Methods of measuring the focal length of the optical element

Yu Yan

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Tutorial
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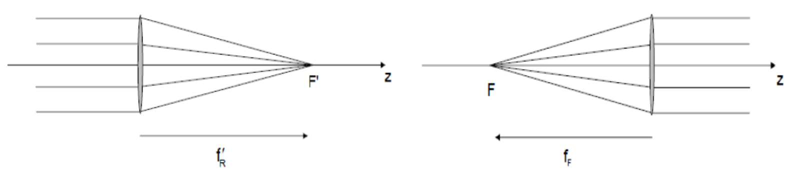
Introduction

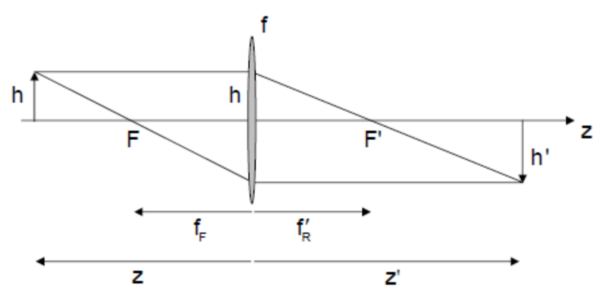
Focal length of optical elements is an essential parameter to optical engineers. In many applications, we need to get this parameter first then continue the following work. However, this parameter is not always known in advance or given in wrong value. In this situation, we need to find out or verify the focal length of the optical elements. This tutorial will discuss the basics of first order optics first and then introduce some basic methods used in practice.

Gaussian Equations

An object at infinity is imaged to the Rear Focal Point of the lens F'. Similarly, An object at front focal point is imaged at infinity. Shown as in the following fig.



Object position and its correspond image position is conjugate. The relationships between these two positions and the magnification can be determined from Gaussian Equation.



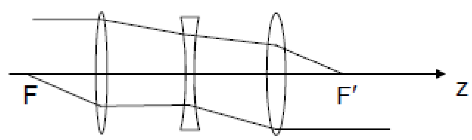
$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \qquad m \equiv \frac{h'}{h}$$

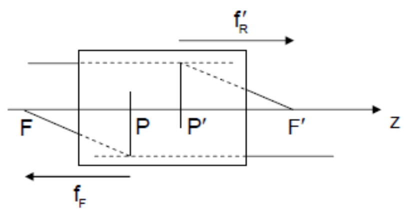
And z', z and f in this equation follow sign convention. Positive if the direction is from left to right and negative if the direction is from right to left. For height, positive if the direction points up, negative if the direction points down.

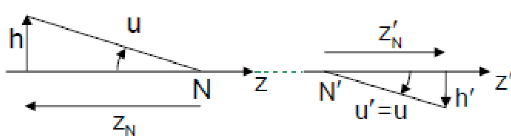
Cardinal Points

The cardinal points lie on the optical axis of the optical system. Each point is defined by the effect the optical system has on rays that pass through that point, in the paraxial approximation. The paraxial approximation assumes that rays travel at shallow angles with respect to the optical axis, so that Aperture effects are ignored: rays that do not pass through the aperture stop of the system are not considered in the discussion below.

The cardinal points are points that can be found in any optical systems and they have their corresponded cardinal planes which are front and rear focal planes, nodal planes and the principle planes. The focal planes are the planes at which all rays parallel to the optical axis in object space will come to focus in image space. The nodal planes are the planes of unit angular magnification. The principle planes are the planes of unit lateral magnification. The front and rear cardinal points are conjugate to one another.





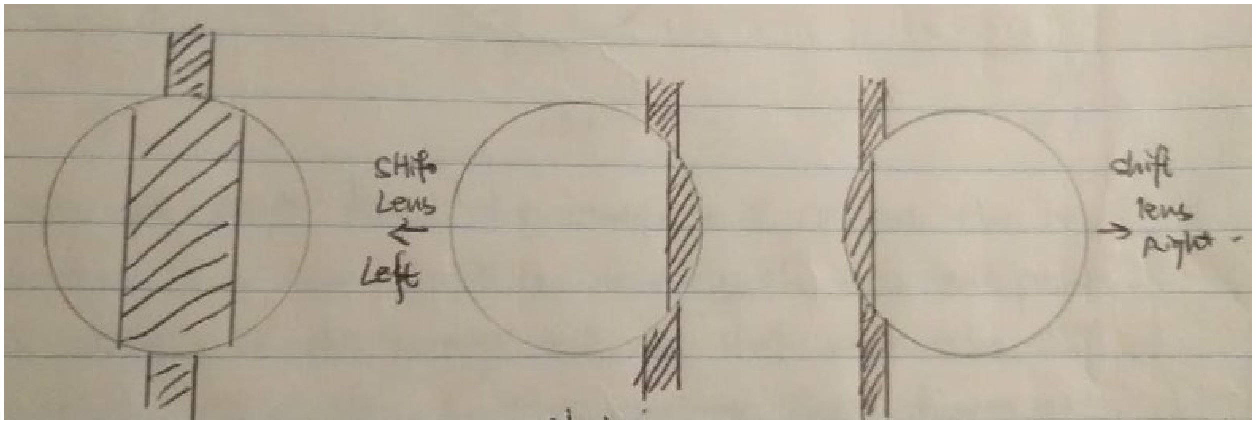


Methods 1. Quick Estimate

One easy way to get a rough lens focal length is to hold it under a ceiling lamp and use the testing optical element to form an image of the ceiling lamp. The optical element is moved vertically back and forth until a sharp image of the ceiling light is projected on the floor (table surface or other things that can be used as a screen). The distance between this element and the floor is focal length. The distance we got in this method is actually the back focal distance. But since this method is always applied to thin lens and the result is just an approximation, we can directly regard it as focal length. The result can be just used to get the concept of the optical elements. More precise measurement need to be done to get the better parameters.

Methods 2. Neutralization Test

The Neutralization test can be used to estimate the power (focal length) of an unknown lens. The basic concept can be illustrated as

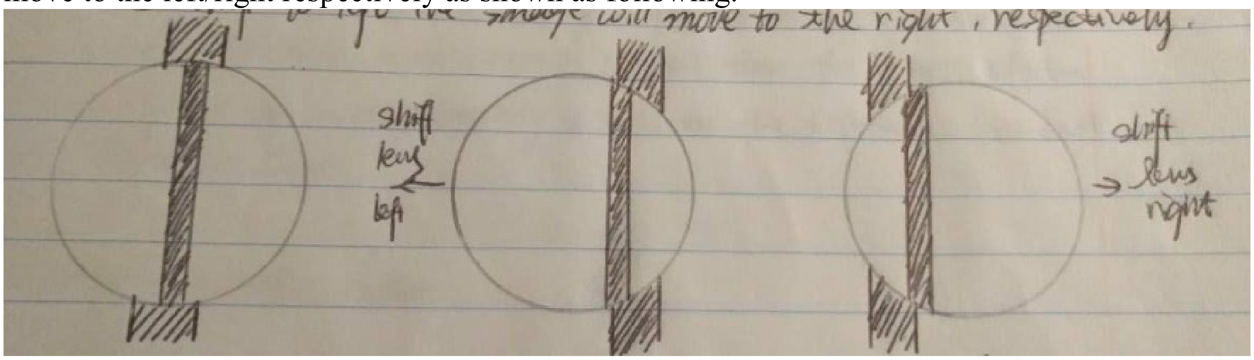


For a positive lens

Place a positive lens above a stick. The image of the stick will be magnified as shown above. If we shift the lens right or left horizontally. The image of the stick will move with the opposite direction as shown above. When the lens shift to right, the image will move to the left. When the lens shift to left the image will move to the right, respectively.

For a negative lens

Place a negative lens above a stick, similar to previous section. The image of the stick will be diminished and if we shift the lens left or right horizontally, the image of the stick will move along the same direction. That's to say when we shift the lens to the left/right, the image will move to the left/right respectively as shown as following.



For zero power lens, the image will reflect the real situation. the image will coincide with the real position of the object and the image will not shift with the shift of the lens.

We can use this phenomenon to exam the power of an unknown lens by directly put an unknown lens against one calibrated lens and process the neutralization test. Based on the phenomenon we get, we can change different reference lens till a zero net power combination is present. The power of the unknown lens approximately equals to the reference lens with a sign change.

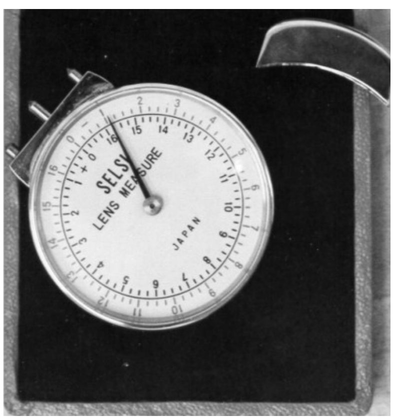
The limitations of this method is that 1) we may not find have a reference lens that match the power of testing lens. 2) this method is also for approximation.

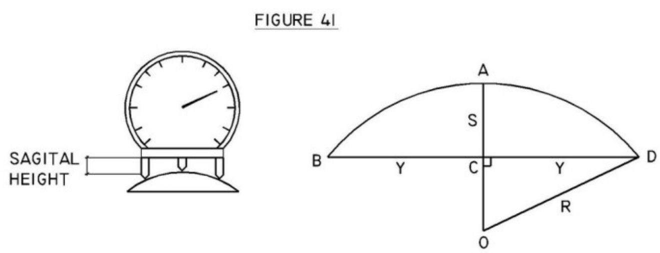
Methods 3. Geneva Gauge

A Geneva Gauge can be used to measure the focal length of a thin lens. It consists of three steel prongs, the outer two of which are fixed and inner prong is free to move along its axis. The middle prong is connected to an indicator gauge through a mechanical linkage.

In use, the gauge is pressed onto one surface of the lens to be tested. the surface power is directly read out from the dial.

The gauge is shown as





The quantity actually measured by the gauge is the sag of the surface. The dial of the gauge is calibrated under the assumption that the refractive index of the glass is 1.523. the power of the surface is given as

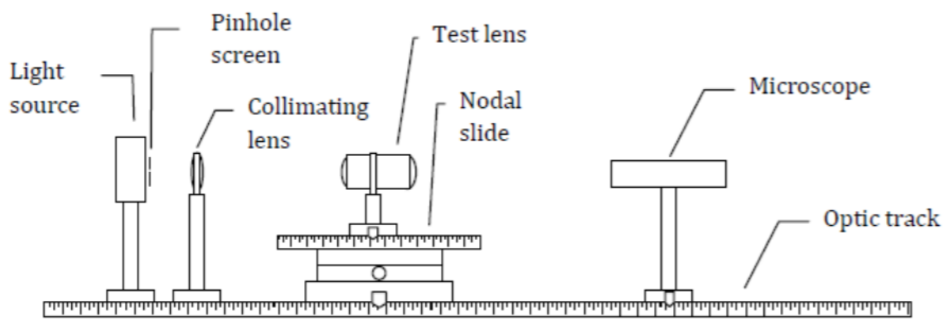
$$\phi = \frac{n-1}{R} = \frac{0.523}{R}$$

When the refractive index of the lens is different from 1.523. the real focal length can be calculated be the following equation.

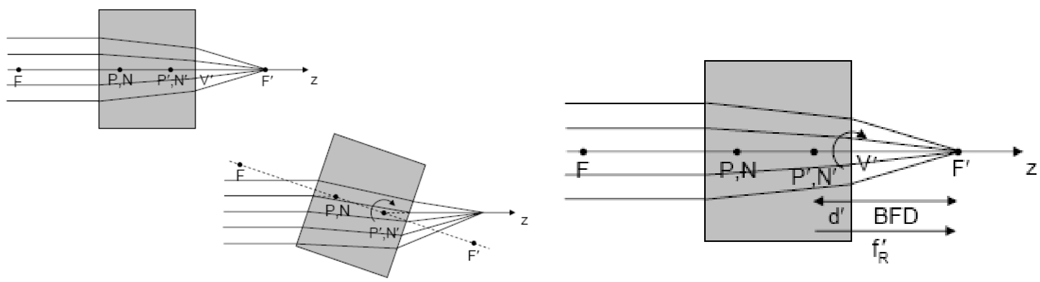
$$f_{true} = \frac{0.523}{n_{lens}-1} f_{measured}.$$

Methods 4. Nodal Slide

A classic nodal slide setup is shown as

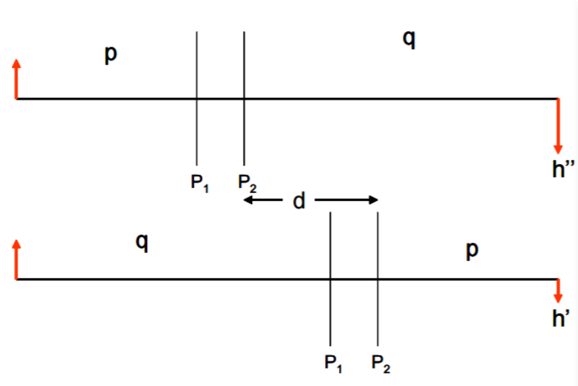


Firstly, we need to adjust the collimation slide to provide parallel light. And then adjust the lens to be measured and observing objective (microscope in the figure) in the same line that we can locate the light point into the objective. Look through the objective and find the smallest light point position and note down the scale on the rail. After determine the focal point position, we need to determine the position of the surface and get the back focal distance. Next, we use the change of rotation axis to determine the distance between the surface and principal/nodal point. By using these two numbers, we can get the properties of the lens measured.



$$f = f'_R = BFD - d'$$

Methods 5. Reciprocal Magnification Derivation



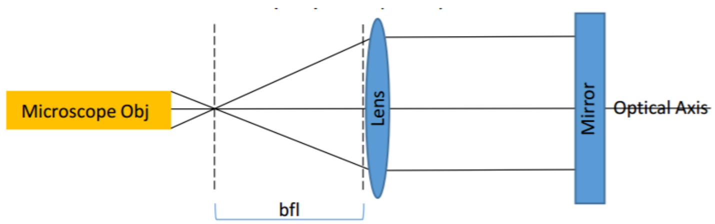
When taking this method, we need to first find one conjugate positions of object and its image. Determine the distance p and q shown in the fig. and then fix the object and image and slightly shift the optical element to find another conjugate configuration and record the distance the element move and then use these three measurement we can calculate the focal length of the optical element.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad \frac{1}{f} = \frac{m-1}{d}\left(\frac{m+1}{m}\right) = \frac{m^2-1}{md}$$

$$p = \frac{d}{m-1} \qquad q = \frac{d}{1-\frac{1}{m}}$$

$$f = \frac{d}{m-\frac{1}{m}}$$

Methods 6. Point Source Microscope(PSM)



The whole setup is shown above. To exam the back focal length (for thin lens, we can regard the back focal length as its focal length), we need to place a mirror behind the lens as shown. Adjust the lens till we observe the smallest spot light in monitor. Use the position of the lens we can get the back focal length of the lens.

References

Fischer, Robert and Tadic-Galeb, Biljana. Optical System Design. New York: McGraw-Hill, 2000.

Greivenkamp, John. Field Guide to Geometric Optics. Bellingham:SPIE, 2003.