# A review of slit diaphragm flexures for optomechanics

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## 1 Introduction

This technical report is a review of Vukobratovich's paper [1], "Slit diaphragm flexures for optomechanics," which discusses the basic design considerations when using a diaphragm flexure for motion control. I will describe the important results from this paper, along with a small amount of background information on flexures. With the results, I will then discuss a few examples of how this type of flexure design may be utilized with a particular focus on the field of optomechanics.

## 2 Background

In order to understand the utility of a diaphragm flexure, a basic knowledge of flexures in general is required. A flexure is an object that controls motion in specific directions through the bending of a material. Typically, flexures can be thought of as a small beam, where it is compliant in the direction of motion. Flexures are used when small, precise motions are required (e.g. 1 mm of travel with an accuracy of micrometers). This is in contrast to rolling element bearings, where the relationship between friction and torque is nonlinear over small motions (i.e. unpredictable motions when a small displacement is required). The bearing design also suffers from hysteresis and rapid wear when small repetitive motions are made. Flexures are a good solution when precision is important, but they also require significantly more physical space to produce the same amount of motion as a roller bearing.

## 3 Diaphragm Flexure

A flexure design form that reduces the volume of the required mechanics is a diaphragm type. As the name implies, the shape of the flexure is a diaphragm, or a piece of material with radial fixtures instead of linear as is true for the traditional beam-like flexures. In this report, we will focus on linear and gimbal motions created with the diaphragm flexure. An example of this kind of flexure operating in these

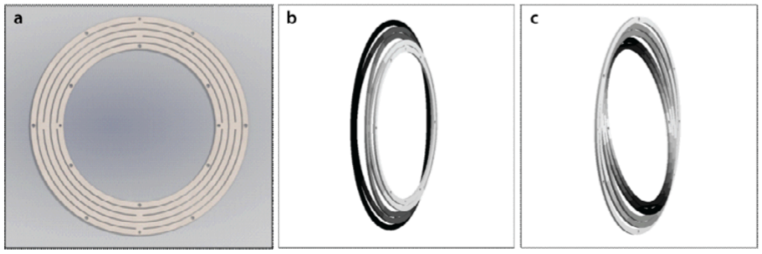


**Figure 1:** Example of a diaphragm flexure (a) shown with axial motion (b) and angular motion (c) [3].

modes is given in Fig. 1. These are slit type diaphragms, where a single flexure consists of two strip flexures combined in series. Another type is the corrugated diaphragm, which has well understood properties.

## 3.1 Axial Motion

For a slit type diaphragm where we are interested in obtaining motion along the axis of the diaphragm, we want to determine the relationship between the required force along the axis for a given displacement. To do this, we first consider this relationship for just a single flexure,

$$F_i = \frac{6EI\delta}{L^3}, \tag{1}$$

where $F_i$ is the applied force, $E$ is the elastic modulus, $I$ is the cross section moment of inertia, $L$ is the length of the flexure, and $\delta$ is the motion of the flexure. When we now consider the whole slit diaphragm, with $n$ individual flexures spaced tangentially by length $c$ at radius $r_f$, we can determine the length of each flexure as,

$$L = \frac{2\pi r_f - nc}{n}. \tag{2}$$

The moment of inertia $I$ is calculated by,

$$I = \frac{bh^3}{12}, \tag{3}$$

where $b$ is the width of the flexure in the radial direction, and $h$ is the thickness of the flexure in the axial direction. Combining Eq. 1–3, the force required to move the center of the diaphragm flexure a distance $\delta$ is,

$$F_A = \frac{n^4 Ebh^3\delta}{(nc - 2\pi r_f)^3}. \tag{4}$$

The stress that is generated by displacing the center is found to be,

$$\sigma_A = \frac{3E\delta n^2 h}{2(nc - 2\pi r_f)^2}. \tag{5}$$

If we then calculate the stiffness of the diaphragm flexure, we find that it is has relatively poor rotational stiffness. Therefore, we can use two axial motion flexures in tandem to produce the desired motion.

### 3.2 Rotational Flexure

Repeating a similar procedure to analyze the rotational flexure, we find that the angle of rotation produced gimbal flexure is,

$$\theta = \frac{TL}{2Gbh\left(\frac{1}{3} - 0.21\frac{h}{b}\left[1 - \frac{1}{12}\left(\frac{h}{b}\right)^4\right]\right)}, \tag{6}$$

where $T$ is the applied torque, $L$ is the length of a single flexure, $G$ is the shear modulus, $b$ is the width of an individual flexure, and $h$ is the axial thickness. The stress induced in the flexure is therefore,

$$\tau = \frac{T(3b + 1.8h)}{2b^2h^2}. \tag{7}$$

This type of flexure is usually used on its own in order to maintain compactness in the axial direction. Both flexure designs are low cost and compact compared to single strip type flexure designs. It should be noted that the equations given above are approximations, so they should be used to start a design but further analysis (e.g. FEA) should be done to verify important parameters.

## 4 Diaphragm Flexure Applications

The diaphragms described above can operate in a passive or active mode. A passive mode can be thought of as an automatic positioning system such that when a force is applied, the system reacts and tries to realign itself. An active flexure is driven by some force to obtain motion from the flexure.

### 4.1 Driven Flexures

In order to create motion, we need to apply a force to the object we want to move. The flexure then ensures that this motion is what we want, and in the direction we want. To apply a force, a piezoceramic (PZT) driver may be used [2]. In this situation, careful analysis of the natural frequency of the flexure and driving PZT must be carried out in order to not excite a undesirable resonant situation. The natural frequency of the system is,

$$\omega = \frac{1}{2\pi}\sqrt{\frac{K_{equiv}}{M_{equiv}}}, \tag{8}$$

where $K_{equiv}$ is the system equivalent stiffness, and $M_{equiv}$ is the system equivalent mass. These parameters will vary for each situation of mounting the PZT on the flexure as well as for the form of the flexure.

## 4.2 National Ignition Facility

The diaphragm flexure designs are very good for applications that require large displacements, simplified assembly, and excellent sensitivity. For example, they have been used in a design for the National Ignition Facility (NIF) in the Gamma Reaction History diagnostic [3]. In this design, a turning mirror is used for system alignment and it needs a gimbal mount. The mount needed to have a small profile, and only provide two angular degrees of freedom. Using a simple hex wrench to turn $1/4-28$ setscrews, a sensitivity of $3.5\mu$m is obtained. This translates to a angular sensitivity on the order of 0.1 mrad about the two axes. Such a simple flexure design yields very good adjustment and sensitivity all while being low cost and low volume.

# 5 Conclusion

In discussing slit diaphragm flexure designs, we have detailed how to perform preliminary calculations for a design, driving the flexure or using it as a passive element, and showed an example of using diaphragm flexures in a precision application. The main benefits of using a diaphragm flexure over typical single strip flexure are the packaging volume reductions. Using the diaphragm type, we can achieve the same range of motion but with a much more compact design that is suitable for optomechanical applications. The equations that we presented are used to start a design, choosing material properties and basic flexure properties such as thickness, number of flexures, and diameter. Further analysis to solve for the resonant modes of the flexure should be carried out if the flexure will be driven, or if it will experience environmental vibrations. Therefore, with this information we believe that diaphragm flexures are best used in designs with rotational symmetry, especially those that will be contained within a barrel assembly. Furthermore, designs that require small and precise motion (e.g. focusing optics) will also benefit significantly from this design form.

# References

[1] D. Vukobratovich, J. P. McNiven, and L. Sinclair, "Slit diaphragm flexures for optomechanics," SPIE **2542**, 2–10, (1995).

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[3] B. Cox, M. Kaufman, "Design considerations of a slit diaphragm flexure used in a precision mirror gimbal," Proc. SPIE **8125**, 81250R-1–11, (2011).