A Brief Review of Non-deterministic Dynamics of a Mechanical System

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Review by Phil Scott

In order to write a brief review of this paper I'd like to simply write a review of each section including the relevant findings and topics discussed. This paper provides some insight into the complex and chaotic nature of systems which we often think of as determinant. This specifically addresses a chaotic behavior in friction forces and moments.

Introduction: The authors assert that most mechanical systems can be thought of as simple deterministic systems. Apply a force, get an acceleration, a deflection, or a moment. At the surface this is mostly true, but as one dives deeper into the analysis systems becomes very complex quite rapidly. This is true of almost any system. Think of the details of friction for example. The first order simply says that the friction force is the normal force times some coefficient. While this simple explanation is fairly accurate, the details depend on the surface characteristics, temperature, lubrication, motion, and a variety of other variable which can complicate the system.

From a mathematical perspective, if there is some discontinuity in the differential equations then there isn't guaranteed to be a unique solution. As an example the author talks about a sliding block. Once it reaches its rest state, all information about how it got there is lost. There is no unique trajectory for the block to reach this state. Another example is chalk being pushed along a chalkboard. The chalk will skip along the board in an unpredictable pattern. This presents an ambiguity in future motion as the motion of the chalk tip can't be determined. This paper presents a physical case where both forward and backward time ambiguities are present and chaotic behavior is observed.

Mechanical Model: The Experiment consists of two plates and a wheel as shown in Figure 1. The bottom plate rotates at a fixed speed while the top plate is linked to the bottom through some friction coefficient and rotates about the same center point. A wheel contacts the bottom plate and is held in

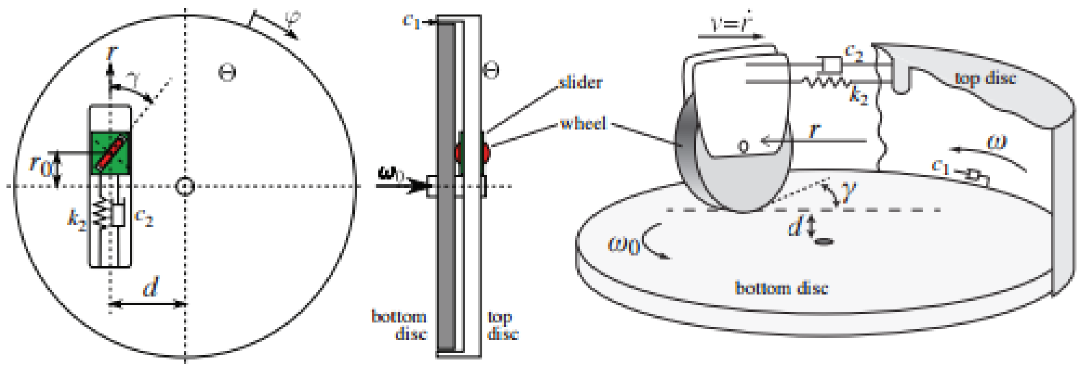

Figure 1: A layout of the experiment. The wheel is shown in red, the slider in green.

place by a slider with a single degree of freedom attached to the top plate. The wheel is set at an angle γ to the slider direction. The friction force will act perpendicular to the wheel axis and will be balanced by a moment created by the friction. The slider displacement r and the rotation speed ω are the important variables.

The equations of motion are given as are the equations for the force and the moment. Relative velocities of the wheel are given for the rolling direction and the lateral direction and the discontinuity in the system is discussed. The discontinuity leads to a set of points which defines a switching surface. The switching surface contains regions that cause the wheel to slip for a time (sliding region) and other regions which will cause the slipping to stop spontaeneously (escaping regions). Points of interest for this topic are where these regions intersect at a point called a singularity.

Singularity of the Friction Force: The friction force and moment balance each other with respect to the pin about which the two discs rotate. Parameters in this section are given a mathematical form which, when solved, shows that there can only be 4 or fewer singularities on the switching surface. Only one of the singularities will create the non-deterministic effect this paper sets out to show and is called a Teixeira singularity.

Teixeira Singularity: The conditions for the singularity are laid out in this section. One possible condition (and the one that is most interesting to this paper) is that conditions near the Teixeira singularity can

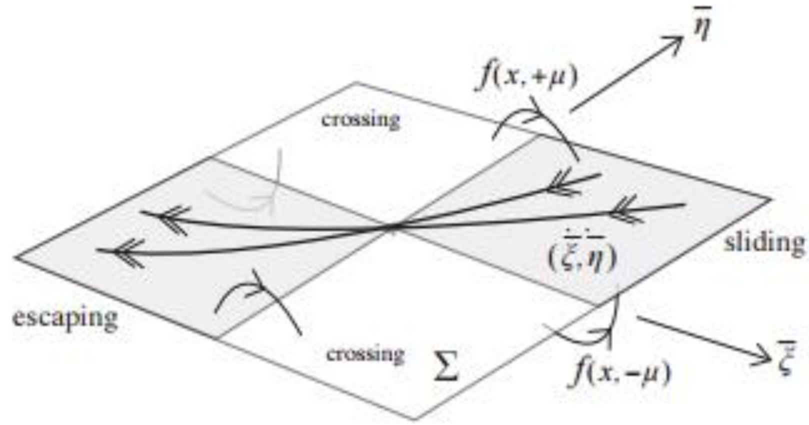

Figure 2: Two trajectories that pass through the singularity between sliding and escaping regions. It can't be determined which trajectories in the escaping region came from which ones in the sliding region.

cause the trajectories all to cross the regions from sliding to escaping at the same point which leads to an ambiguity in what the initial trajectory was and what the future trajectory will be. This means that the system is no longer deterministic, but that the trajectory along the surface will evolve into one of infinite possible trajectories after passing through the singularity. There are cases where the trajectory will loop through this singularity repeatedly, but the trajectory cannot be determined beyond knowing that it lies in a set of solutions. Figure 2 helps illustrate two trajectories passing through the singularity along the switching surface.

Simulations of the Mechanical Model: Given the equations laid out in this paper, there are large regions of parameter space in which the singularity can be found and where chaotic behavior is possible. One set of such conditions was chosen and the trajectories of the wheel were analyzed. It is shown that trajectories that enter the singularity result in a whole series of possible solutions which all find their way back to the singularity.

Special Cases: Singularity Without Non-determinism:

1) If $\gamma=0$ from Figure 1, then the wheel is aligned with the slider. This causes no slipping of the wheel if the top plate and bottom plate rotate in unison. A singularity can be found, but it is well defined and non-determinism goes away under this condition.
2) If $\gamma=\pi/2$ then the wheel is mounted perpendicular to the slider. Here the wheel will slip, but there is no singularity present, so the system is deterministic.

Closing Remarks: The authors present the conditions for their simulation to model reality by stating "The essential characteristics of the model are a coupling of linear and angular motions, and the sudden change of force that occurs in dry friction." Anywhere where these conditions are present there can be non-deterministic behavior of the mechanical system. They go on to state that these conditions do not address wheel shimmy or flutter as the physical characteristics of the problem are different. Lastly the authors state that trajectories cannot be followed through the singularity. One could choose the trajectory based on statistics, but there's no obvious way to choose an appropriate distribution. Likewise the singularity could be smoothed by applying various functions, but then the behavior is largely influenced by the details of smoothing, but that these are a set of solutions contained in the discontinuous model used in this paper. Appendices follow and will not be reviewed here.

My Conclusions: The setup seems somewhat contrived, but I did find it interesting to see chaotic behavior in a simple mechanical system. I would like to find examples of this in common practice, but can't think of any off the top of my head. I think that the author's showed their point well and that the paper was well written if a bit mathematically intimidating.