Pierre Meystre

When atoms become waves

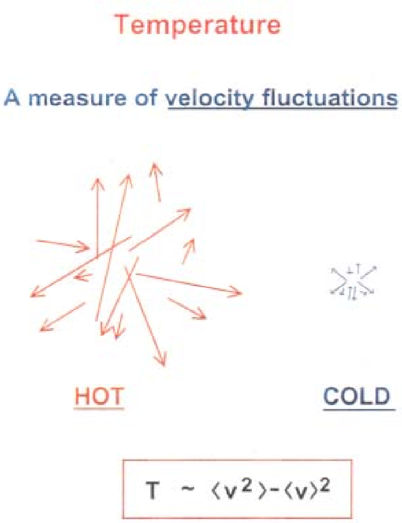

Figure 1: Schematic illustration of a gas at high and low temperatures.

Scientific developments have lead to the realization of ultracold temperatures, where our preconceived ideas and intuition about atoms need to be revised. Pierre Meystre discusses how atoms should be thought of as waves, either individual ones as in the emerging field of „atom optics", or collective ones as in Bose-Einstein condensations and „atom lasers".

The scales of temperature that we are used to, be they Celsius or Fahrenheit, find their origin in phenomena and perceptions from everyday life. For instance, 0° Celsius is defined by the melting point of ice, and 100° Celsius corresponds to the boiling point of water. Similarly, 0° Fahrenheit is the freezing point of saturated salt water, and 100° Fahrenheit is the approximate human body temperature.

Toward Absolute Zero

While these temperature scales are of undeniable use, physicists have found it important to introduce a different scale, the Kelvin (or absolute) scale. The reason that this is meaningful is that from its microscopic definition, it is quite clear that there is an absolute lowest possible temperature. In the Kelvin scale, it is called absolute zero, $T = 0$ K. This temperature is known to be roughly equal to −273.15° Celsius. Measured in the Kelvin temperature scale, room temperature is of the order of 300 degrees.

The existence of an absolute zero is a simple consequence of the definition of temperature. To see how this works, let us backtrack for a moment and consider a closed container filled with a sample of gas, for instance Oxygen. If it were possible to observe the individual molecules forming this gas under a microscope, one would notice that they move in random directions at velocities that are different for all particles in the sample, and rather large at room temperature, of the order of several hundreds of meters per second. This is illustrated in *Fig. 1*. Indeed, if one were to measure all individual velocities and average them, one would find that this average is equal to zero, or, in the form of an equation, $\langle v \rangle = 0$. This is because the direction of motion of the molecules is random, so that they are as likely to move, say, to the right as to the left.

If, on the other hand, one were to average the square v^2 of these velocities, one would obviously find a result larger than zero, $\langle v^2 \rangle > 0$, as follows from the fact that the square of a real number is always positive, and the average of positive numbers is itself positive. The only case when $\langle v^2 \rangle$ can be equal to zero is when all individual velocities are themselves equal to zero. Now, as it turns out, the absolute temperature *T* of a sample is proportional to this quantity $\langle v^2 \rangle$

$$T \propto \frac{\langle v^2 \rangle}{k_B},$$

where k_B is Boltzmann's constant. In other words, *T* is a measure of the velocity fluctuations in the sample. There are some constants appearing as proportionality factors in the relationship between *T* and $\langle v^2 \rangle$, but they need not concern us here, except for the fact that they are positive. We see, then, that the absolute temperature must by definition be larger than zero, and in addition, that if *T= 0*, then all particles in the sample must be at rest.

With these general considerations in mind, let us then turn to *Fig. 2*, which sets the stage for our discussion. This figure is a logarithmic scale of absolute temperatures, where the temperature is decreased by a factor of 10 from one point to the next lower one. 5000 K is as good a place as any to start. This is roughly the temperature at the surface of the sun. Decreasing this temperature by a factor of hundred or so (two orders of magnitude in physics jargon), we reach much more friendly temperatures, of the order of 300 to 400 K. This is the range of temperatures we are most familiar with, and where almost all life as we know it takes place, between the boiling and freezing points of water. As human, we are very sensitive to very small variations of temperatures in that range. A one or two degrees change in our body temperature and we feel very uncomfortable, a few tens of degrees change in the outside temperature and we switch from

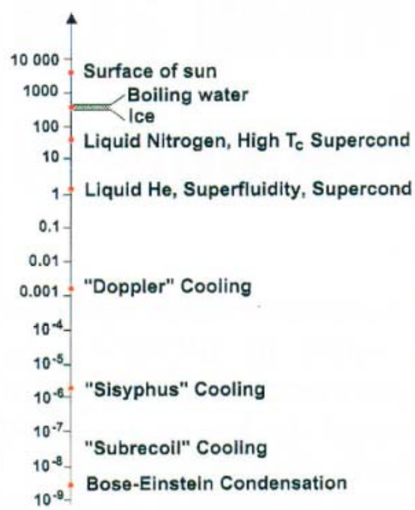

Figure 2: The Kelvin temperature scale

air conditioning to heating our houses.

Moving to lower temperatures, below the freezing point of water, an interesting place to stop is about 70 K. This is the freezing point of Nitrogen, and also the regime where the phenomenon of „high temperature“ superconductivity was recently demonstrated. I put „high temperature“ in quotation marks here, because it may sound somewhat paradoxical that temperatures nearly 200 degrees below the freezing point of water should be considered „high“. But indeed, such temperatures are downright balmy in the strange world of low-temperature physics.

Another two of orders of magnitude down the temperature scale leads us to a few degrees Kelvin, the domain of traditional low-temperature physics. Such temperatures are associated with phenomena such as superconductivity and superfluidity. These are manifestations of quantum physics quite foreign to our everyday experience, where electric conductors lose all resistance, fluids climb on walls, etc. Alas, we won't have time to stop in this fascinating place, but must move on to even lower temperatures, and to a world which was experimentally inaccessible until just a few years ago. Thanks to advances in cooling techniques, it is now possible to cool atomic systems to one millionth of a degree Kelvin, and even lower. At these extreme temperatures, the world is an utterly strange place where our everyday's common sense is useless, quantum physics rules with its counterintuitive laws, and atoms behave as waves.

But before proceeding, let as pause for a moment and reflect on how far we have traveled. Room temperature is about 300 K, and we are now at about a hundredth of a millionth of a degree. These are ten orders of magnitude, or a factor of ten billions, below room temperature. Ten orders of magnitude are very hard to fathom: but it might be helpful to note that a tenth of a billionth of the Munich-Los Angeles distance is about 1 mm! If room temperature corresponded to Los Angeles and 0 K to Munich, our trip down the temperature scale would have taken us to within 1 mm of our goal.

Particles and waves

The world in which we live is governed by the laws of classical physics. The underlying quantum laws, while always present in principle, are completely masked by the velocity and other thermal fluctuations that we have mentioned earlier, and which are invariably present at normal room temperatures. From this point of view, temperature may be thought of as a source of noise which washes out the fragile effects of quantum physics. There are of course notable exceptions, and indeed a number of quantum devices have changed our lives in profound ways in the last few decades. Important examples are the transistor and the laser, which are at the heart of the information revolution. But in these devices, great care must be taken to control the detrimental effects of thermal fluctuations, which rapidly mask and destroy the subtle interference effects which are the hallmark of quantum physics. In contrast, thermal noise is completely absent from the ultracold world, where quantum dynamics and the „paradoxes" associated with it become fully apparent.

One of the most unsettling aspects of quantum mechanics may well be the so-called wave-particle duality. We are all familiar with the notions of waves and particles. We have observed water waves since throwing pebbles in ponds as children, and we have learned in high-school that sound and light also consist of waves. Waves are characterized by a wavelength and a frequency: the wavelength is the distance between two crests of the wave, and the frequency is the number of crests that an observer at rest sees passing in front of his eyes every second. Light waves have very short wavelengths, of the order of a millionth of a meter or less, and very high frequencies, of the order of 100 000 billions per second. The frequency υ and wavelength λ of light are always related (in vacuum) by the simple relation $\lambda = c/\upsilon$, where c is the speed of light, $c \simeq 300\,000$ *km/sec*. Blue light consists of waves of higher frequency and shorter wavelength than green light, and similarly green light consists of waves of higher frequency and shorter wavelength than red light.

In contrast, we like to think of atoms, electrons, etc. as particles, somewhat like tiny little balls[1]. And as we do for marbles or tennis balls, we describe particles in terms of their velocity $\mathbf{v}$ and their kinetic energy E, the energy and velocity being related by the well-know expression

$$E = Mv^2/2,$$

where M is the mass of the particle[2]. Actually, physicists like to use the particle momentum $\mathbf{p} = M\mathbf{v}$ instead of its velocity, because as we shall see, this is a more fundamental and useful quantity.

One of the most profound revolutions brought about by quantum mechanics is that it does away with the distinction between particles and waves. The first blow to classical physics was given by Einstein, who proposed in his explanation of the photoelectric effect that it would be useful to think of light as made out of massless particles, to which an energy and a momentum should be associated according to the relations

$$E = h\upsilon,$$

and

$$p = h/\lambda.$$

A few years later, de Broglie went one step further and proposed that electrons, and for that matter all massive particles, should likewise be thought of as waves of wavelength Λ given by

$$\Lambda = h/p.$$

This wavelength is now referred to as the de Broglie wavelength. In these equations, the fundamental constant h is called Planck's constant. The considerable intuition of Einstein and de Broglie eventually lead to the development of modern quantum mechanics and quantum electrodynamics by Heisenberg, Schrödinger, Dirac and many others. It is not the place here to go into these developments. Rather, it will be sufficient to emphasize that in quantum mechanics, the distinction between particles and waves loses much of its significance. Depending upon the situation at hand, it is useful to think of light as made of particles (now called photons) and of matter as particles, or of both matter and light as waves, or any other combination.

Which brings us to the title of this article...

To understand why it is so useful to think of ultracold atoms as waves, let us relate their de Broglie wavelength Λ to temperatures. This is easily done.

We have seen that the temperature of a sample is proportional to its mean squared velocity $\langle v^2 \rangle$, or, as a result of the proportionality between velocity and momentum, to its mean squared momentum $\langle p^2 \rangle$. Since the de Broglie wavelength is itself inversely proportional to p, we find that the (thermal) de Broglie wavelength of a sample is inversely proportional to the square root of its temperature,

$$\Lambda \propto \frac{1}{\sqrt{T}}$$

Therefore, the colder the sample, the larger its de Broglie wavelength. *Figure 3* shows the de Broglie wavelength of a typical atom as a function of its temperature. At room temperature, it is very small, of the order of a tenth of a billionth of a meter, or an Angstrom. It is impossible to observe an object of this size with visible light, because light cannot resolve details much smaller than its own wavelength, which is thousands of times larger in the present case! To image an atom in such detail one would need instead to use an X-ray source. Unfortunately X-ray photons carry so much energy that they would instantly destroy the atoms they are meant to image. This is why at room temperature, the wave nature of atoms is normally irrelevant, and it is

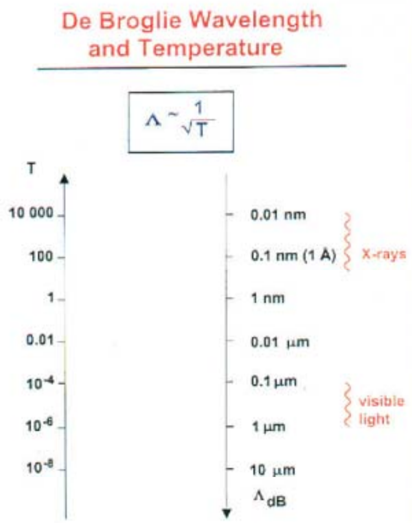

Figure 3: Correspondence between Kelvin temperature and the thermal de Broglie wavelength of a typical atom.

most useful to think of them as particles.[3]

But as temperatures are lowered toward the ultracold regime of a millionth of a degree Kelvin or less, the thermal atomic de Broglie wavelength becomes very large, so large as to be comparable to, or even larger than the wavelength of visible light. Visible light can impinge on atoms without destroying them. Optical and atomic physicists have over the years developed exceedingly sophisticated methods to control the way light interacts with atoms, and have in the process gained a detailed understanding of their interaction. It is readily possible to apply this understanding to the preparation, manipulation, and imaging of atoms in that ultracold regime where the distance between crests of the „matter waves" is comparable to the distance between crests of the optical waves, and where their wave nature can therefore be observed directly. And instead of conventional optics, one can now study „atom optics" and its applications. In complete analogy with optics, they go from the development of atom optical elements, atom mirrors and lenses, etc. to atomic imaging and to novel devices such as „atom lasers."

Laser cooling

We have indicated in the preceding section that visible light provides a perfect tool to prepare, manipulate, and detect matter waves. We now discuss how light also provides a powerful and inexpensive tool to cool atoms to extremely low temperatures, using techniques generally called laser cooling. In order to understand how this works, it is necessary to review some of the basic elements of the way light and atoms interact.

1. Atom-light interaction

While it would be far beyond the scope of this article to describe the theory of light-matter interactions, it will be necessary in order to proceed to introduce a few of its most basic elements.

Atoms consist of a rather heavy nucleus formed of neutrons and protons surrounded by a cloud of much lighter electrons that move on „orbitals" of prescribed energy. Normally, the electrons occupy the orbits with the lowest possible energy. But when an atom is irradiated by light it can absorb a photon, whereby an electron is promoted from a „low" orbit of energy E_1 to a „higher" orbit of energy E_2. Simultaneously the light intensity is decreased since one photon has been absorbed.

All physical interactions must proceed in such a way that two fundamental conservation laws are satisfied,

- conservation of energy,
- conservation of momentum.

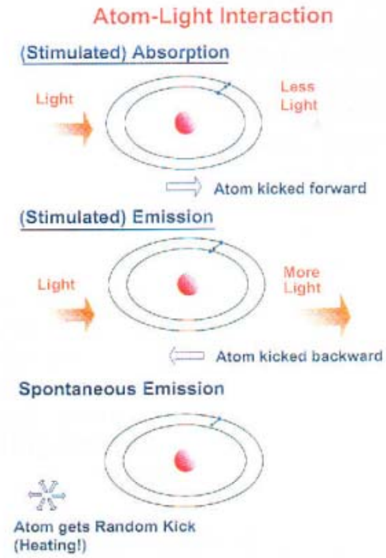

Figure 4: Schematic representations of absorption, stimulated emission and spontaneous emission of light by an atom.

The law of conservation of energy implies that the difference in energy $E_2 - E_1$ between the initial and final electron orbits must equal the energy $h\upsilon$ of the absorbed photon,

$$E_2 - E_1 = h\upsilon.$$

Conservation of momentum furthermore implies that when absorbing a photon of momentum h/λ, the atom must gain a momentum

$$\Delta p = M\,\Delta\upsilon = h/\lambda.$$

In other words, its velocity undergoes a kick $h/M\lambda$ along the direction the absorbed photon came from. This is illustrated in *Fig. 4*.

In addition to absorbing light, *Fig. 4* also illustrates how an atom can also emit light as an electron jumps down from a high orbit of energy E_2 to a lower orbit of energy E_1. There are two ways in which this can happen, *spontaneous* and *stimulated emission*. In the first case, a photon of energy

$$h\upsilon = E_2 - E_1$$

is emitted in a random direction, without any apparent outside help to trigger the emission. Because of momentum conservation, the atom experiences a kick in momentum by the amount $M\Delta\upsilon = h/\lambda$ in the direction opposite to the direction of photon emission. This recoil is similar to the recoil experienced by a gun when shooting a bullet.

In contrast to spontaneous emission, which can happen without any light initially present, stimulated emission, the reverse mechanism of absorption, requires the presence of a light beam. In this process, a photon is emitted in the direction of that light beam, with an atomic recoil Δp in the opposite direction. As a result of this process the number of photons, or in other words the intensity of the light beam, is increased. This mechanism is at the heart of Light Amplification by Stimulated Emission of Radiation, or LASER action, which we shall return to at the end of this article.

2. Doppler cooling

Armed with this basic understanding of atom-light interactions, we can now discuss the simplest form of laser cooling, Doppler cooling. We proceed by first considering an atom initially at rest, and irradiate it with two laser beams, one from the right and the other from the left, see *Fig. 5*. The frequency υ of this light is chosen in such a way that the corresponding photon frequency $h\upsilon$ is less than the energy difference $E_2 - E_1$ between the electronic orbitals we are considering. Because it is not possible then to absorb a photon and promote an atom from the low-energy to the higher-energy orbital in an energy-conserving way, i. e. with $h\upsilon = E_2 - E_1$, the absorption of light by the atom is impossible – or, strictly speaking, very unlikely – and nothing happens.

Suppose now that instead of being at rest, the atom is moving at some velocity υ toward one of the light beams, and away from the other. To that atom, the frequency of the light beam it is moving toward appears higher, the light seems „more blue". In contrast, the atom perceives the frequency of the light it is moving away from as lower, or „more red". This is a simple consequence of the Doppler effect, which we are all familiar with from listening to fire trucks passing by on the street. When they move toward us, the pitch of their siren is higher (higher frequencies) than when they move away from us (lower frequencies).

This same Doppler effect also happens with light, the frequency shift perceived by the moving atom being proportional to its velocity. As a result, the apparent energy of the photons in the beam the atom is running against will be closer from $E_2 - E_1$, and energy conservation can now be more nearly satisfied when one photon is absorbed from the beam. Hence, the absorption of these photons becomes possible, with a concomitant velocity kick in the direction of the light beam and a slowing down of the atom. In contrast, the apparent photon energy in the beam propagating in the same direction as the atoms is even further from $E_2 - E_1$ than for atoms at rest, hence, the atom is even more unlikely than before to absorb light from that beam.

As a result of this imbalance between the absorption of photons coming from the left and from the right, the velocity of the atom is reduced. The same argument can be made if the atom is initially moving in the other direction, except that the roles of the two laser beams are reversed. Hence no matter which direction the atom is moving in, it will be slowed down and cooled as a result of its interaction with the light beams.

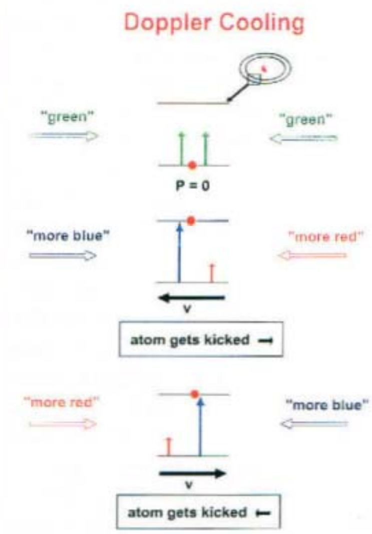

Figure 5: Schematics of Doppler cooling. See text for details.

While it might appear that this technique of Doppler cooling, perhaps with some improvements, could be used to completely stop atoms, this turns out not to be the case. This is because we ignored the effects of spontaneous emission. As we have seen, in this process an atom emits a photon in a *random* direction, with a concomitant kick in atomic velocity in the opposite direction. These random velocity kicks correspond exactly to what we saw to be a non-zero temperature in the preceding section. This shows that spontaneous emission is actually a source of heating, which turns out to limit the temperatures that can be reached via Doppler cooling to about a thousandth of a degree Kelvin, the so-called Doppler limit. To go past this limit, it is necessary to somehow circumvent the detrimental effects of spontaneous emission.

3. Below the Doppler limit

The optical method of choice to cool an atomic sample below the Doppler limit is called Sisyphus cooling. To understand how this works requires a more detailed understanding of light-atom interactions than can be given here. For our purposes, it is sufficient to note that by a clever choice of electronic orbits and laser beam arrangements, it is possible to force the atoms to move in much the same way as marbles on a corrugated roof. But the trick here is that there are two such „roofs", and the atoms can jump from one to the other at essentially no cost, but only if they are located near maxima of the surface, see *Fig. 6*. As a result, the atoms are forced to always move „uphill", very much like Sisyphus of the Greek legend. In the process, they lose most of their energy, and wind up with a mean squared velocity $\langle v^2 \rangle \cong h^2/\lambda^2$, which corresponds to the so-called *recoil temperature*

$$T = \frac{h^2}{2k_B M\lambda^2}.$$

This temperature is of the order of a millionth of a degree Kelvin for typical alkali atoms such as Sodium.

The fact that the wavelength λ of light appears in this equation provides one with a hint about what limits the temperatures that can be achieved via Sisyphus cooling: it is the light shining on the atoms! When it comes to atomic cooling, light is both an atom's best friend and its worth enemy. To go past the recoil limit, it turns out to be necessary to develop cooling methods where the atoms are no longer subject to the detrimental heating effects of light. Several optical methods achieving this goal have been developed, relying on subtle quantum effects such as the existence of so-called dark atomic states, which are atomic states that remain unperturbed by light. But another method, evaporative cooling, has proved more successful so far in achieving relatively high atomic densities at extremely low temperatures.

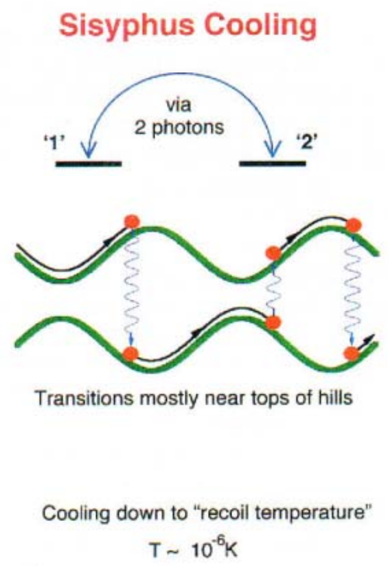

Figure 6: Schematics of Sisyphus cooling. See text for details.

4. Evaporative cooling

In contrast to the cooling mechanisms that we have discussed so far, evaporative cooling is quite familiar from everyday life. Indeed, anybody who has ever had a cup of coffee or tea has witnessed the effects of evaporative cooling: as time goes on, coffee cools down as the warm molecules escape from the cup, and the remaining ones „rethermalize" at a lower temperature. The evaporative cooling of an atomic sample works in much the same way, except that the temperatures involved are of course much lower.

We have seen that in an atomic sample at non-zero temperature, the various particles move around at more or less random velocities, the sample's temperature being proportional to $\langle v^2 \rangle$. Now, high-velocity particles can clearly more easily escape from a trap, being it a coffee cup or one of the magnetic traps that are used in modern experiments to contain atoms. Once the fast atoms have escaped, the mean squared velocity of the remaining sample is lower, hence so is its temperature. If one keeps the escape process going by gradually reducing the depth of the trap, as illustrated in *Fig. 7*, atoms with lower and lower velocities will be able to escape, and the temperature of the remaining sample will keep decreasing. According to this scheme, one should be able to reduce the temperature of the sample to arbitrarily low temperatures. Of course, there are several catches, the most important one being that one wants to still have a sufficient atomic density available when the desired temperature has been reached. In addition, collisions, which are essential in reestablishing a thermal equilibrium in the sample each time the fast particles escape, become less and less frequent as the sample becomes colder and more rarified, so that the cooling can stop altogether if things are not just right. But experimentalists have found clever ways around these difficulties, at least for alkali atoms such as Sodium and Rubidium. The technique of evaporative cooling has proved extraordinarily success-

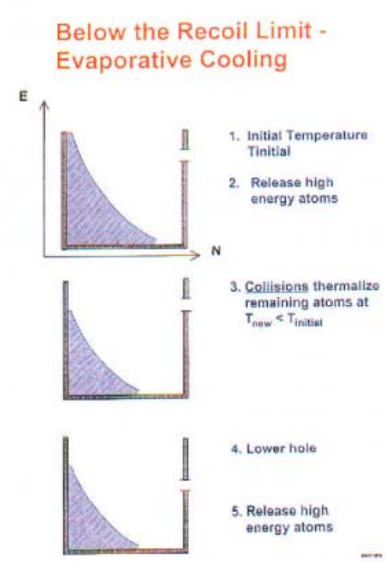

Figure 7: Schematics of evaporative cooling. See text for discussion.

ful in reaching the exceedingly low temperatures, of the order of a billionth of a degree Kelvin, at which the quantum effects we turn to next manifest themselves.

Atoms as waves

1. Atomic diffraction

We have mentioned that the de Broglie wavelength of atoms becomes longer, the lower their temperature. For temperatures of the order of a millionth to a billionth of a degree Kelvin, this wavelength becomes comparable to, or even longer than an optical wavelength. In that limit, the wave nature of atoms becomes particularly easy to observe in the laboratory.

To see how one such experiment works, consider a situation where a light beam is shined on a beam of ultracold atoms, as illustrated in *Fig. 8*. We assume here that the energy $h\nu$ of the photons is equal to the energy difference $E_2 - E_1$ between the electronic orbitals of interest, in contrast to the situation of Doppler cooling discussed earlier, so that a very cold atom can easily absorb a photon without having to rely on the Doppler effect. An atom with an electron on a low orbital of energy E_1 can therefore absorb a photon, thereby being promoted to the higher orbital of energy E_2, and receiving a kick in momentum $\Delta p = h/\lambda$, or, stated otherwise, a kick in velocity $\Delta v = h/M\lambda$. From that instant on the atom, instead of moving perpendicularly to the light beam, slightly changes its direction of motion... except that quantum mechanics does not give us any way to know exactly *when* the absorption event occurs, only the probability that it occurs at a given time. Hence all we know, really, is that the atom will keep going straight with some probability $P_{no\text{-}kick}$, and will move at some angle with another well-prescribed probability P_{kick}. More precisely – and this point is of fundamental importance – what the laws of quantum mechanics allow us to determine is the so-called *probability amplitudes* for „no-kicks" and „kicks". Probability amplitudes are often described by the Greek letter ψ, vis. $\psi_{no\text{-}kick}$ and ψ_{kick} in the present case.

Quantum mechanics also provides us with a set of rules that describe how probability amplitudes evolve and are to be recombined, much like probability theory teaches us how to combine the probabilities of various alternatives in a stochastic process. For our present purpose, it is sufficient to note that probability amplitudes are analogous to the partial waves that we observe, say, when we throw pebbles in water. Each pebble is the source of a small wavelet which propagates in circles away from the pebble. When two wavelets meet, they interfere and typically produce a complicated interference pattern at the surface of the pond. The partial atomic waves $\psi_{no\text{-}kick}$ and ψ_{kick} can be thought of in much the same way as these partial waves, the „pebble" initiating the wavelet ψ_{kick} being the absorption of a photon by the atom.

We can easily see what happens at later times to the partial wave ψ_{kick}. Either the atomic electron remains in the orbital of energy E_2, or it returns to the lower orbital via stimulated emission of a photon. In that case, the atom will suffer a momentum kick $-h/\lambda$, so that it will wind up moving again perpendicularly to the light beam, see *Fig. 8*.[4] We conclude, then, that as a result of successive absorption and stimulated emission events, the atom finds itself in a combination of partial waves of the form $\psi_{no\text{-}kick}$ and ψ_{kick}, and its velocity has two components, one perpendicular to the laser beam, and the

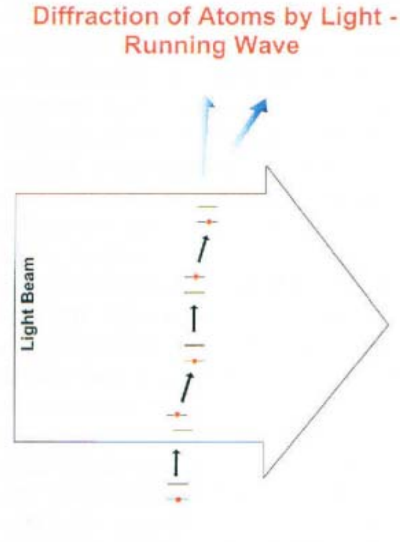

Figure 8: Velocity of an atom as it successively absorbs and emit light from a running wave.

other at a small angle from it. Quantum mechanics indeed allows such a curious object as an atom „moving with two velocities at a time!" *Fig. 9* illustrates this behavior: as a function of the time the atom interacts with light, and the probability that it moves with one or the other velocity oscillates, with significant time intervals where „the atom moves with two velocities."

The situation becomes even more interesting if instead of in-

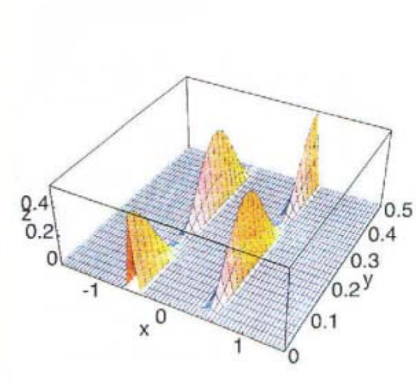

Figure 9: Probability for an atom to move perpendicularly to the light beam, or at an angle from it, as a function of time. (Time is labeled *y* in the figure.)

teracting with just one light beam, the atom interacts with two counter-propagating lasers, see *Fig. 10.* The electron can still be excited by absorbing a photon, but since that photon can be provided either by the left or the right propagating beam, the atom can be given a velocity kick in either direction, $\Delta v = \pm h/M\lambda$. Let us label ψ_1 and ψ_{-1} the partial waves corresponding to these two alternatives. The excited electron can then fall back to its lower orbital via the stimulated emission of a photon into either of the two laser beams, thereby receiving a velocity kick in the direction opposite to the beam involved. Hence, the partial wave ψ_1, for example, evolves into the sum of a partial wave ψ_0 perpendicular to the laser beams and of a partial wave ψ_2 with velocity $2h/M\lambda$,

$$\psi_1 \rightarrow \psi_0 + \psi_2.$$

Similarly,

$$\psi_{-1} \rightarrow \psi_0 + \psi_{-2}.$$

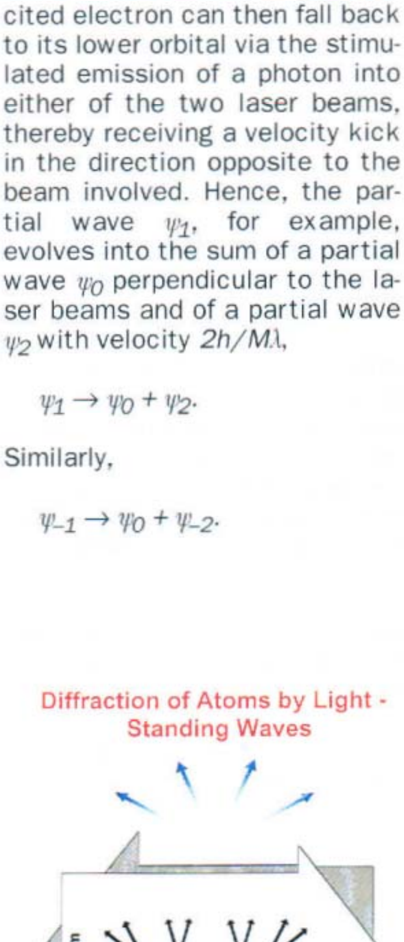

Figure 10: Velocity of an atom as it successively absorbs and emit light from a standing wave, the superposition of two counter-propagating running waves.

The important point of these two equations is that the atom can evolve back into the partial wave ψ_0 via two different paths. This is very much like the example of water waves produced by throwing two pebbles in the pond. When the two partial waves reach the same point they *interfere*, leading to the complicated pattern of crests we mentioned earlier. The same thing happens with de Broglie waves: Their superposition leads to complicated *quantum interference* patterns in the total matter wave $\psi = \psi_{-2} + \psi_{-1} + \psi_0 + \psi_1 + \psi_2$. And the situation becomes more and more complicated as subsequent absorption and stimulated emission events take place. Taking into account the fact that according to the laws of quantum mechanics, the probability to find the atom at a given position is given by the square of the absolute value of ψ, evaluated at that location, yields as a function of time to the density pattern shown in *Fig. 11.*

2. Atom optics

The reader familiar with classical optics will have realized from the previous discussion that the motion of an atom interacting with two counter-propagating laser fields is completely analogous to the diffraction of light by a mechanical grating. The correspondence between the two situations has lead physicists to introduce the new field of *atom optics*, which is exactly the same as optics, except that electromagnetic waves are replaced by de Broglie matter waves.

It would be unfair to give the impression that this idea finds its origin in atomic physics: matter-wave optics was first developed for electrons and for neutrons. However, the fact that electrons are charged particles makes it somewhat cumbersome to work with them, and neutron

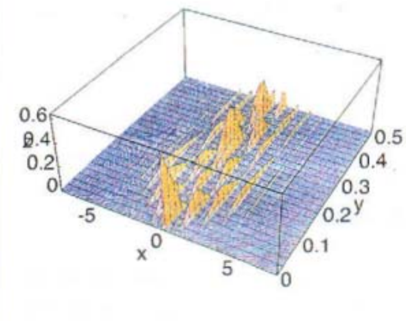

Figure 11: Probabilities for various atomic velocities as a function of time (labeled *y*) for an atom absorbing and emitting light from and into a standing wave.

sources require access to reactors, which are not readily available. In contrast, atomic sources are easy to come by, the cooling of atoms is, as we have seen, relatively straightforward, and in addition, the internal electronic structure of the atoms provides one with considerably more flexibility than is the case with electrons and neutrons.[5]

Atom optics is rapidly developing into a mature subfield of atomic physics and quantum optics. Basic atom optical elements such as atomic mirrors, atomic beam splitters, atomic gratings, etc. have been demonstrated. These elements can consist either of mechanical nanostructures or of optical fields. This latter method relies on the use of the mechanical effects of light on atoms that we have just outlined. A particularly fascinating aspect of atom optics is that it completely reverses the roles of light and matter from the usual situation: in conventional optics one manipulates the propagation of light with mechanical devices such as mirrors, lenses, and so on. In atom optics, the roles of matter and light are interchanged: the propagation of matter waves is modified and controlled by the use of optical devices such as matter wave mirrors, lenses, etc. We shall see in the next section how the analogy between conventional and matter-waves optics can be carried out even further to study nonlinear atom optics and „atom lasers", which are the matter waves analog of optical lasers.

Quantum statistics

We have mentioned repeatedly that at ultra-low temperatures the de Broglie wavelength Λ of atoms becomes extremely large. What this means, roughly speaking, is that the atoms become „huge", or more precisely strongly delocalized. Returning once more to the water waves analogy, we realize that it is meaningless to even define the wavelength λ of waves if the distance between crests is larger than the dimensions of the pond, which must be at least several λ before this makes sense. By analogy, it is reasonable to say that the „size" of the atomic wave is at least several de Broglie wavelengths Λ.[6] What this means in practice is that atoms begin to feel that they are not isolated, even at low densities. It is as if the atoms had long tentacles that allow them to probe their environment and notice the presence of other atoms. When this happens, yet another law of quantum mechanics starts to take effect, the law of quantum statistics.

According to quantum mechanics, all particles belong to one of two kinds, *bosons* or *fermions*, after the scientists who first introduced them, S. N. Bose and E. Fermi. The major difference between bosons and fermions is that fermions are subject to the Pauli exclusion principle, while bosons are not. Put in simple terms, this means that no two fermions can be in the same state at the same time, while bosons do not suffer such a restriction. On the contrary, they have a tendency to accumulate in the same state. Fermions are „repelled" by each other, while bosons are „attracted" under the effect of the so-called „exchange force" sketched in *Fig. 12*. As we shall now see, this leads to spectacular consequences in the case of bosons.

1. Bose-Einstein condensation

We have seen in the discussion of evaporative cooling that as the depth of the atomic trap is lowered, atoms with high velocity escape and the remaining ones rethermalize at a lower temperature. Their de Broglie wavelength Λ increases, and they start to feel the presence of their neighbors. One of two things can then happen, depending upon whether the atoms are bosons or fermions: Fermions start to „repel" each other so as to make sure that no two of them are in the same state. Bosons, on the other hand, begin to „attract" each other, attempting to all occupy the same state, in the present example the lowest energy state of the trap. At absolute zero temperature, all atoms eventually „condense" into that state. This is the phenomenon of *Bose-Einstein condensation*, which was already predicted by Einstein. Bose-Einstein condensation was first observed in an isotope of Helium called ^4He by P. L. Kapitza as early as 1938, at a temperature of about 2.17 K. In 1971, another form of Bose-Einstein condensation was observed in the ^3He isotope, at a much lower temperature of a few milliKelvin.

In both cases, though, the densities required to achieve condensation were rather high, corresponding to liquid densities. In contrast, ultra-low temperatures in the sub-microKelvin range lead to the onset of quantum statistical effects at much lower densities, a billionth or so of liquid density. One advantage of working at such low densities is that the atoms in the sample interact only weakly with each other. While the dynamics of liquids is dominated by frequent collisions that can involve many partners, collisions in rarified gases are very seldom, and typically involve two collision partners only. From this point of view, the low density, weakly interacting systems that can be generated at ultra-low temperatures are much „cleaner" and easier to understand. In addition, while the fraction of condensate in quantum liquids such as ^4He is always very small, of the order of a few percent, it is possible to put practically the entirety of the sample in the condensate state at low densities. This is of considerable advantage for applications such as „atom lasers."

Fig. 13 illustrates the transition of an ultracold atomic system from a conventional gas to a Bose-Einstein condensate as its temperature is decreased. The three pictures, obtained by the group of W. Ketterle at MIT, show the velocity distribution in the atomic sample, along two directions in the sample and for three different temperatures. They are color-coded in such a way that blue corresponds to a

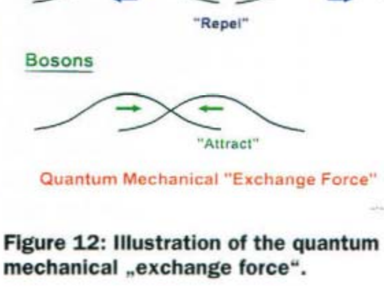

Figure 12: Illustration of the quantum mechanical „exchange force".

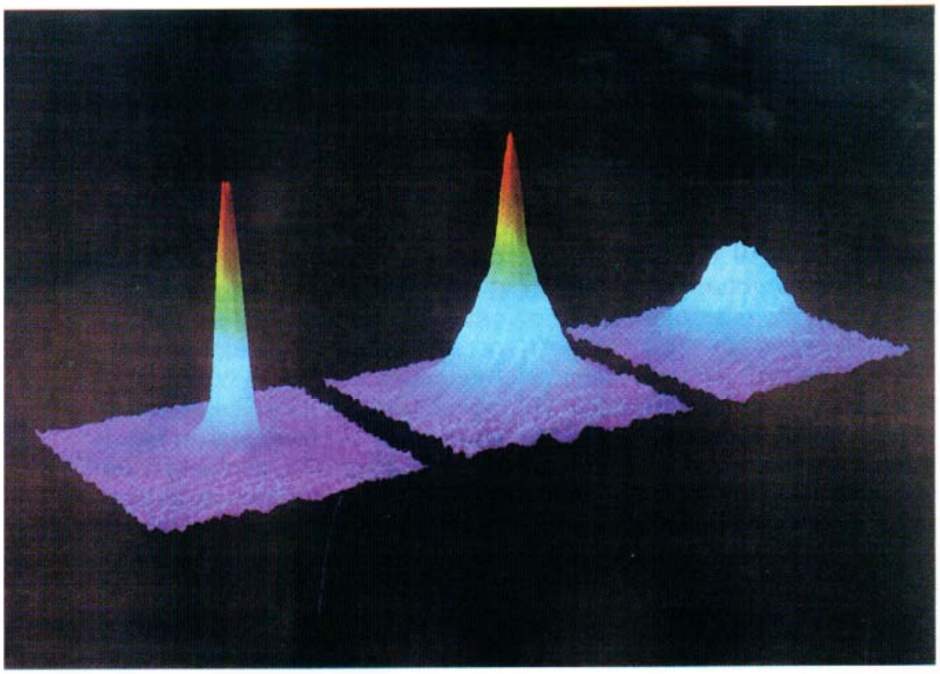

Figure 13: Bose condensation of a Sodium sample as its temperature is decreased. (Courtesy W. Ketterle, MIT)

low density – few atoms have the corresponding velocity – and red to a high density – many atoms have the corresponding velocity. Zero velocity is at the center of the pictures. The right picture corresponds to a relatively high temperature, above the transition from „normal" gas to condensate. Here, the velocity distribution is quite broad, with a smooth distribution decreasing from the maximum at $v = 0$. For lower temperatures, illustrated in the middle picture, the shape of the velocity distribution undergoes a qualitative change. The velocity distribution comprises now two distinct contributions, a broad one quite similar to that of the preceding case, and superimposed to it a sharply peaked one, also centered at $v = 0$. This contribution corresponds to the fraction of atoms that form a condensate at the bottom of the trap, or more precisely in the lowest energy level of the trap. In the left picture, which corresponds to the lowest temperature, the broad distribution has all but disappeared, all atoms finding themselves in the condensate.

All atoms in the condensate are in the same quantum mechanical state. They have completely lost their individual identity and can no longer be distinguished from one another, even in principle. As such, the condensate can be thought of as forming a new state of matter, a quantum gas behaving as a macroscopic system of a million or so atoms, all evolving in a coherent way. The condensate forms a *single* quantum mechanical entity: to describe it mathematically, it is not necessary to keep track of the individual wavelets ψ of the atoms forming the condensate. Rather, it is sufficient to describe the whole condensate as one wavelet, which is often labeled with the Greek letter Ψ, the upper case version of ψ. The behavior of the atoms at extremely low temperatures is truly and remarkably simple: a single function, rather than millions of them – one for each atom – is required for their description.

In many ways, it is possible to think of a condensate as a „macroscopic atom" which can be split and recombined in much the same way single atoms were split and recombined in atom optics. For instance, it is possible to cut a condensate into two parts, using a laser beam as a „knife", and to later on recombine the two parts. The two condensates behave as waves, very much like ultracold atoms, and their wavelets Ψ_1 and Ψ_2, when recombined, form an interference pattern as illustrated in *Fig. 14*. Such interference patterns are a clear and unambiguous proof of the macroscopic coherence of Bose-Einstein condensates.

2. Atom lasers

We have established that ultra-cold atoms do behave as waves, and in addition, that at high enough density they lose their individuality, condense into a single state and act „as one" – provided that they are bosonic. As it turns out, a very similar situation occurs with light. In the particle picture, light consists of photons, which are bosons fully characterized by their momentum p and energy E. One can then ask whether it is possible to bring a large number of photons into a single quantum state, i.e. to force them to all have the same energy and momentum, thereby losing their individuality. The answer to that question is „yes": this is precisely what is achieved in a laser. We have hinted earlier at the way lasers work: Atoms with an electron in a high orbit of energy E_2 emit a photon via stimulated emission, with the same energy $E = E_2 - E_1$ and momentum p as the photons inducing the emission. As a result, the photon number is increased by one.

Stimulated emission is the more likely, the larger the number of photons of energy E and momentum p already present. This is very much like the „attraction" of atomic bosons in-

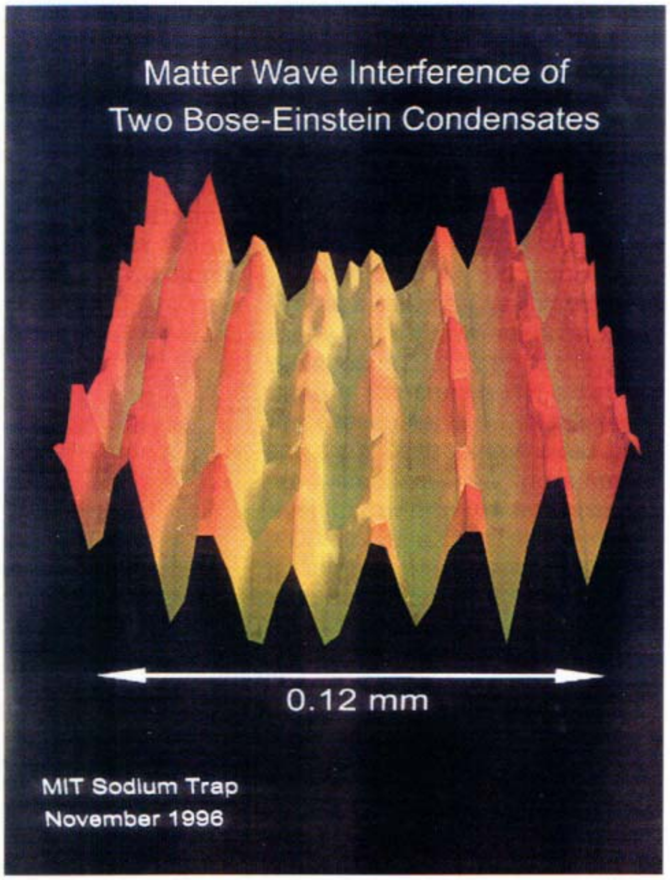

Figure 14: Matter-wave interference of two Bose condensates. (Courtesy W. Ketterle, MIT)

to a given quantum state, which is proportional to the number of atoms already in that state as we have seen. In both cases, this behavior is an unavoidable consequence of quantum statistics, sometimes called Bose enhancement.

In conventional sources such as light bulbs, the number of photons of a given energy and momentum is always very small, because photons are emitted in all possible directions, that is, with all possible momenta. In that case Bose enhancement is negligible. An essential ingredient of a laser must be a device where the number of photons of a given momentum and energy can somehow become larger than one. This is achieved by using an „optical resonator", which is a „photon trap" where only photons of very specific momenta and energies can be stored. The basic elements of a laser are therefore a „pumping mechanism" that excites electrons into a high orbit of energy E_2 from which they can fall to a lower orbit of energy by stimulated emission, an optical resonator that can store photons of the appropriate energy and momentum, and a mechanism to extract some of the light from the resonator in the form of a useful coherent light beam.

These considerations naturally lead one to wonder whether it is possible to build an „atom laser," a device that generates a coherent beam of atoms in a single quantum state. Our previous discussion makes it clear that by achieving Bose-Einstein condensation, this goal is already almost achieved: Bose enhancement leads to the creation of a macroscopic population of atoms in the lowest energy state of the atomic trap, much like a macroscopic population of photons of energy E and momentum p is created in a laser cavity. An outcoupling mechanism to produce a useful coherent atomic beam outside the trap has also recently been demonstrated. *Fig. 15* shows the pulsed coherent atomic beam emitted by this first, still somewhat primitive „atom laser".

Despite the analogies we have emphasized so far, there seem to be important distinctions between Bose-Einstein condensation and optical lasers: Most importantly perhaps, lasers rely on stimulated emission, that is, they *need* atoms to operate. In contrast, Bose-Einstein condensation does not rely on the presence of light, the atoms do the job by themselves. Or do they ?.....

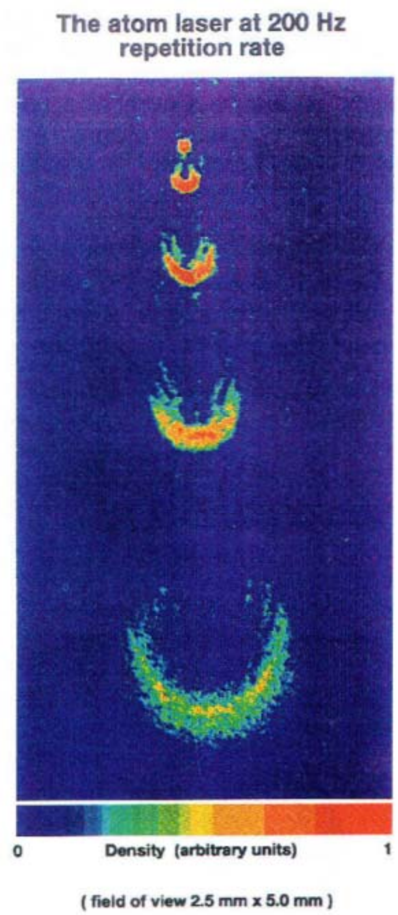

Figure 15: Pulsed output from the MIT atom laser (Courtesy W. Ketterle, MIT)

The reason why lasers depend on atoms for their operation is that photons do not interact with each other: Two beams of light can cross without influencing each other's trajectory. Atoms are needed to change this state of affairs and to modify the frequency and direction of propagation of light. The situation is of course different for atoms. They do collide, and two beams of atoms cannot cross without influencing each other. As it turns out, is this simple difference which makes it possible to produce an „atom laser" without the help of light.... But then again, this is not quite correct, because at the most fundamental level, the reason atoms collide is that they communicate via electromagnetic (optical) interactions. It's just that we are so familiar with the idea of collisions that we don't usually think of them in these terms. So, atoms do need light after all, and as was the case earlier for atom optics, there is indeed a complete role reversal between atoms and light when going from a conventional laser to an „atom laser".

Conclusion and outlook

The goal of this paper was to discuss how at ultracold temperatures our preconceived ideas and intuition about atoms need to be revised. Atoms behave as waves, either individual ones as in atom optics, or collective ones as in Bose-Einstein condensation and in atom lasers. In the last few years, considerable advances have been made in learning how to generate, propagate and manipulate these waves. Progress is swift, and it may already be time to start thinking about applications. In particular, it is now quite clear that a practical atom laser is only a few years away, if that much. The possible uses of such a coherent atomic beam are difficult to assess, as most laser applications were not immediately and obviously apparent. After all, who would have predicted in the early 1960's that the widest application of lasers would be in compact disk audio systems? While clear and obvious applications of atom lasers will include precision nanofabrication and atom holography, the most interesting ones will probably come as surprises. It will be exciting indeed to follow the development of this field and to see whether atom lasers will have a technological impact comparable to that of conventional lasers.

Acknowledgments

I have benefited over the years from numerous discussions with many collaborators and colleagues. Special thanks are due to Prof. W. Ketterle, who generously provided me with a number of the illustrations reproduced in this article. I wish to thank Professor H. Walther and the Max Planck Institute for Quantum Optics for their warm hospitality and the creative environment they provide. It was not possible in this brief overview to present an accurate historical perspective and to give proper credit to specific contributors. I apologize for this shortcoming.

1 Actually, we visualize atoms as tightly bound groups of particles, one of them forming the nucleus and the others being electrons orbiting around it somewhat like planets orbit around the sun.
2 We use boldfaced letters to indicate vectors, i.e. quantities characterized by a length and a direction. Examples of vector quantities are velocity and momentum. Normal letters are used for the length of vectors and for scalar quantities such as the mass M and the temperature T. For instance, the length of the vector $\mathbf{p}$ is p.
3 By using carefully prepared atomic beams, it is actually possible to directly observe the wave nature of atoms at room temperature. To a large extent, such experiments are equivalent to considering atomic beams that are extremely cold in just one dimension.
4 We neglect for simplicity the effects of spontaneous emission in the present discussion.
5 At the extremely low energies we are considering here, neutrons can be considered for all practical purposes as elementary particles with no internal structure.
6 The idea of an „atomic size" is used very losely here. Strictly speaking, the size of the atom is always the same, independently of its temperature. The „size" we are refering to here is the size of the wave packet ψ that describes the atom in quantum mechanics.

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