

Advanced Experiments With Diffraction Gratings

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The following treatment is a continuation of a separate discussion, "How a Diffraction Grating Works, (without equations)". In that discussion we relied entirely on pictures to convey a gut feeling for how a diffraction grating separates the different color components, or spectrum, of multicolored light, such as white light.

The present treatment outlines a number of experiments you can perform with a diffraction grating to learn what it can do. With a little mathematics and some crude angle estimates a great deal can be measured, predicted, and verified.

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How A Diffraction Grating Works (With Equations)

In the introductory discussion, "How a Diffraction Grating Works (without equations)", we saw that only in certain directions do the light waves from every grating slit add constructively. E.g. peaks lying on peaks and valleys on valleys. In order to determine the directions where light waves add constructively, we have to introduce some symbols and use some trigonometry. Referring to Fig. 1, let Θ_1 represent the direction of a diffracted beam, λ the wavelength of the light, and d the slit separation (sometimes called the grating constant).

These are related by the expression

$$d \sin \Theta_1 = 1\lambda \quad (\text{first order}) \quad (1)$$

Fig. 2 shows how it is possible to have more than one order of diffraction. E.g. there can be many directions where the light waves add constructively. Fig. 3 portrays the second-order situation. We see that

$$d \sin \Theta_2 = 2\lambda \quad (\text{second order}) \quad (2)$$

In general, for each color λ there are many constructively adding directions:

$$d \sin \Theta_n = n\lambda \quad (\text{nth order}) \quad (3)$$

or more simply

$$\boxed{\sin \Theta = n\lambda} \quad (\text{nth order}) \quad (4)$$

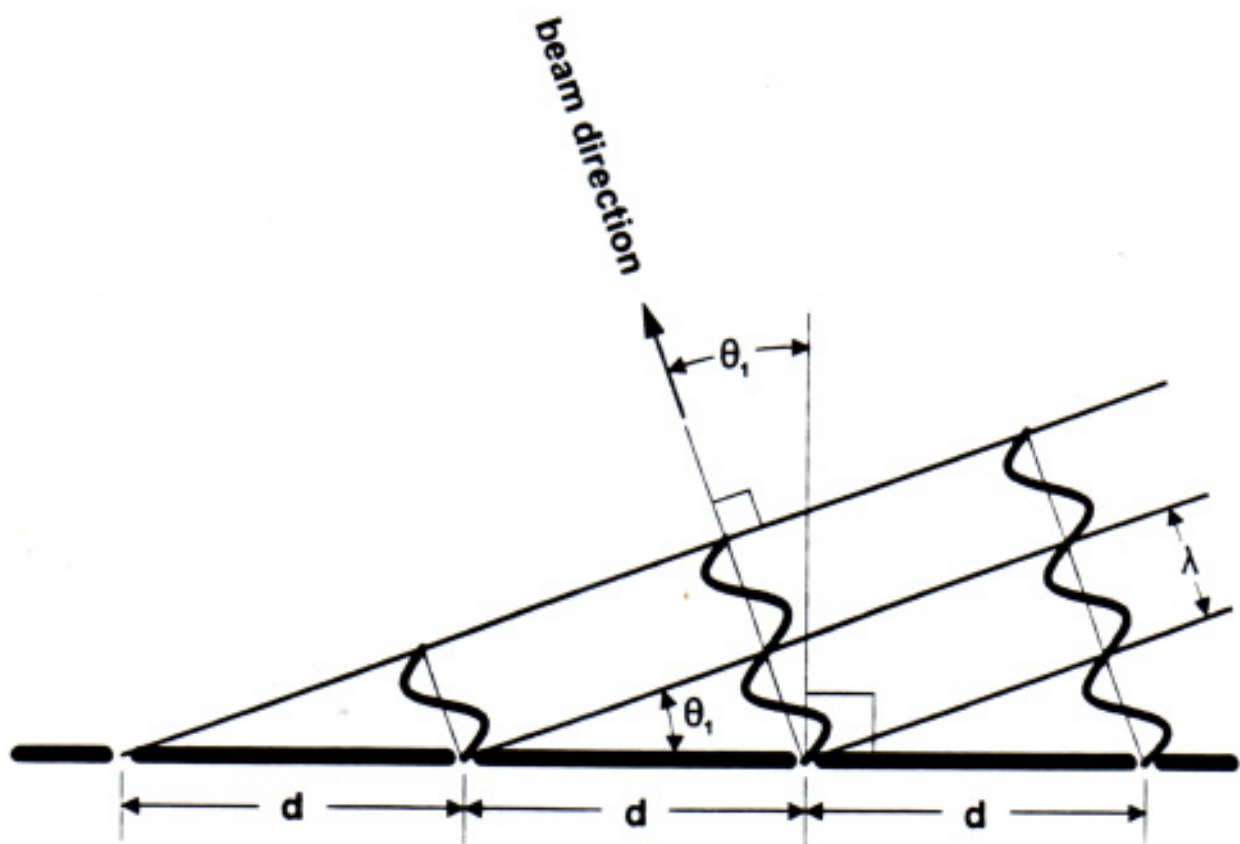


Fig. 1. Relationship between the direction of a diffracted beam Θ_1 , wavelength λ and slit spacing is $d \sin \Theta_1 = \lambda$.

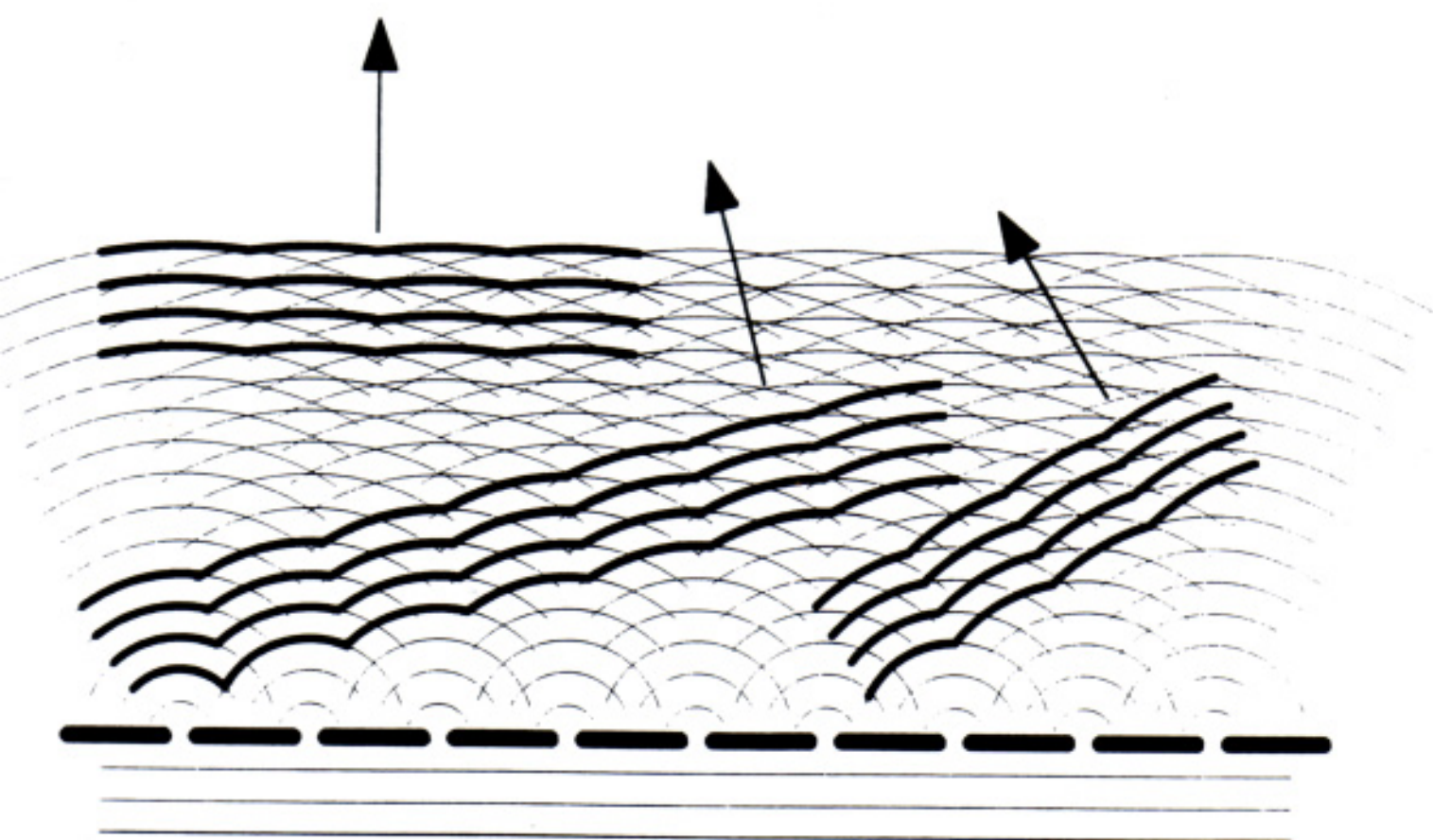


Fig. 2. Directions where cancellation is avoided. Left to right, the zero, first and second orders.

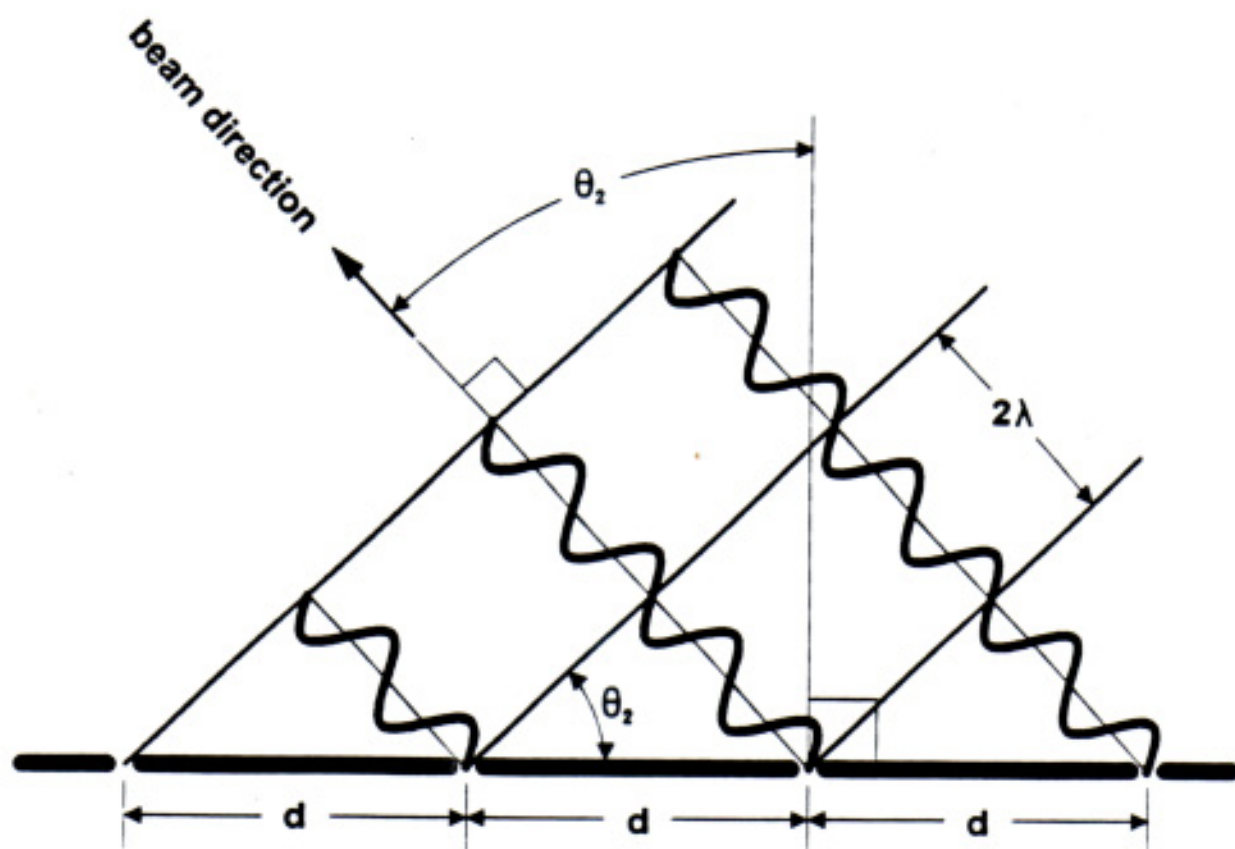


Fig. 3. For the second order, the relationship between the beam direction Θ_2 , wavelength λ , and slit spacing d is $d \sin \Theta_2 = 2\lambda$.

Measuring The Grating Constant D

It's surprisingly easy to estimate experimentally the spacing d between the grating slits. Select a distant light source such as a fluorescent lamp. Hold the grating up to your eye and look at the lamp through the grating. To estimate an angle, Θ , imagine how much distance to the side corresponds to how much distance away



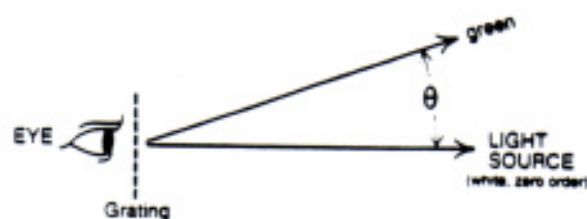
The angle Θ (in radians) $\approx \frac{\text{distance to the side}}{\text{distance away}}$

Example:

If distance to the side = 10 cm and distance away = 100 cm then $\Theta \approx \frac{10}{100} = .1$ radian.

In this way, estimate the angle between the zero order (white image) and the green part of the spectrum ($\lambda \approx 500$ nm) as shown in Fig. 4.

Example: Suppose $\Theta \approx 30^\circ$



$$\begin{aligned} \text{Then } d \sin 30^\circ &= 1\lambda \\ d (1/2) &= 500\text{nm} \\ d &= 1000\text{nm} \\ &= 1000 \times 10^{-9} \text{ meter} \\ &= 10^{-6} \text{ meters} \\ &= 10^{-3} \text{ millimeters} \\ &= .001 \text{ mm} \end{aligned}$$

Fig. 4.

Resolving Power

The resolving power of a diffraction grating refers to its ability to separate two colors that are very similar, but not quite the same. Suppose the colors are separated in wavelength by a small amount $\Delta\lambda$.

We can define a quantity,

$$\text{Resolving Power} = \frac{\lambda}{\Delta\lambda} \quad (5)$$

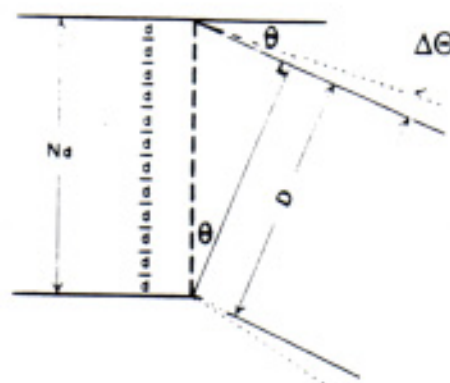
and can now use Eqn. (4) to

- predict what you ought to be able to resolve (theoretical resolving power)
- compare this with what you can actually resolve (experimental resolving power)

Theoretical Resolving Power

Equation (4) tempts us to believe that a diffraction grating can separate different colors exactly as predicted by trigonometry ($d \sin \Theta = n\lambda$). However, because of edge effects, in the real world there's no such thing as a truly parallel beam, e.g. the beam would have to be infinitely wide and thus too wide to pass through our grating. Since our beam must necessarily have a limited width, which we shall call D , the beam will actually spread. This means that, even for a single wavelength or color, we'll inevitably have a range of angles instead of just the angle Θ . This spreading $\Delta\Theta$ is shown in Fig. 5.

Fig 5.



Because of this angular spreading, the grating won't do a perfect job of separating the colors by their angles as predicted by the grating formula

$$d \sin \Theta = n\lambda. \quad (4)$$

To find out how much color spreading $\Delta\lambda$ results from a small angular spreading, $\Delta\Theta$, we take a differential of Eqn. (4).

$$\Delta\lambda = \frac{d\lambda}{d\Theta} \Delta\Theta \quad (5)$$

$$d \cos \Theta \Delta\Theta = n\Delta\lambda \quad (6)$$

Later, we'll show in the appendix that

$$\Delta\Theta = \Delta\Theta_{\text{diffraction}} \approx \frac{\lambda}{D} \quad (7)$$

but for the moment, let's accept it and see what Eqn. (7) predicts for $\Delta\lambda$ corresponding to the angular spread $\Delta\Theta_{\text{diffraction}}$:

Substituting Eqn. (7) for the angular spread into Eqn. (6) we get

$$d \cos \Theta \frac{(\lambda)}{(D)} \approx n \Delta \lambda \quad (8)$$

Finally, we see from Fig. 5 that the beam width D is related to other parameters as follows:

$$D \approx N d \cos \Theta \quad (9)$$

Therefore, substituting equation (9) into equation (8),

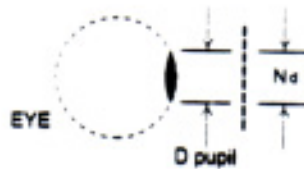
$$\lambda \approx n N \Delta \lambda \quad (10)$$

After rearranging terms,

$$nN \approx \frac{\lambda}{\Delta \lambda} = \text{Resolving Power} \quad (11)$$

In order to estimate the theoretical resolving power of our grating, we need to realize that N represents not the total number of rulings in the whole grating, but rather the number actually doing the job, e.g. the number right in front of our eye's pupil. (see Fig. 6).

Fig. 6.



For example, if our eye's pupil diameter D_{pupil} measures 5 mm, then

$$N = \frac{D_{pupil}}{d} = \frac{5\text{mm}}{.001\text{mm}} = 5000$$

Thus the theoretical resolving power for the first order, ($n=1$),

$$\frac{\lambda}{\Delta \lambda}_{\text{theoretical}} = N = 5000$$

This means that if our grating is perfect, for $\lambda = 500 \text{ nm}$ we should be able to resolve two spectral lines $\Delta \lambda = 0.1 \text{ nm}$ apart.

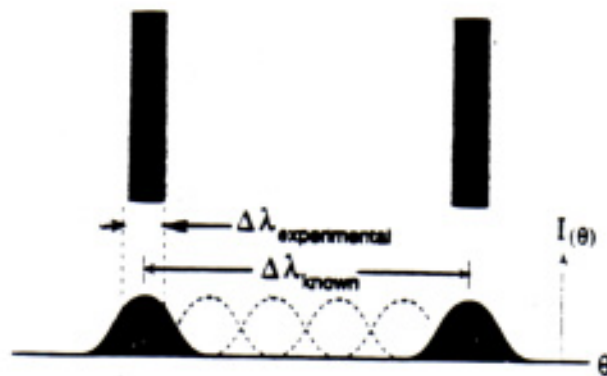
Experimental Resolving Power

Now, let's go out and see what we can actually resolve with our grating. To estimate $\lambda/\Delta\lambda$ experimentally, we need to find a pair of spectral lines of *known wavelength difference*, $\Delta\lambda_{\text{known}}$, and estimate how many more such lines could be squeezed in between. Fig. 7 shows an example.

Fig. 7.

Suppose a pair of lines appears like this

How many more
could be squeezed in?
Let's call it four.



Then, for this case,

$$\Delta\lambda_{\text{experimental}} = \frac{\Delta\lambda_{\text{known}}}{5}$$

A convenient pair of lines to use are the yellow mercury lines seen in a fluorescent lamp spectrum. Their wavelengths are 577 and 579 nm, so that $\Delta\lambda_{\text{known}} = 2$ nm. Then in our example,

$$\Delta\lambda_{\text{experimental}} = \frac{2 \text{ nm}}{5} = 0.4 \text{ nm}$$

Alternately, a different $\Delta\lambda_{\text{known}}$ may be found in the spectrum of a low pressure sodium lamp. The sodium "D line" is really a pair of lines with wavelengths 589.0 and 589.6 nm, or $\Delta\lambda_{\text{known}} = 0.6$ nm.

Another thing you can verify while you're trying to resolve line pairs is that, according to Eqn. (11) they should be n times better resolved in the n^{th} order. When you look at them try a low pressure sodium or mercury (fluorescent) lamp. Note that their relative sharpness (resolution) is really 2 times better in the second order than in the first order, and three times better in the third order than in the first, just as predicted by Eqn. (11).

Seeing Fraunhofer Lines In The Sun's Spectrum

Night Spectra Quest offers a tough challenge. One where success requires considerable experimental skill and patience: seeing the narrow black absorption lines in the sun's spectrum.

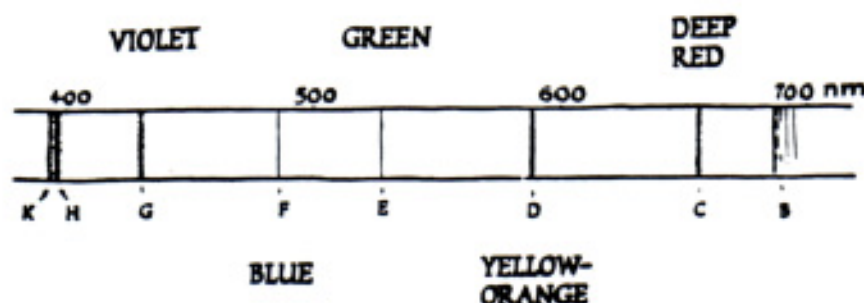
When we compared the sodium high pressure vs. low pressure spectra, (f vs. g), we learned that the reason the sodium yellow line is black in the high pressure case is self-absorption. Somewhere between the source and the viewer the light got absorbed. This was noted in the sun's spectrum by J. Fraunhofer in 1814. He explained the black lines as evidence that sodium, calcium, and hydrogen are present in the sun's atmosphere. Fig. 8 shows a simplified spectrum of the strongest Fraunhofer lines in the visible. Note also that some absorption is due not to molecules in the sun's atmosphere, but to molecules of oxygen in the earth's atmosphere.

There are two experimental problems in observing the Fraunhofer lines using **Night Spectra Quest**:

- Because you will be looking for very narrow black lines in bright daylight, you'll need a dark background, such as the shadow of your hand.
- Because the lines are so narrow, you'll want to be sure they're really there and not just your imagination. It will help if the source can appear slit-shaped, like a fluorescent lamp. Then you'll be confident the long, narrow black lines are real. (Also, you can check the colors which are indicated in the spectrum in Fig. 8).

The trick then is how to make a slit-shaped image, starting with a nearly parallel beam of sunlight. Fortunately, you can afford to waste a lot of light, because the sun is so bright. One solution is to use a cylindrical mirror, such as the shiny chrome trim on automobiles. If you go out into a parking lot you can find many to choose from. One winner is the curved chrome trim around some auto windshields. If you stand in the right place, you'll see a brilliant line of sunlight that works beautifully! Fig. 9 shows why. This is an example of "toroidal optics," so named because the mirror surface is part of a toroid, or donut shape.

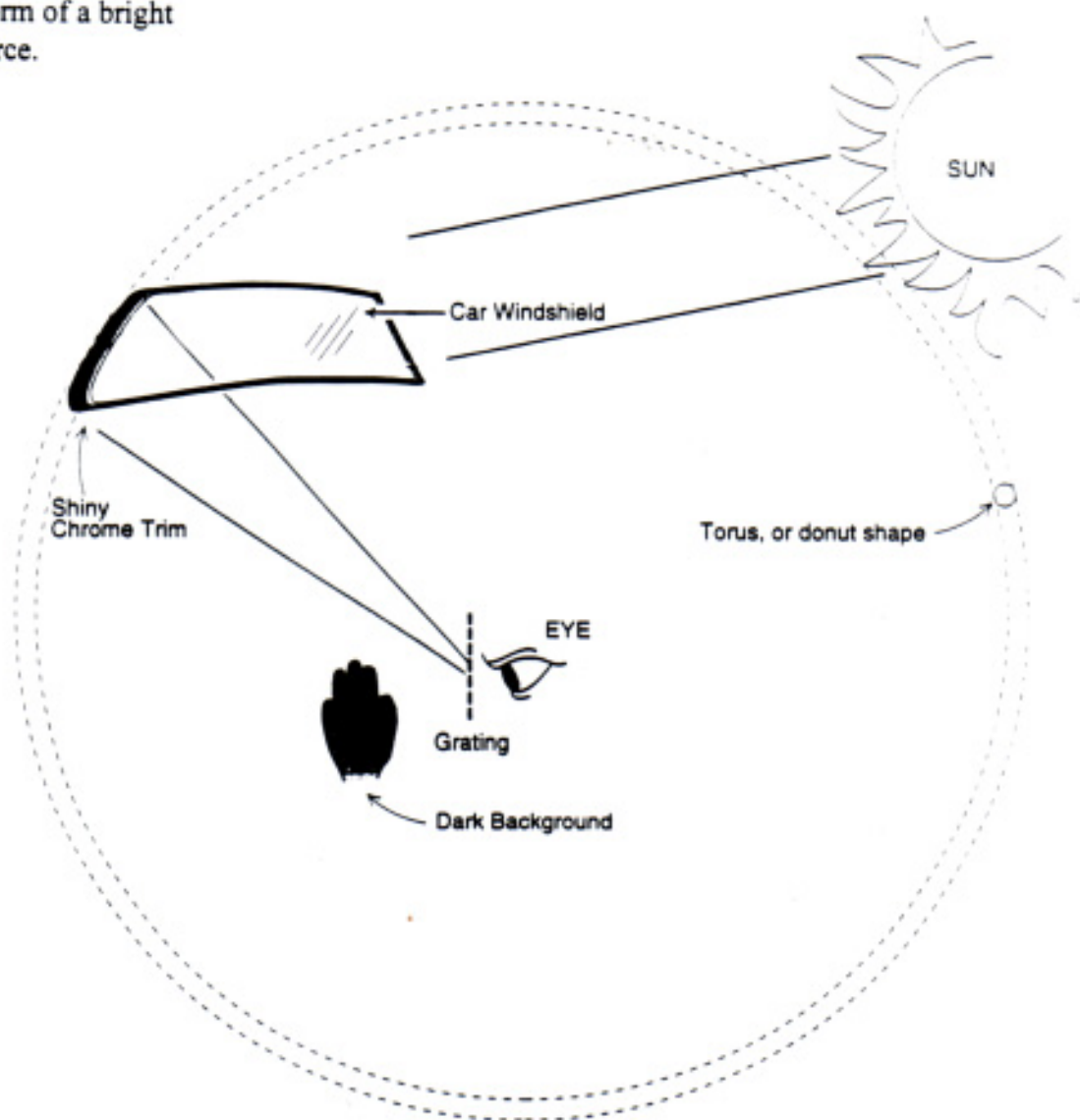
Fig. 8.



Fraunhofer lines of the solar spectrum. B = terrestrial oxygen, C = hydrogen, D = sodium, E = calcium, F = hydrogen, G, H, I = calcium

Fig. 9.

How to see sunlight
in the form of a bright
line source.



Caution

Never look directly at the sun with **Night Spectra Quest** or your eye.

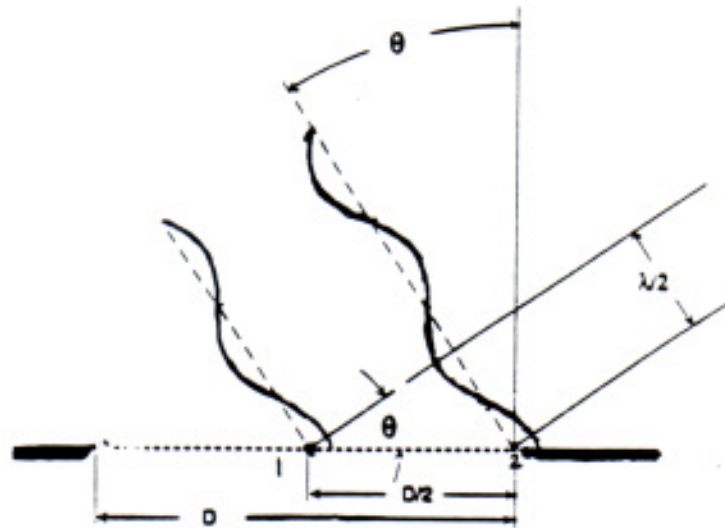
Appendix

How A Light Beam Of Limited Width Spreads By Diffraction

Fig. 10 shows how a water wave or light wave propagates past a periodic structure of equally spaced narrow slits. Now let's replace the periodic structure with a *single* slit of width D . With this wide slit the wavelet sources (all in phase) are now packed together as close as you please. (See Fig. 10).

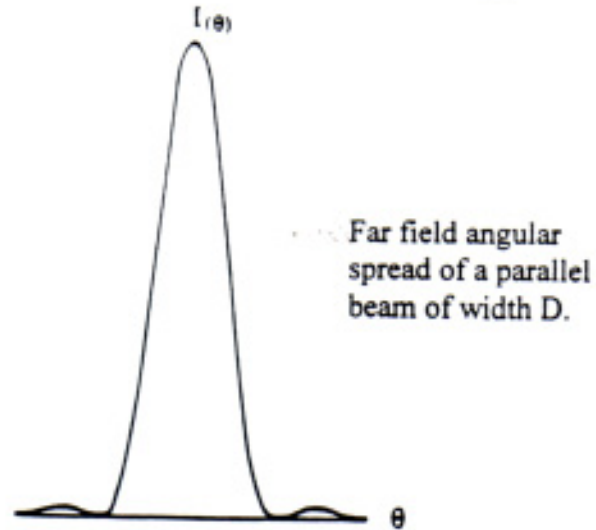
Fig. 10.

In the far field, all the light in the direction, θ interferes destructively, resulting in $I(\theta) \rightarrow 0$ (See Fig. 11).



It turns out that far away from the structure (far field) the angular light distribution $I(\theta)$ looks like Fig. 11.

Fig. 11.



We will not derive this shape, but will settle for understanding why $I(\theta) \rightarrow 0$ and for what angle θ . $I(\theta) \rightarrow 0$ when the angle θ is such that the peaks of half the wavelets lie on the valleys of the other half of the wavelets. Referring to Fig. 10, this happens if the peak of the wavelet originating from the exact center (point 1) lies on the valley of the wavelet originating from the right-hand edge (point 2). Then the contribution from every point along the right hand half of the slit has a corresponding point along the left hand half whose contribution cancels it. The net result is total cancellation.

From Fig. 10 we see that

$$\frac{D}{2} \sin \theta = \frac{\lambda}{2} \quad \text{is the condition for which } I(\theta) \text{ first goes to zero.}$$

For small angles,

$$\sin \theta \approx \theta$$

Therefore

$$\theta \approx \frac{\lambda}{D} \quad \text{is the approximate diffraction spreading of a beam of width } D.$$