25**, 1878, (1986).

FABRY-PEROT LASER OSCILLATOR

The word laser is an acronym standing for Light Amplification by Stimulated Emission of Radiation. The device we are familiar with today is really a laser *oscillator*. To make an (electrical) amplifier into an (electrical) oscillator one needs feedback whereby a fraction of the

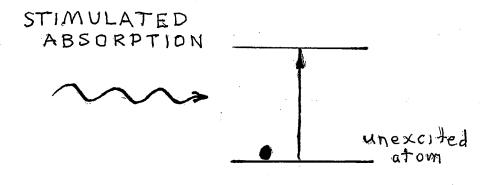


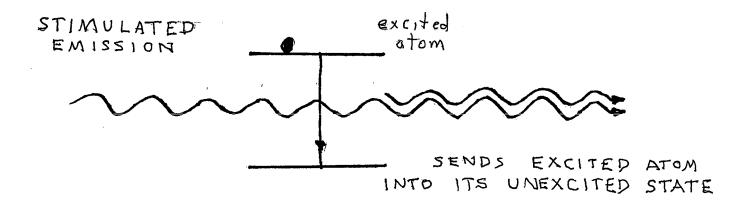


output signal is fed back to the input with a phase identical to the input signal. If the amount of feedback is large enough, then the input signal can be removed and the output signal will continue. This is an oscillator.

What keeps it going is the power pumped into the amplifier. What starts the oscillator is (thermal) noise (all phases), some of which finds itself fed back in the same phase as the input signal.

A *light* amplifier works by stimulated emission, which is the reverse of (stimulated) absorption.





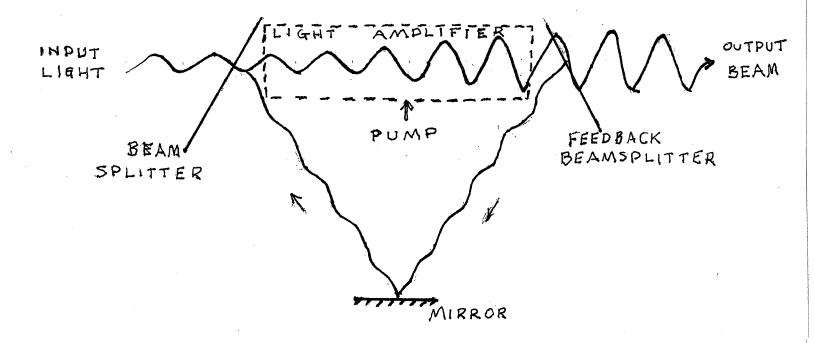
The stimulated emission is in the same phase and direction as the stimulating light. If there are more atoms in their excited state than in their unexcited state (population inversion) then we have a light amplifier.

There is an additional process going on; excited atoms may emit *spontaneously* (all phases and all directions) without needing light to make it happen. This is called spontaneous emission. Spontaneous emission is what starts the oscillator. It also reduces the gain-producing population inversion.

SPONTANEOUS

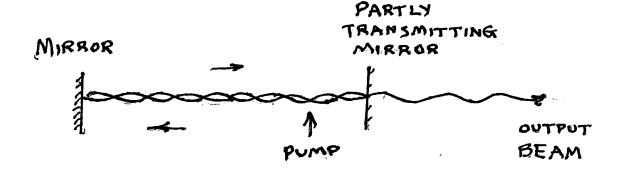
EXCITED ATOM
DE-EXCITES SPONTANEOUSLY

To see how the amplifier/oscillator principle may be used with light, consider the feedback resonator shown below:



RING RESONATOR WITH PATHLENGTH ADJUSTED TO FEED BACK LIGHT IN PHASE WITH INPUT LIGHT

Part of the invention of the laser was seeing that this could be accomplished by the Fabry-Perot resonator:



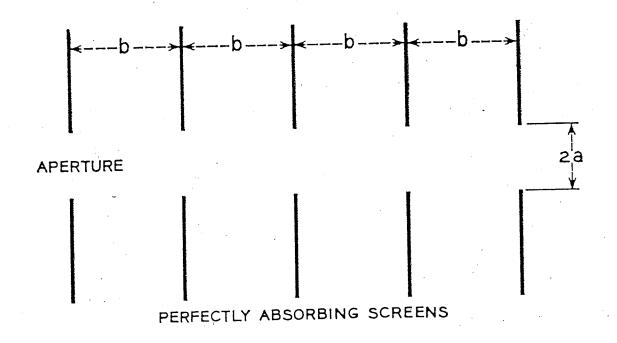
It is not easy to find a system that can be pumped to provide a population inversion. Especially on a continuous (not just pulsed) basis. The first continuous laser (HeNe) had extremely low gain, so that losses were a worry because unless the gain per pass exceeds the loss per pass, the oscillation simply dies out. These losses include absorption, scattering, and diffraction.

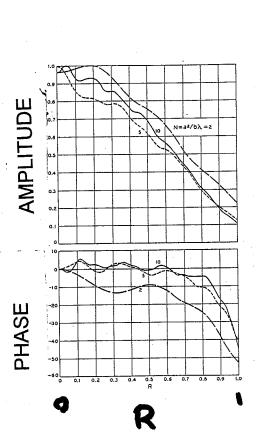
A lot of effort went into designing and producing very flat mirrors as discussed earlier. Also a lot of effort went into understanding diffraction losses and mirror alignment tolerances. Figure I shows Fox & Li's computer analysis of diffraction losses for two flat mirrors.

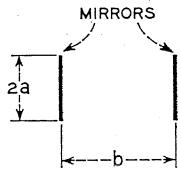
There was an effort at making self-aligned resonators (see Figure II and Figure III). These could work, but were too difficult to make.

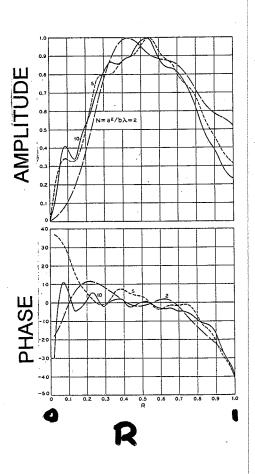
FOX & LI 1961

Figure |



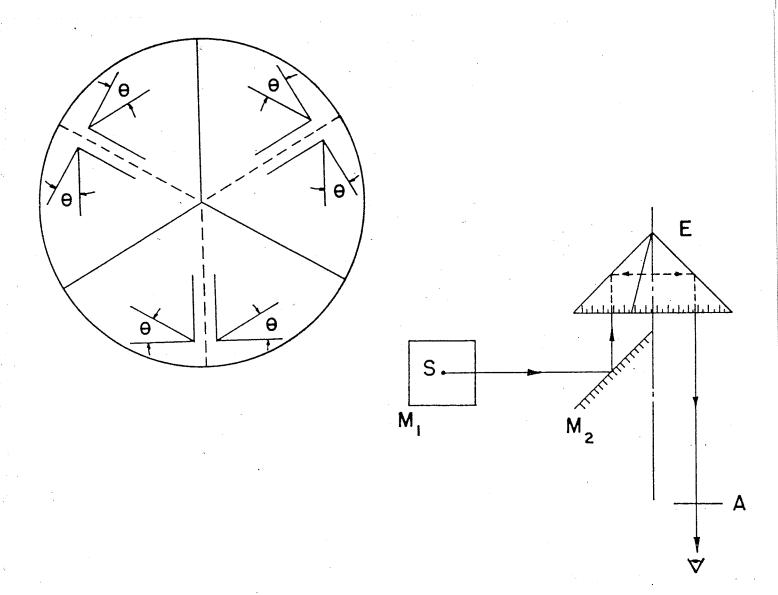


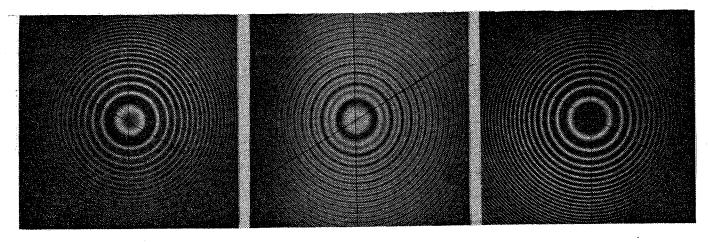




CUBE CORNER FP

Figure ||

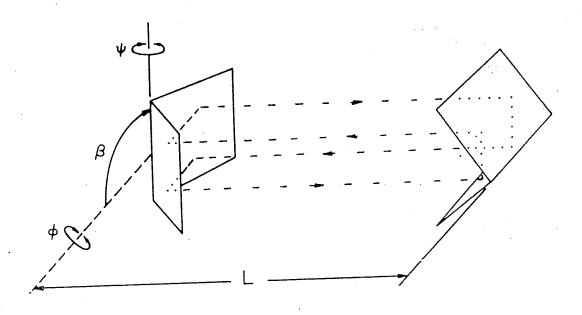


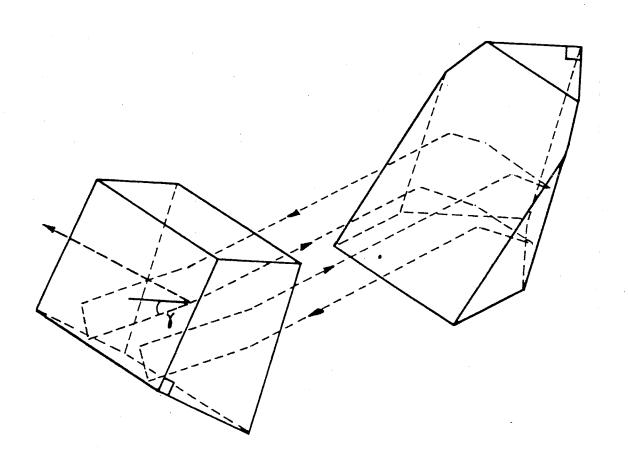


Unpolarized

CROSSED ROOF FP

Figure III



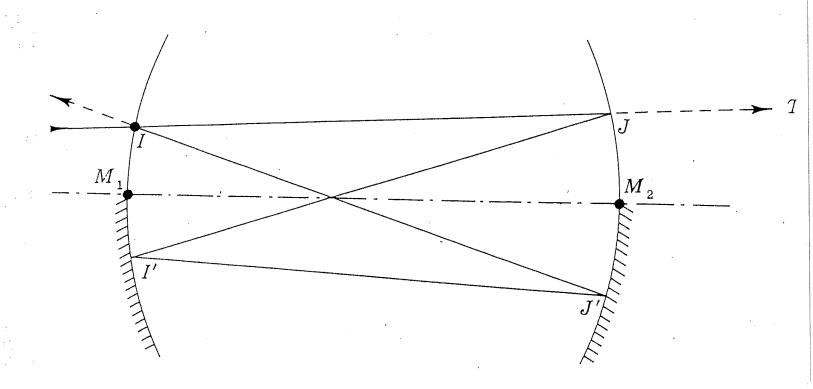


In 1956, Pierre Connes introduced a novel Fabry-Perot configuration that was easy to make and easy to align. Figure IV shows the ray optics arrangement for his Spherical Mirror FPI. It is effectively a folded FP cavity whose resonance condition is $m\lambda = 4R$ instead of $m\lambda = 2L$. Mirror adjustment consists only in making the distance $M_1M_2 = R \pm \sim 10 \,\mu m$!

Little to worry about angular orientation of the mirrors. For extended sources etendue, or throughput $A\Omega$, is limited only by spherical aberration so that throughput is vastly superior to that of (single mode) plane FP interferometers. The reason is that extended sources (for astronomy applications) mean higher order modes, half of which are simultaneously resonant in a (degenerate) confocal resonator.

Unfortunately, it was not clear how to use this for a laser. Accurately setting the mirror separation would be very inconvenient. Where to locate the amplifying medium?

SPHERICAL MIRROR FPI - Figure IV



Then came a pleasant surprise for laser scientists. Boyd & Gordon discovered that Gaussian Spherical Waves (GSW) are (analytic) solutions to the diffraction problem for curved mirror pairs. E.g. GSW wavefronts propagate like this:

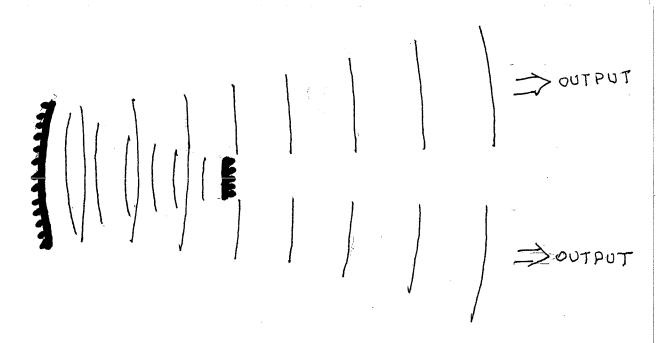
The drawbacks of Connes' SFP went away when diffraction is considered, mirror separation is no longer critical, and location of the gain medium is clear. Losses are low compared to flat mirror pairs, and alignment sensitivity is much relaxed.

Laser research accelerated dramatically.

DIFFRACTION OUTPUT COUPLING

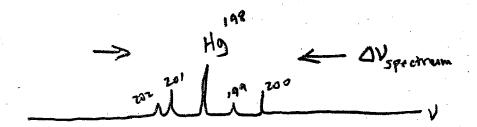
Not all diffraction losses are bad. They are useful in obtaining high laser output power in a nearly Gaussian shaped beam from a laser medium that has a relatively high gain and a large amplifier width-to-length ratio.

An example is shown below:

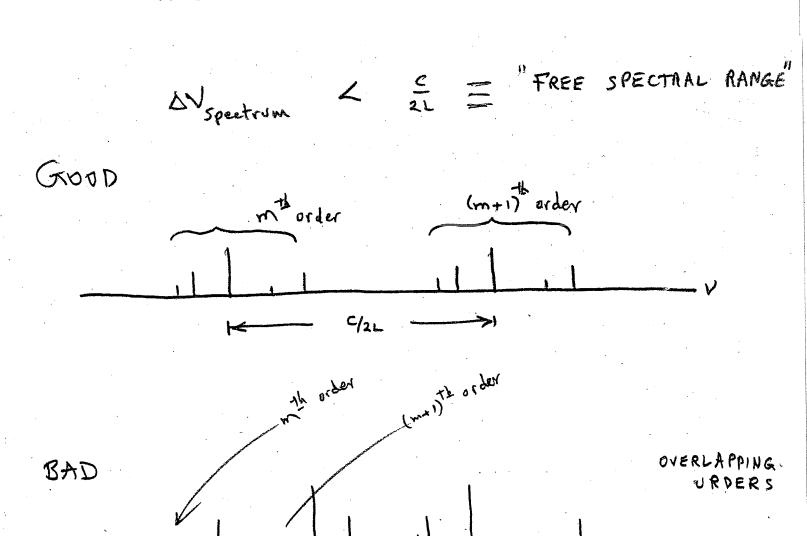


(Hg) SPECTROSCOPY

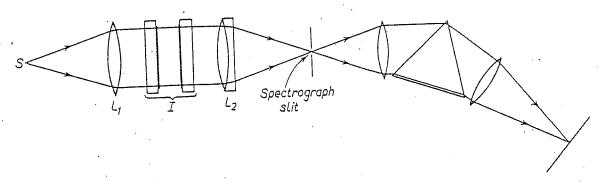
Suppose you want to look at the light from several different isotopes of mercury (Hg) hyperfine spectrum



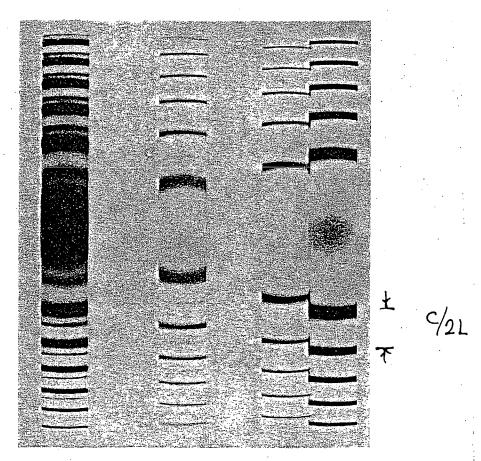
Since the FP resonance condition, $m \lambda/2 = L$, or $v_m = m c/2L$, the spectrum will be repeated for each order. In order to avoid overlapping orders one generally shortens length L such that:



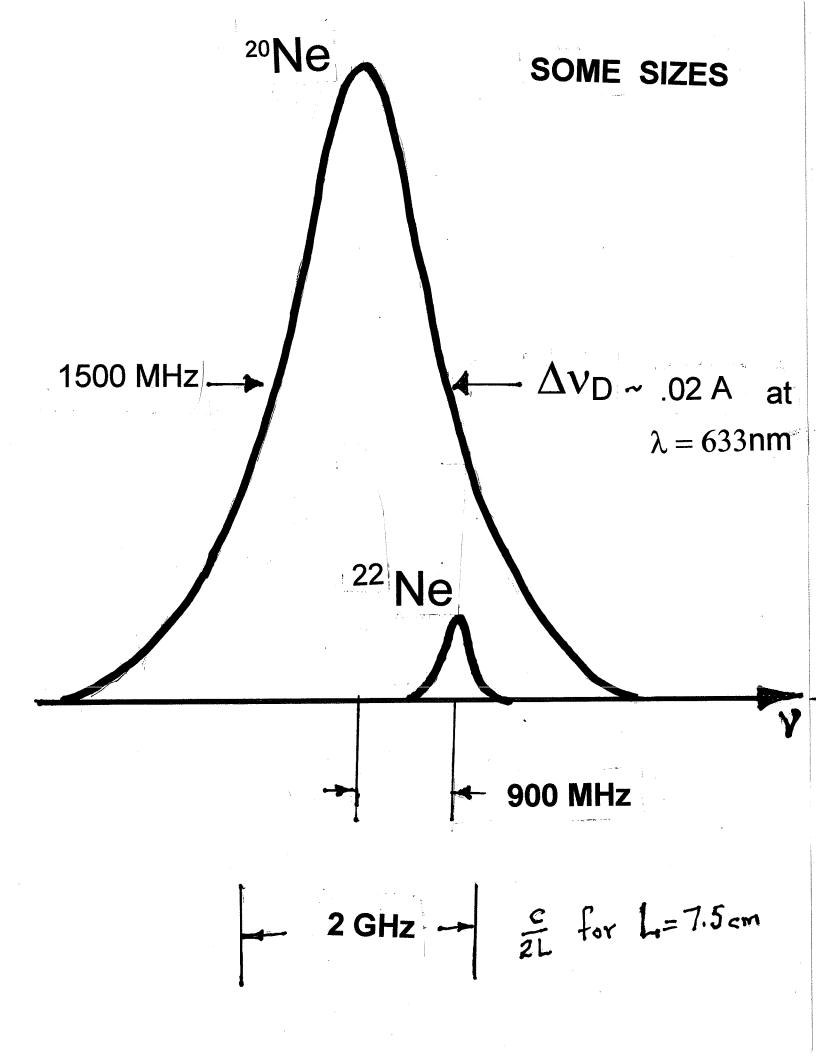
If it is desired to look at fine spectral details of *many* spectral lines, one can separate the lines by crossing FP with another dispersing device as shown below.



FABRY-PEROT interferometer crossed with a prism spectrograph

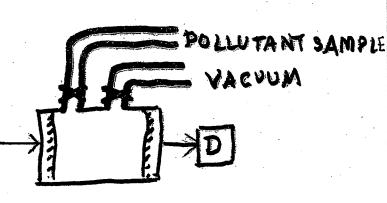


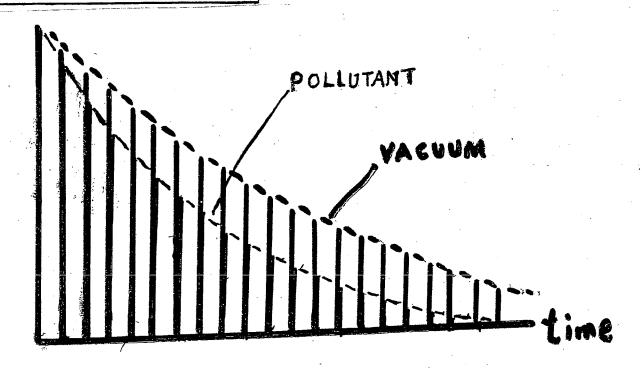
FABRY-PEROT fringes from lines in the emission spectrum of helium



CAVITY RINGDOWN TIME TOMEASURE AIR POLLUTANTS

Tunable pulsed laser Locked to pollutant Absorption line

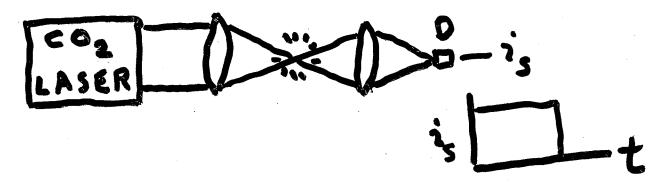




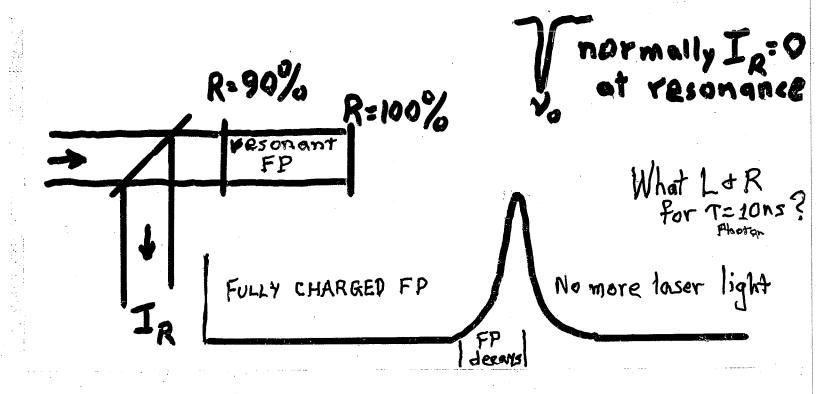
PULSE SHAPING

Needed particular pulse duration for laser induced fusion (10.6 mm)

Noted that CW CO, laser selfextinguishes fast due to air breaking



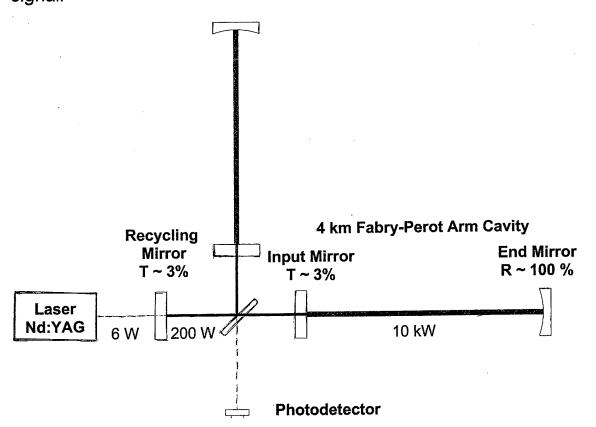
Used beam reflected from resonant FP



LIGO MICHELSON FP INTERFEROMETER FOR DETECTION OF GRAVITY WAVES

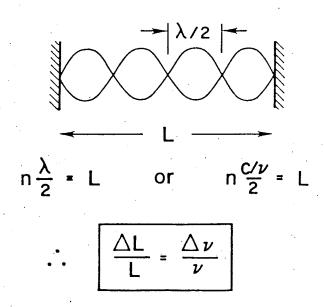
The laser light bounces back and forth many times in the resonator. By reading the "dark port" - which is the one where the "crest" of the light phase from one arm cancels the "trough" of the light phase from the other - the measurements are made to keep that at *maximum darkness*.

Fabry-Perot cavities store the beams and increase effective pathlength of the Michelson interferometer. After equivalent of 75 trips down the 4 km length to the far mirrors and back again, the two separate beams leave the arms and recombine at the beam splitter. The beams returning are kept out of phase so that when the arms are both in resonance their light waves subtract and no light reaches the photodetector. When a gravitational wave passes through the interferometer, the distances along the arms are shortened and lengthened, so some light arrives at the photodetector, indicating a signal.



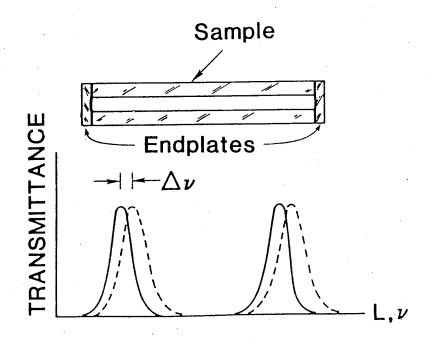
LASER HETERODYNE FP MEASURES CHANGES IN L

FABRY-PEROT RESONATOR

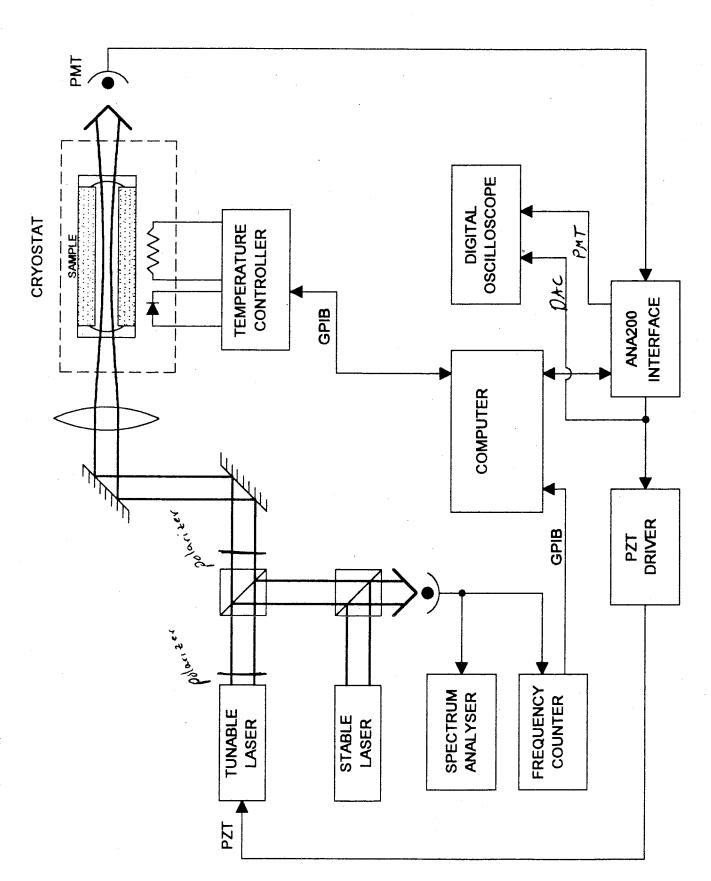


Master equation for the laser interferometric method

ULTIMATE LIMIT IS LASER STABILITY

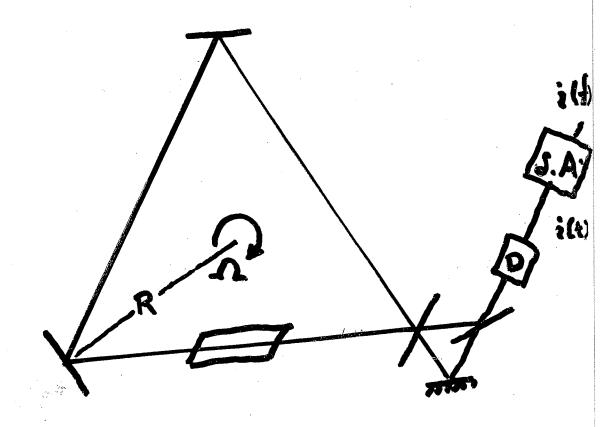


Configuration and transmittance of Fabry-Perot sample/resonator



Arrangement for thermal expansivity measurements

ROTATION RATE SENSING



Call the ring perineter P

$$\Delta V_{best} = \frac{4\Delta\Omega}{\lambda P}$$

Dearth = 8×10-5 rads/sec

$$\Omega_{\text{state}}$$
 of the art = 10⁻⁹ Ω_{cont}

FREQUENCY STANDARDS & ATOMIC CLOCKS

A good way to make a frequency standard is to lock a stable tunable laser to a narrow linewidth atomic transition that is quite insensitive to environmental influences, such as temperature, pressure, electric, and magnetic fields, etc. To make a clock, all you need is a stable oscillator of a known frequency and a way to count that frequency.

The Fabry-Perot resonator is useful here in two respects: In order to demonstrate the tunable (Fabry-Perot) laser's stability one generally compares against a temperature-controlled passive Fabry-Perot cavity.

The Fabry-Perot provides the short term reference; the atom provides the long term reference. This is only necessary because the spectroscopy doesn't have enough signal-to-noise in short times to provide a feedback signal to stabilize the laser's quick fluctuations.

As of 2007, a favorite atomic transition is atomic strontium cooled to $2\mu K. \left({}^1S_0 - {}^3P_0 \right)$. $\lambda = 698nm$, with natural linewidth $\sim 1Hz$. A frequency standard has been demonstrated * whole linewidth is $\sim 1Hz$.**. Future progress in this field requires pushing laser to even higher stability.

This calls for improving the thermal noise limitation of passive Fabry-Perot cavities that can be done in three ways:

- 1. lower temperature of passive F-P cavities;
- 2. longer cavities;
- 3. substrate and delectric coating materials with lower mechanical loss.
- * A.D. Ludlow, X. Huang, M. Notcutt, T. Zanon-Willette, S.M. Foreman, M.M. Boyd, S. Blatt, and J. Ye, "Compact, thermal-noise-limited optical cavity for diode laser stabilization at 1×10^{-15} ," *Optics Letters*, **32**, 641-643, (2007).
- ** Observed finesse = 250,000, implying R = 99.999%. $\tau_{photon} \sim 20 \mu \sec$, $L_{coh} \sim 6 km$