Quantum Information Processing in Optical Lattices: Cold Atomic Qubits in a Virtual Crystal of Light

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I. Introduction

Every student of modern physics comes away from their first course on the subject with the same unsettling feeling – quantum mechanics is just plain weird. Electrons are both particles and waves, wave functions "collapse" when they are observed, nonlocal correlations cannot be explained by a local realistic theory, "indivisible" photons simultaneously take two paths in an interferometer, or in Schrödinger's most extreme brain-twister, cats can be forced into a virtual purgatory, being both dead and alive. We are generally taught to just accept these paradoxes and move on. Quantum theory has been put to test and has predicted the proper statistics for every experiment from the lowest to the highest energies in the universe. It explains the conductivity of silicon and the scattering cross sections of elementary particle collisions in our most powerful accelerators. Reconciling the bizarre aspects of quantum theory is better left for the philosophers. Scientists and engineers should get on with more pragmatic work.

If we're taught one thing as scientists, it is never to stop asking, "why;" "what does it mean;" "if I push that line of reasoning to its logical conclusion what are the implications?" It turns out that implications of quantum weirdness are of profound pragmatic significance. Quantum mechanics is a theory of probabilities, and probabilities in modern Bayesian statistics are statements of information – predicting outcomes given our prior knowledge – what we know and what we don't know. Following this line of reasoning to its logical conclusion we are lead to a radically new idea – *quantum information*. The lesson here is one we were taught by the late Rolf Landauer, "information is physical". The laws of information processing cannot be divorced from the physics of the devices that carry out the tasks.

Contrary to common belief, quantum theory is not a paler version of pristine classical physics, filled only with pesky uncertainty and uncontrolled random behavior. Quantum information processing (QIP) opens the door to tasks which are otherwise impossible. Examples included perfectly secure secret-key distribution based on the principle of information-gain vs. wave function disturbance, and improved efficiency for complex communication tasks based on the principle of quantum entanglement and teleportation. Perhaps most exciting is the prospect of new computers which can implement algorithms unfathomable with the worlds most powerful "classical" supercomputers. Peter Shor's algorithm to factor numbers into their prime constituents is an example of this sort; it was the spark that caused the major explosion in quantum information science research that we see today. What lies ahead on Moore's roadmap of ever shrinking microprocessor components is not just a tinier version of devices where switches open and close valves for classical currents, but a broad new principle based on quantum theory.

What will it take to unleash the great power of quantum information? The most general task requires input of classical information into a quantum system, processing that quantum system in some way, and then reading out the result (Fig. 1). These tasks involve state preparation, quantum control, and measurement. Much progress has been made on these fronts, especially in atomic-molecular-optical systems. The quantum optics community has for years been engaged in experimental studies at the foundations of quantum mechanics, based on the preparation, manipulation, and measurement of individual photons and atoms and more generally, precision spectroscopy. These techniques are now "online", ready for application to QIP. Indeed, some tasks, such as quantum key distribution, are close to practical reality based on advances in photon sources and detectors.

Given this progress one might imagine that a quantum computer is around the corner. Unfortunately, that is far from true. Full QIP, such as quantum computation, requires quantum control and measurement of a *many-body* system. This is a daunting task. It is tantamount to engineering our own version of "Schrödinger's Cat". Luckily, quantum information theorists have taught us that it is not insurmountable. The control problem can be decomposed into basic building blocks known as quantum logic gates, in analogy to classical Boolean circuits. If state preparation, gate operation, and measurement can be performed with sufficiently fidelity, QIP is robust, secure from the ravages of errors through special quantum-error-correction protocols. Nonetheless, the gap between the fault-tolerant thresholds and the state of the art in experiment is huge. Further progress requires new implementations and experimental tools for preparation, control, and measurement, along with new theory, to help build a bridge that can span the gorge



Fig. 1. Quantum information processing involves three crucial steps: (1) Preparing an input quantum state based on some classical information, (2) processing the system to some final state through the murky quantum world, (3) measurement of the output state.

Our research at the University of New Mexico and University of Arizona is focused on developing these tools. The system we study is the "optical lattice", ultracold atoms trapped in the microscopic potential wells created by the interference pattern of a set of intersecting laser beams. Atoms are effectively sucked into the nodes or antinodes of a standing wave (generally three-dimensional) in the same way that a polystyrene ball is sucked into the focus of a laser beam as used in "optical tweezers". The resulting trap is very tight due to the sharp gradients in the field that varies on the scale of the optical wavelength, but shallow due to the weak force acting on the small neutral atoms. The atoms must be ultra-cold (microKelvin regime) in order to remain in their virtual egg crate (see Fig.6). Fortunately, the tools of laser cooling provide a good starting point to prepare the system. With these trapped atoms we can work to prepare, manipulate, and measure them, thereby growing our quantum toolbox and build the bridge to QIP.

II. Quantum Control of Atomic Motion in Optical Lattices

Motion of a particle in a double-well potential has long been used as a paradigm for quantum control and quantum coherent dynamics. We have implemented a unique version of this basic system using cesium atoms trapped in a 1D lattice. This problem represents control of an ensemble of uncoupled atomic quantum systems; it is not intended to lend itself directly to quantum information encoding and processing which requires atom-atom coupling, but its successful realization involves manipulation and observation of spin- and center-of-mass degrees of freedom, in the presence of non-trivial interactions between them. This makes it a perfect training ground for the development of control and measurement tools needed to undertake the much harder challenge posed by few-atom control processes such as quantum logic. From another perspective the cold atom/optical double-well system is interesting because it is mesoscopic, in the sense that the characteristic action can be varied continuously across the quantum classical border (the range $0.1-10\hbar$), and therefore is subject to rapid decoherence if the lattice and its environment is poorly controlled. It is also *complex*, in the sense that the atomic wavepackets are spinors that represent highly entangled states of the atomic center-ofmass and external degrees of freedom, and that the coupling of the two leads to dynamics whose classical counterpart exhibits deterministic chaos. Last but not least, we have a series of powerful laboratory techniques available that allow us to prepare well defined pure states, subject them to controlled unitary evolution, and accurately measure the quantum state along the way.



Fig. 2. A one dimensional optical lattice with polarization gradients forming separated standing waves of right and left helicity, and trapping spin-up and spin-down states of an atom. The application of a transverse magnetic field couples the two states leading to double-well potentials. Tunneling is accompanied by a changing in the atomic spin.

The lattice potential is formed by two linearly polarized, counter-propagating beams whose polarization vectors are misaligned by an angle θ (see Fig. 2). The field can be decomposed into two circularly polarized standing waves of opposite helicity (σ_{1}) whose nodes are separated by $\Delta z = \lambda(\theta/2\pi)$. Cesium, like all the alkalis, has one valence electron with spin 1/2. Under appropriate conditions, the spin-up (down) state will be trapped near the nodes of the $\sigma_{_{+(-)}}$ standing wave. Now imagine adding a transverse magnetic field. Without the lattice, the atomic spin would Larmor precess about the field at the frequency $\Omega = {}_{B}B_{\perp}$. In our system, however, because the atom's internal state is correlated with different potential surfaces, spin precession is accompanied by motion of the atoms between the two wells. For small enough magnetic fields, and low enough energy, the atom must *tunnel* to get from one well to the other. This tunneling is accompanied by a rotation of the atomic spin. These two degrees of freedom become entangled. Thus, we can think of the spin as a "tunneling-meter". We measure the spin at different times, for many runs of the experiment, using a Stern-Gerlach apparatus. The oscillation of the atomic magnetization is then evidence of coherent tunneling back and forth in the double-well. In the actual experiment, the real Cs atom is not spin 1/2 due to the hyperfine interaction with the nuclear spin. Our atoms are prepared in a state with total angular momentum F=4, giving a total of 2F+1=9 sublevels. Nonetheless, the physics of the full system has the same qualitative features as the simple spin-1/2 model.

To observe coherent tunneling we first prepare a dilute vapor of ~ 10^6 noninteracting Cs atoms, each in an initially localized quantum state, say the left ground state $|\psi_L\rangle$, and then observe the subsequent quantum evolution of the ensemble. Figure 3 shows a typical oscillation of the atomic magnetization as a function of time. Our data fits well to a damped sinusoid, and allows us to extract a good measurement of the tunneling frequency. We find generally excellent agreement between the measured frequencies and a bandstructure calculation of the energy spectrum, with no free parameters, over a wide range of experimental conditions. In this first experiment the tunneling oscillations dephase on a timescale of a few hundred microseconds, most likely due to variations in the tunneling frequency across the atomic sample. A next generation of the experiment is now underway, in which we hope to increase the dephasing time by an order of magnitude by better control over lattice beam and magnetic field inhomogeneities. With better homogeneity and larger detuning we hope to explore coherent dynamics on a time scale much longer than the Rabi period.



Fig. 3. Typical oscillation of the atomic magnetization as a function of time. The solid line is a fit to a decaying sinusoid.

At a quarter of the oscillation period, the system is in a superposition state reminiscent of Schrödinger's cat. The positive atomic magnetization is analogous to the undecayed nucleus correlated to the live cat (here, the left localized wave packet), and the negative magnetization is analogous to the decayed nucleus that triggers the poison vial and thus correlated to the dead cat (right localized wave packet). Using our Stern-Gerlach apparatus we can measure the individual magnetic sublevels of our atoms to reveal these correlations. Figure 4 shows a plot of the theoretically calculated squared wave function components at 1/4 of the tunneling period, with position and spin probability distribution projected on the walls. This clearly demonstrates correlation between spin and motion in the "Schrödinger-cat" state, with depressed probability in the "classically forbidden zone". Comparing theory and experiment we see strong qualitative agreement and quantitative agreement to within experimental uncertainties.



Fig. 4. At a quarter period of the tunneling oscillation the atomic state is a Schrödinger Catlike superposition, where the internal and external degrees of freedom are strongly entangled. This figure shows the position dependent probability density for each of the nine magnetic sublevels of the F=9 hyperfine ground state of ¹³³Cs. The marginal probability distributions in position or spin are shown in shadow, projected on the wall. In addition, we show for comparison the experimentally determined *m*-state distribution, measured with a Stern-Gerlach apparatus.

III. Quantum Control of Many Atoms

III.A. Quantum Logic Gates

Quantum information processing borrows much of its language from classical information theory. In the latter, the fundamental unit of information is the "bit", short for "binary digit" 0 or 1. In quantum information, the fundamental unit is the "qubit", short for "quantum bit" $|0\rangle$ or $|1\rangle$. The logical basis $\{|0\rangle, |1\rangle\}$ are two orthogonal states of

a quantum system, for example, the spin-up and spin-down states of a spin 1/2 electron, or two well defined energy levels of an atom. However unlike its classical counterpart, the qubit can be in a continuum of states which have no classical description, the coherent superpositions $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$. Of course, when the qubit is measured we will find only one classical bit of information, $|0\rangle$ or $|1\rangle$, with probabilities $|\alpha|^2$ and $|\beta|^2$ respectively. Nonetheless, the quantum bit can in some sense store both $|0\rangle$ and $|1\rangle$ simultaneously, in the same way that the photon can take both paths in an interferometer. The interference of probability amplitudes is at the heart of the working of the quantum computer.

The quantum computer involves many qubits, at least one for every classical bit of information we want to process, and lots more when we include redundant encoding needed for error correction. The state of the many-body system will generally be a highly *entangled state*, of the sort behind the famous Einstein-Podolsky-Rosen paradox and Bell's theorem for the two-body problem. A state with N qubits will require 2^N probability amplitudes. This exponential explosion of Hilbert space, without an exponential explosion of physical resources (here, the number of qubits) is one of the key features that distinguishes quantum computers from classical analog wave computers (e.g. Fourier image processors). It allows us to interfere an *exponential* number of paths of our virtual interferometer. Each path can be a classical outcome (like the prime factors) which gets processed simultaneously. This picture of computation is known as "quantum parallelism" (see Fig. 5).



Fig. 5. Quantum Parallelism: A quantum computer acts like a multi-port interferometer, operating simultaneously on many classical outcomes. Shown here is a schematic of a three qubit system which can encode 8 possible outcomes (0-7). Interference between paths, driven by the quantum algorithm, leads to constructive interference for the desired answer to a computational problem (here 0), which can then be read out with high probability.

How does one run a quantum algorithm? Again, we borrow from classical information theory. A collection of classical bits can be processed thorough a Boolean circuit consisting of a collection of logic gates acting in sequence only on small subsets of bits. Examples of gates include the single-bit gate *NOT* and the two-bit gates *AND*, *OR*. An arbitrary transformation between input and output can be constructed from a small number of "universal gates". In QIP we have analogous constructs. Quantum logic gates map input states to output states (they are unitary transformations). Thus, the analogy to the classical *NOT* is the quantum map $NOT|0\rangle = |1\rangle$, $NOT|1\rangle = |0\rangle$. However there are a multitude of nonclassical single-qubit gates such as \sqrt{NOT} which maps $\sqrt{NOT}|0\rangle = (|0\rangle - i|1\rangle)/\sqrt{2}$, $\sqrt{NOT}|1\rangle = (|1\rangle - i|0\rangle)/\sqrt{2}$. It is a nice exercise to prove to yourself that $\sqrt{NOT} \circ \sqrt{NOT} = NOT$ (up to an overall negligible phase). Another

example is the Hadamard $H|0\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $H|1\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. Though both \sqrt{NOT} and *H* map the logical basis into 50-50 superposition, the relative phases make them quite different form each other. You can show yourself that $H \circ H = I$, the identity operator.

To deal with many-body control we will need a nontrivial two-qubit gate that *entangles* the two qubits. A nice example is the "controlled not" (*CNOT*) which applies a *NOT* on a target qubit conditional on the state of the control qubit. In the logical basis, treating the second qubit as the target, the truth table is $|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$, $|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$, $|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$, $|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$. In classical theory, this is the familiar truth table for *XOR*. But in the quantum world there are new possibilities. Suppose the control qubit is in a superposition state. Then after the gate, $(\alpha|0\rangle + \beta|1\rangle)|0\rangle \rightarrow \alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle$, which is an *entangled state*! Another example of a two-qubit gate which has no classical analog is the "*CPHASE*", $|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$, $|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$, $|1\rangle|0\rangle \rightarrow |1\rangle|0\rangle$, $|1\rangle|1\rangle \rightarrow -|1\rangle|1\rangle$. This seemingly innocuous operation puts a phase -1 on the $|1\rangle|1\rangle$ state, but it is anything but trivial. It too can produce entangled states. It is a good exercise to show yourself that the following circuit is correct: $CNOT(a,b) = H(b) \circ CPHASE(a,b) \circ H(b)$, with *a* and *b* labeling the two qubits.

This last relation is an example of an extremely important result in quantum information theory. A few logic gates can form a *universal set*. Arbitrary many-body transformations can be built out of a sequence of a few single qubit gates plus *one* entangling two-qubit gate, such as *CPHASE*. This greatly simplifies the task of implementation. Arbitrary many-body states can be built out of a sequence of single-body and two-body interactions! The single-body transformations can be performed today with great precision due to decades of advances in NMR, microwave, and laser spectroscopy. The new challenge is to design coherent *two-body* interactions, of the sort *CPHASE*. This is the task we have been exploring in the context of our optical lattices.

III.B. Entangling Dipole-Dipole Operations

Neutral atoms don't naturally interact with each other unless they're forced to through some kind of "collision". Collisions tend not to be controlled, the key word for QIP. The challenge then is to design a kind of controlled collision that achieves the two-qubit quantum logic gate. Our approach has been to consider a photon-mediated collision. Classically this is the electric dipole-dipole interaction. To see that this collision might be controlled, consider the following scaling argument. In the near field, the strength of the dipole-dipole coupling energy goes like $V_{dd} \sim \frac{d^2}{r^3}$. At the same time, an optically excited dipole can radiate spontaneous emission. The spontaneous emission rate is bounded, and goes like $\hbar\Gamma \sim \frac{d^2}{\lambda^3}$ (in energy units), where $1/\lambda$ is the radiation wave number. Spontaneous emission is generally an uncontrolled process and must be avoided for proper performance of the gate. Thus, a figure of merit for the gate can be defined,

$$\kappa = \frac{V_{dd}}{\hbar\Gamma} \sim \left(\frac{\hbar}{r}\right)^{3}.$$

We therefore see that if the atoms can be localized to distances small compared to the radiation wavelength, $\kappa >> 1$, they can be coherently coupled by a photon.



Fig. 6. Controlled collisions for quantum logic can be performed in a three dimensional optical lattice. Along the quantization z-axis, polarization gradients trap two different groupings of atoms, labels by their internal states. By rotating the polarization vectors, atoms can be brought together pair-wise for coherent interaction. In our protocol an additional laser field acts to excite entangling electric dipole-dipole interactions.

The optical lattice provides an ideal platform to put these ideas to the test. Atoms trapped near the bottoms of the potential wells are highly localized. Furthermore, atoms can be moved together pair-wise using the polarization gradients as in Fig. 2. We imagine then encoding a qubit in the internal hyperfine ground states of the atom (like those used in an atomic clock). An entangling quantum logic gate would then be performed as follows (Fig. 6). Atoms are trapped in a three dimensional trap consisting of linearly polarized standing waves along the three Cartesian axes. Rotating the polarization vectors along one axis, we bring an ensemble of atoms together in neighboring planes, ready for interaction. A separate "catalysis" laser excites dipoles. For appropriately chosen pulses the required phase is imprinted jointly on the two-atom target state, thereby achieving a *CPHASE*. Errors can occur in this gate due to spontaneous emission. If the gate operating time is Δt , the probability of spontaneous emission goes like, $P_{error} \sim 1 - e^{-\Gamma\Delta t}$. The gate operating time scales like $\Delta t \sim \hbar/V_{dd}$, so

$$P_{error} \sim 1 - \exp(-\hbar\Gamma/V_{dd}) \approx \frac{1}{\kappa}.$$

For a very large figure of merit, the error probability can be made to approach zero as required.

A more complete theoretical model involves a detailed study of the atomic/molecular energy level structure (when two atoms get close enough to couple, they look like a molecular dimer) and proper treatment of the quantized motional states in the trap. We have performed such a study and found excellent prospects for high fidelity operation in realistic scenarios. We are currently working to test these ideas in the laboratory. Of course, many technical challenges remain, including proper filling of the lattice, and individual addressing and measurement of the individual atomic qubits. We are confident these hurdles can be overcome once the more fundamental task of controlled collisions have been implemented, as is our focus.

IV. Summary

Information is physical. One cannot divorce the information processing capabilities of a device from the physics that governs the operation of the device. Rolf Landauer's realization has important consequences for the foundations of information theory and new possible technologies. This is particularly true when the devices are nanoscopic; the physics of the quantum world is so profoundly different from our "classical" macroscopic description that nanotechnology is likely to have elements unlike anything we've seen to date. Quantum information provides the potential for secure secret-key distribution, efficient communication, and new computational power beyond the hope of any device acting under the laws of classical probabilities.

Bringing the full theoretical promise of quantum information into the laboratory is a grand challenge. It requires quantum control of a many-body systems with a very large number of degrees of freedom. It demands a degree of isolation from the perturbing effects of noise and manipulation with a precision never before achieved. Nonetheless, the exciting prospects for new fundamental breakthroughs has sparked worldwide research efforts to implement quantum information processors in a variety of systems, ranging from superconducting SQUIDS, to single electron quantum dots in semiconductors, to trapped ions.

We at the Universities of New Mexico and Arizona have conducted theoretical and experimental research to implement QIP with trapped ultracold neutral atoms in optical lattices. This flexible system allows us to fully control the atomic external and internal degrees of freedom, by building on the strong tradition of precision spectroscopy in the atomic-molecular-optical community. In developing our toolbox, we have observed mesoscopic quantum coherence of Cs atoms tunneling in engineered double-well potentials. This system represents a clean arena in which study the basics of control and measurement. In addition we are working towards implementation of universal quantum logic based on controlled atomic collisions. These two-atom interactions have a strong kinship with control of a molecular dimer. This represents a new frontier in the manipulation of atomic-molecular-optical systems, with new possibilities for basic science and technologies.

Further Reading

For an overview of the field see *Quantum Computation and Information*, by M. A. Nielsen and I. L. Chuang (Cambridge University Press, New York, 2000). For a review of optical lattices, control and logic therein see:

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