mode-locked Er:YAG laser, although an antireflection-coated laser rod was used.[6] Therefore another line-narrowing mechanism could also be responsible for the increased mode-locked pulse duration.

In conclusion, we have shown that acousto-optic modulators based on TeO_2 can be used for active mode locking in the mid infrared. The high figure of merit of TeO_2 allows us to achieve a moderate modulation depth without damage to the transducer crystal even for Er lasers operated around the 3-μm laser wavelength. Mode-locked pulse durations < 700 ps have been obtained with an Er:YLF laser. Further work is needed to determine the limits of acousto-optic mode locking of Er lasers with TeO_2 modulators, since the laser system was not optimized for generation of bandwidth-limited pulses.

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Generation of a frequency comb with a double acousto-optic modulator ring

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We use an acousto-optic modulator ring setup to impose an asymmetric frequency comb on a dye laser. Applications include laser cooling of stored heavy ions.

Modern electro-optic and acousto-optic devices offer many possibilities for tailoring the frequency distribution of laser light to meet specific needs in physics and engineering. By various techniques, a single-frequency laser can be homogeneously broadened[1,2] or sidebands that are separated from the carrier by a radio frequency can be generated.

Dye lasers can be frequency modulated by using an intracavity electro-optic modulator.[3] This modulation generates a frequency comb that extends over hundreds of modes, and it has a range of gigahertz to a few terahertz. The comb is nearly flat, with intensity the highest close to the edges and rapidly decreasing to zero beyond the edges. However, the mode spacing in the comb must equal the laser-cavity free spectral range, which is typically a few hundred megahertz.

Another approach to frequency-comb generation is successive frequency translation by using an acousto-optic modulator (AOM) in a ring configuration.[4] In this way we produce a frequency spectrum with a sharp cutoff. In our setup (Fig. 1a) two AOM's are inserted into an optical ring. Though the configuration of our device is that of a resonator, it is here considered only as a multipass device. Still, the resonator criterion of mode stability applies, and the lens sequence must be chosen properly. We chose a lens sequence (Fig. 1b) with four necessary properties:

(1) A unity ray matrix for one complete round trip, which provides mode stability.

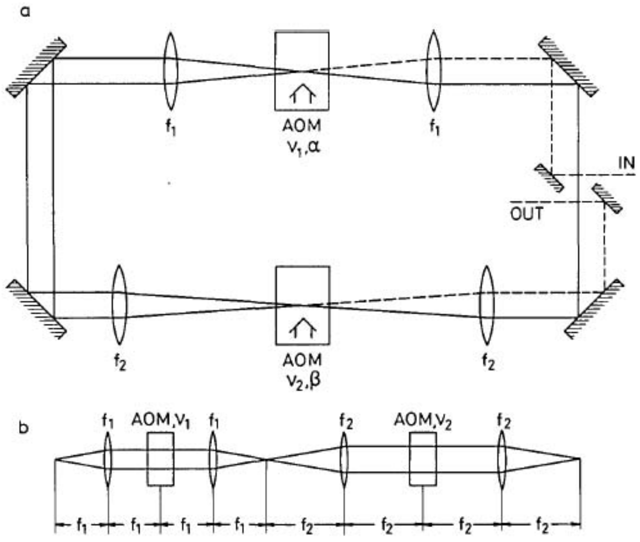

Fig. 1. a, Beam path through the AOM ring. b, Periodic lens sequence of the AOM ring.

(2) Sharp foci at the input–output of the ring, which gives good spatial separation between input–output and the circulating beams.

(3) Collimation through AOM's which gives good deflection efficiency and mode quality.

(4) Focal lengths and AOM frequencies so that $f_1\nu_1 = f_2\nu_2$. This provides stable circulation both for deflected and undeflected beams, because the different angular deflections in the two AOM's are converted into the same spatial offset.

The ring as depicted in Fig. 1 with AOM frequencies $\nu_1 \neq \nu_2$ produces a frequency comb with shifts of $n\nu_1 + m\nu_2$, with $n, m \geq -1$, except for the frequency $-\nu_1 - \nu_2$, which is not generated. With deflection efficiencies α and β and with round-trip transmission T the intensity of the (n, m)th mode is

$$\begin{aligned} I_{n,m} &= T^{n+m+2}\,\alpha^n(1-\alpha)^m\,\beta^m(1-\beta)^n \\ &\times \left\{\varphi(n+1)\varphi(m+1)\binom{|n+m|}{|m|}\right. \\ &\times [\alpha^2\beta^2 + (1-\alpha)^2(1-\beta)^2] \\ &+ \varphi(m+|n|-n)\varphi(m+1) \\ &\times \binom{|n+m|}{|m-1|}\alpha^2(1-\beta)^2 \\ &+ \varphi(n+1)\varphi(n+|m|-m) \\ &\times \left.\binom{|n+m|}{|m+1|}(1-\alpha)^2\beta^2\right\}, \end{aligned}$$

$$n, m \geq -1, \quad \varphi(n) = \begin{cases} 0 & \text{for } n = 0 \\ 1 & \text{otherwise} \end{cases}.$$

The calculated frequency comb for $T = 1.0$ is shown in Fig. 2. The parameters used in this and the following calculations and measurements are $\nu_1 = 44.766$ MHz, $\alpha = 0.85$, $\nu_2 = 32.000$ MHz, and $\beta = 0.70$. For laser cooling on a long-lived transition, even the weak sidebands may be useful because the saturation intensity is low.

In the above expression, interference effects are neglected. Generally, a sideband has contributions with different path lengths through the ring. These add coherently at the output, and the power in different sidebands depends on the laser frequency and the mechanical stability of the ring. In principle, the coherent addition can be used to suppress unwanted sidebands. In our setup, however, we have not attempted to control interference effects, as we are primarily interested in the sharp cutoff at $-\nu_{\max}$ and the power distribution over several modes. The detailed shape is unimportant for our application. Furthermore, many frequency components are formed primarily from one particular path, and the intensity formula is therefore correct for these frequencies.

We have used self-heterodyne detection on a fast photodiode to observe the generated frequency comb. The self-heterodyne rf spectrum consists of two series of lines, with frequencies $|i\nu_1 + j\nu_2|$ $(i, j \geq 0)$ and $|i\nu_1 - j\nu_2|$ $(i, j > 0)$. The rf power in the (i, j)th pair of lines is proportional to

$$P_{ij}^{+} = \left[\sum_{n=-1}^{\infty}\sum_{m=-1}^{\infty}(I_{n+i,m+j}I_{n,m})^{1/2}\right]^2, \qquad (i, j) \geq (0, 0),$$

$$P_{ij}^{-} = \left[\sum_{n=-1}^{\infty}\sum_{m=-1}^{\infty}(I_{n+i,m}I_{n,m+j})^{1/2}\right]^2, \qquad (i, j) > (0, 0),$$

where P_{ij}^{+} and P_{ij}^{-} refer to the series that contains $+/-j\nu_2$.

As in the calculation of sideband intensities, interference effects are ignored. Every line contains contributions from many intermode beats that are adding coherently, and the details of the self-heterodyne spectrum change with laser frequency and alignment; however, a substantial fraction of the lines are insensitive to these effects. Figure 3 shows the calculated self-heterodyne spectrum for $T = 1.0$. Figure 4a shows the experimental spectrum, which qualitatively fits with our calculations. There are significant deviations primarily caused by losses in our mirrors, which make $T < 1$. It must be noted that optical misalignment and mode degradation upon deflection in the AOM's will generate sidebands of successively poorer spatial mode quality. These sidebands will beat inefficiently with each other, and the self-heterodyne spectrum will underestimate the intensity at large shifts.

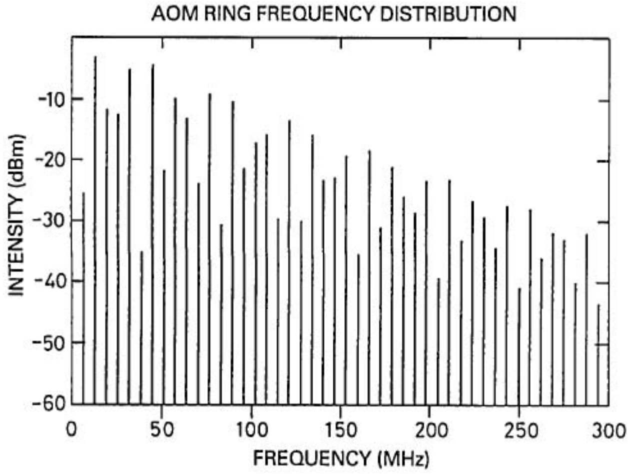

Fig. 2. Calculated frequency comb generated by a loss-free AOM ring.

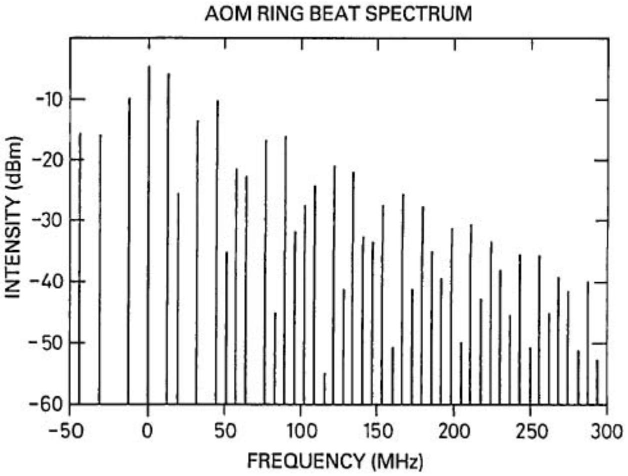

Fig. 3. Calculated self-heterodyne spectrum from a loss-free AOM ring.

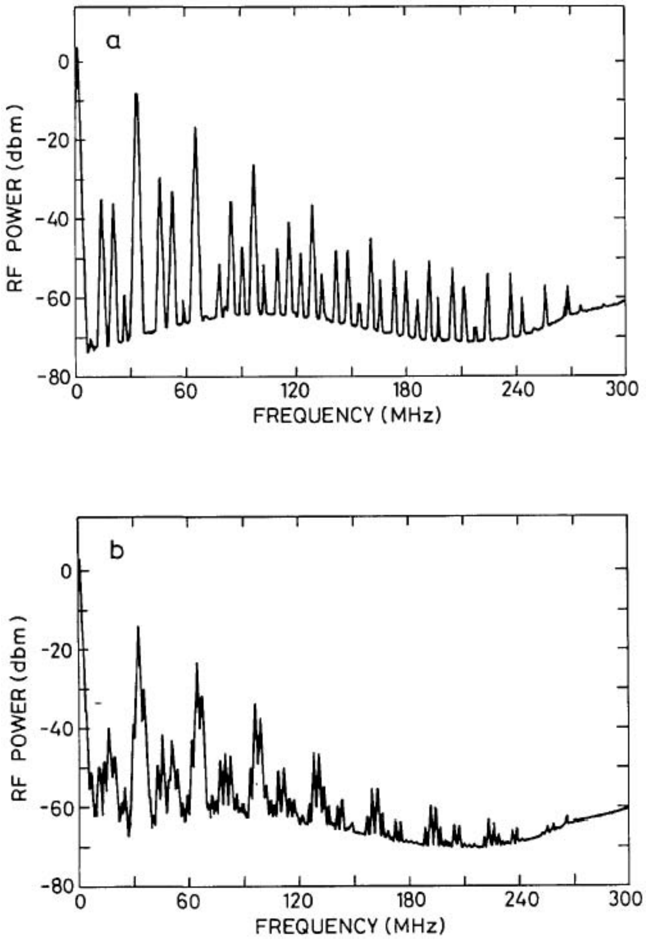

Fig. 4. a, Measured self-heterodyne spectrum from an AOM ring. b, Measured self-heterodyne spectrum from an AOM ring, with additional on–off modulation of the AOM's.

If the sidebands in the frequency comb from the AOM ring are too widely spaced, a further broadening can be obtained by switching the AOM's on and off at a fraction of the acoustic frequency. This modulates the distribution of intensity among the sidebands in the comb, which generates even more sidebands. Figure 4b shows a self-heterodyne spectrum, which is taken with on–off modulation applied to the AOM's.

One can of course choose any combination of AOM frequencies, if the lens sequence is chosen accordingly. Similarly, the direction of propagation of the acoustic waves can be chosen independently for the two AOM's in the ring. It is then possible to produce different symmetric or asymmetric even- or odd-spaced combs. In this way the setup may complement other modulation techniques in applications ranging from optical communication to atomic spectroscopy.

Our motivation for synthesizing the frequency comb described above originates in the need for laser cooling of hot, fast ion beams that are stored in heavy-ion storage rings.[5] The situation in a heavy-ion storage ring is different from that in an atomic beam or a trap. There are violent collisions and coherent plasma waves in the circulating ion beam, which make ions escape the laser-cooling force even late during the cooling process. This calls for a strong force, which has a sharp cutoff to one side (to generate low temperatures), and a long tail to the other side to recapture ions.

We have tested the setup for laser-cooling purposes by producing laser-induced acceleration of a stored 100-keV beam of $^{166}\mathrm{Er}^{+}$. The beam was stored in ASTRID, a storage ring for heavy ions, at the Institute of Physics, University of Aarhus. For laser cooling we used the weak $J = 13/2$ (ground state) to $J = 15/2$ closed transition at 5416 Å. Owing to a large Doppler broadening, a single-frequency laser interacts with only 10^{-3} of the total Doppler profile despite the fact that it has much more than 10^{3} times the saturation intensity. Using the AOM ring increases this fraction and significantly enhances laser-induced acceleration. The velocity change of the ion beam is estimated from the change in ion revolution frequency as measured with a Schottky pickup. With a single-frequency laser, the laser-induced velocity change during the 2-min storage time of the beam is hardly measurable. With the AOM ring setup under identical circumstances, laser-induced acceleration is enhanced by a factor of 5 (+9, −3), to the point of making the velocity change easily observable.

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