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Design of reflective projection lens with Zernike polynomials surfaces ☆

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Abstract

Wide view angle reflective projection lens by using Zernike polynomials surfaces are designed and presented. Detail description and analysis of Zernike polynomials surfaces are provided. Wave front aberration is analyzed and used to design the Zernike polynomials reflective lens. Two types designs of reflective projection lens with Zernike polynomials mirrors are presented, lenses are telecentric in object space. Lens's optical performance are analyzed, under the condition of F-number = 2.5 and field of view $2a = 130°$, both of design's modulation transfer function are over 55% at 60 lp/mm, moreover, the design of three Zernike polynomials mirrors has simpler structure and higher performance. The MTF is over 60% at 100 lp/mm and distortion is less than 2%, which can satisfy the requirement of the high definition projection display system.

Keywords: Optical design; Projection lens; Zernike polynomials; Reflective

1. Introduction

Projection lens is one of most important parts in projection display system, such as LCD/DLP/LCoS projector. Since projection lens affects the definition and optical performance of the whole projection display system greatly, the design of projection lens has become the focus of the research in the projection system. At present, the trend of projection lens is towards to short focus, wide angle of view, large F-number and high definition [1], which can satisfy the requirement of short projection distance and high definition of system. Normally, the projection lens is often designed by refraction lens, however, when refraction projection lens is used in short focus and wide field angle, the various of aberration is difficult to reduce because of rapid increase of chromatic aberration and axial coma aberration owing to large field of view and F-number. Moreover, for refractive projection lens, chromatic aberration inevitably occurs due to characteristics of optical glass. And, when a great mount of refraction lenses are introduced, manufacturing problems in terms of cost, weight, and surface accuracy may occur. Thus, the design concept of reflective projection lens is accepted by more and more lens designer. Many reflective projection lens design are reported [2,3]. Ogawa studied a reflective projection lens layout with four aspheric mirror [4], the detail design and manufacture performance were put forward. These designs have proved that the reflective projection lens by using aspheric surfaces is successful in projection lens. Recently, to simply the structure of projection lens and improve the optical performance, some designs are reported by using other optical element such as fresnel plate [5,6] and so on.

Zernike polynomials are widely used for specifying and balancing of aberrations [7]. The main properties of Zernike polynomials are that they are orthogonal and normalized over a unit circle, and that the first two non-trivial components represent tilt and defocus, thus Zernike polynomials can be used for fringe analysis [8], widely used in investigation of atmospheric turbulence and in adaptive optics [9,10]. Weighting of a pupil can also be expressed in terms of Zernike polynomials. However, using Zernike polynomials to design projection lens is seldom reported.

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In this paper, by using Zernike polynomials mirrors, wide view angle off-axis reflective projection lens are designed. The detail description and performance of Zernike polynomials surfaces are analyzed. Wave front aberration is used to design the Zernike polynomials reflective lens. Two types of reflective projection lens are designed with Zernike polynomials mirrors, one layout is three aspheric mirrors + 1 Zernike polynomial mirror, another is three Zernike polynomials mirrors, lenses are telecentric in object space. Projection lens's optical performance are analyzed, under the condition of F-number = 2.5 and angle of view $2a = 130°$, both of projection lens's MTF(Modulation Transfer Function) are over 60% at 60 lp/mm on magnification side, moreover, the MTF of three Zernike polynomials mirrors is over 60% at 100 lp/mm and distortion is less than 2%.

2. Zernike polynomials surface and aberrations

Eq. (1) shows the expression of Zernike polynomials [7]

$$Z(x,y) = \frac{cr^2}{1+\sqrt{1-(1+k)c^2r^2}} + \sum_{i=1}^{M} a_i r^{2i} + \sum_{i=1}^{N} A_i E_i(x,y) \tag{1}$$

where, $Z(x,y)$ is Sag, k is conic constant, c is the curvature of the reflective surface, r is the height above the optical axis, then $r^2 = x^2 + y^2$, a_i are, respectively, even aspheric coefficients, A_i is coefficient of Zernike polynomial, polynomials can be written,

$$\sum_{i=1}^{N} A_i E_i(x,y) = A_1 x^1 y^0 + A_2 x^0 y^1 + A_3 x^2 y^0 + A_4 x^1 y^1 + A_5 x^0 y^2 + \cdots + A_N x^{j-k} y^k \tag{2}$$

The number of polynomial item is $N = \frac{1}{2} j(j+1t) + k$

Generally, A_i are called the Zernike coefficients. Zernike polynomials also can be expressed in polar coordinates (ρ, θ)

$$Z(x,y) = \frac{cr^2}{1+\sqrt{1-(1+k)c^2r^2}} + \sum_{i=1}^{N} B_i F_i(\rho,\theta) \tag{3}$$

where $0 \leqslant \rho \leqslant 1$, $0 \leqslant \theta \leqslant 2\pi$.

The Zernike circle polynomials are unique in that they are the only polynomials in two variables ρ and θ. The orthonormal Zernike polynomials are usually used to describe the Seidel aberrations of optical system.

As description in Eq. (1), compared with even aspheric surface polynomials, Zernike polynomials have more parameters, it means Zernike polynomials have more freedom for aberration correction in optical design.

Shown in Fig. 1, consider a ray reflected at point $\mathbf{Q_R}$ by Zernike polynomial reflective surface **Z**, the direction of incoming ray and reflective ray are **In** and **Out**. According to the reflection law, the relations among **In**, **Out** and the surface normal unit vector **N** can be established,

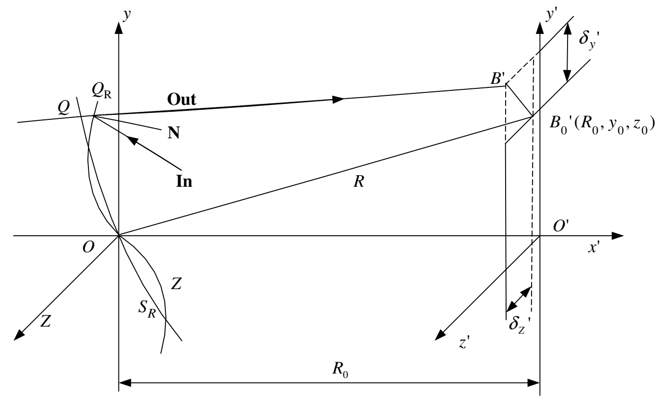

Fig. 1. A ray reflected by Zernike polynomial reflective surface.

$$\mathbf{Out} = \mathbf{In} - 2(\mathbf{N} \cdot \mathbf{In})\mathbf{N} \tag{4}$$

Introduce two sets of mutually parallel Cartesian rectangular axes, with origins at the axial point **O** and **O′** of the surface and image plane, and with the x directions along the axis of the system, if $\mathbf{B}_0'$ is the Gaussian image, establish a reference sphere S_R which is centered on the Gaussian image point $\mathbf{B}_0'$ and passes through **O**, **Q** is the points of intersection of the ray $\mathbf{B'Q_R}$ with the reference sphere S_R, then:

$$\begin{aligned}\mathbf{OQ_R} + |\mathbf{Q_R B'}|\mathbf{Out} &= \mathbf{OO'} + \mathbf{O'B_0'} + \mathbf{B_0'B'} \\ &= R_0\mathbf{i} + y_0\mathbf{j} + z_0\mathbf{k} + \delta_y'\mathbf{j} + \delta_z'\mathbf{k}\end{aligned} \tag{5}$$

where, $\mathbf{OQ_R} = \mathbf{OQ} + \mathbf{QQ_R}$, then

$$\mathbf{QQ_R} = R_0\mathbf{i} + (y_0 + \delta_y')\mathbf{j} + (z_0 + \delta_z')\mathbf{k} - |\mathbf{Q_R B'}|\mathbf{Out} - \mathbf{OQ} \tag{6}$$

In terms of the point characters $\mathbf{Q_R}$ and $\mathbf{B'}$,

$$\begin{aligned}\mathrm{d}w &= \mathrm{d}|\mathbf{QQ_R}| \\ &= \mathrm{d}|R_0\mathbf{i} + (y_0 + \delta_y')\mathbf{j} + (z_0 + \delta_z')\mathbf{k} - |\mathbf{Q_R B'}|\mathbf{Out} - \mathbf{OQ}|\end{aligned} \tag{7}$$

So, the wave aberration is given by,

$$\begin{aligned}W &= \int_{\sum R} \mathrm{d}w = \int_{\sum R} \mathrm{d}|\mathbf{QQ_R}| \\ &= \int_{\sum R} \mathrm{d}|R_0\mathbf{i} + (y_0 + \delta_y')\mathbf{j} + (z_0 + \delta_{z'})\mathbf{k} - |\mathbf{Q_R B'}|\mathbf{Out} - \mathbf{OQ}|\end{aligned} \tag{8}$$

At the same time, the surface normal unit vector **N** is,

$$\mathbf{N} \cdot \sum \mathbf{R} = 0 \tag{9}$$

For the existence of a free form surface, it has a necessary and sufficient condition for the existence of a surface everywhere perpendicular to a vector field **N**, is that this vector field satisfy

$$\mathbf{N} \cdot \text{curl}(\mathbf{N}) = 0 \tag{10}$$

In order to determine the initial shape of the Zernike polynomial optical surface, we have to solve the set of differential equations:

- Eq. (4), which specifies the reflective law.
- Eq. (7), which connects the wave front with the curvature of the incoming wave front and the curvature of the optical surface.
- Eq. (9), which specifies the relationship of the normal vector with optical surface.
- Eq. (10), the integrability condition.

Numerical methods [11,12] is used to solve the differential equations, first the equations should be made discretization and difference method are used. In Fig. 2, light Ray_{n-1} and Ray_n reflected by surface Z arrive at image point. Because the reflector is second differential term, so lights between Ray_{n-1} and Ray_n would be reflected into the line between Ray_{n-1} and Ray_n. If Ray_{n-1} and Ray_n are close enough, the wave front caused by the reflected light could be considered as similar. Then lights Ray_n can be calculated by Ray_{n-1} using the equations mentioned above. The initial merit for optical surface can be derived by numerical method above, this initial merit commonly has great aberrations and can not be used in design directly. The least squares optimization method is used to induce optimization merit after acquired initial merit, more detailed description shown in [13–15].

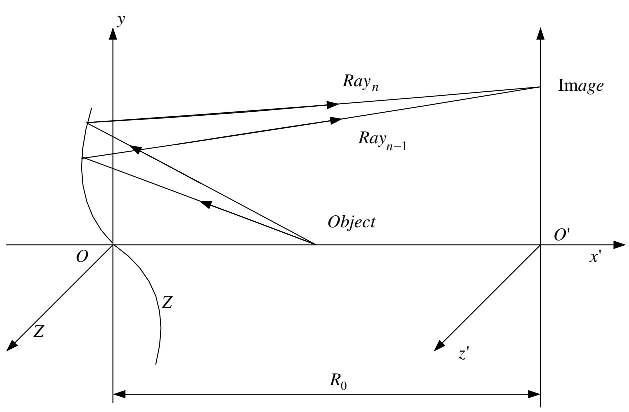

Fig. 2. Scheme of reflective ray.

3. Design of reflective projection lens with Zernike polynomial surface

3.1. Design with three even aspheric mirrors + 1 Zernike polynomial surface

Firstly, a projection lens with four even aspheric reflective surface was designed, layout of design is shown in Fig. 3a. Light flux starting from object plane, which is a picture-forming device, such as DMD, LCD or LCoS panels, then reflected by the four even aspheric reflecting mirrors M1, M2, M3, M4, projected on the screen. In this design, M1 and M3 are concave mirrors, M2 and M4 are convex mirrors, projection distance is 250 mm and the chief ray condition is telecentric in object space. The parameters of four even aspheric surfaces are given in Table 1.

The object's size is 0.42 in. and projection magnification is 143. With F-number = 2.5 and F.O.V $2a = 130°$, the average MTF of four even aspheric surfaces design is over 70% at 60 lp/mm in the center, and average MTF is about 50% at 60 lp/mm on the margin. The spatial frequency 60 lp/mm is measured in the object space. Fig. 3b shows the MTF results.

Based on the even aspheric mirrors design, one Zernike polynomial mirror is used to substitute the even aspheric to reduce the aberration, result shown in Fig. 4a is the layout of design, M1/M2/M3 is the same with even aspheric design, M4 is the Zernike polynomial reflective surface. The chief ray condition is telecentric in object space and F-number is 2.5. Fig. 4b is the average MTF of the design, compared with four even aspheric surface design, the MTF of center and 0.7 field are almost the same, but on the margin-1 field, the MTF is improved from 50% at

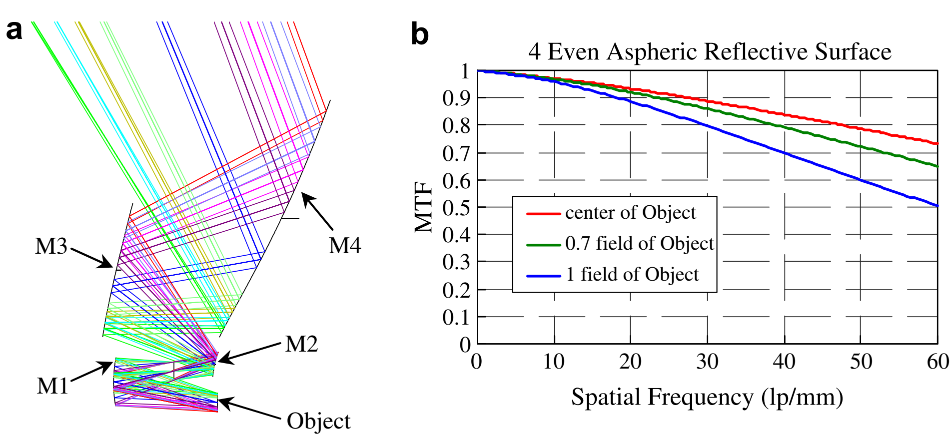

Fig. 3. Layout and MTF of projection lens with four even aspheric mirrors.

Table 1
The structure data of projection lens with four even aspheric mirror

	Type	Position (mm)			Tilt (°)	*R*	*k*	*a2*	*a3*	*a4*	*a5*
		x	*y*	*z*							
Object	Plane	0	0	0	12	83.902	0.589	−1.387e−07	7.168e−10	−1.824e−012	2.133e−015
M1	Even aspheric	−94.81	12.64	0	1.45	83.902	0.589	−1.387e−07	7.168e−10	−1.824e−012	2.133e−015
Stop	Plane	−45.94	16.78	0	0	–	–	–	–	–	–
M2	Even aspheric	−5.73	20.12	0	12.1	63.585	−10.972	4e−08	−3.06e−09	2.46e−10	7.11e−16
M3	Even aspheric	−103.17	80.62	0	6.62	138.041	−0.169	−3.03e−07	2.166e−11	−1.384e−15	4.63e−20
M4	Even aspheric	87.42	102.2	0	20.44	27.797	−3.4227	−8.018e−08	5.919e−012	−2.918e−16	8.9356e−21

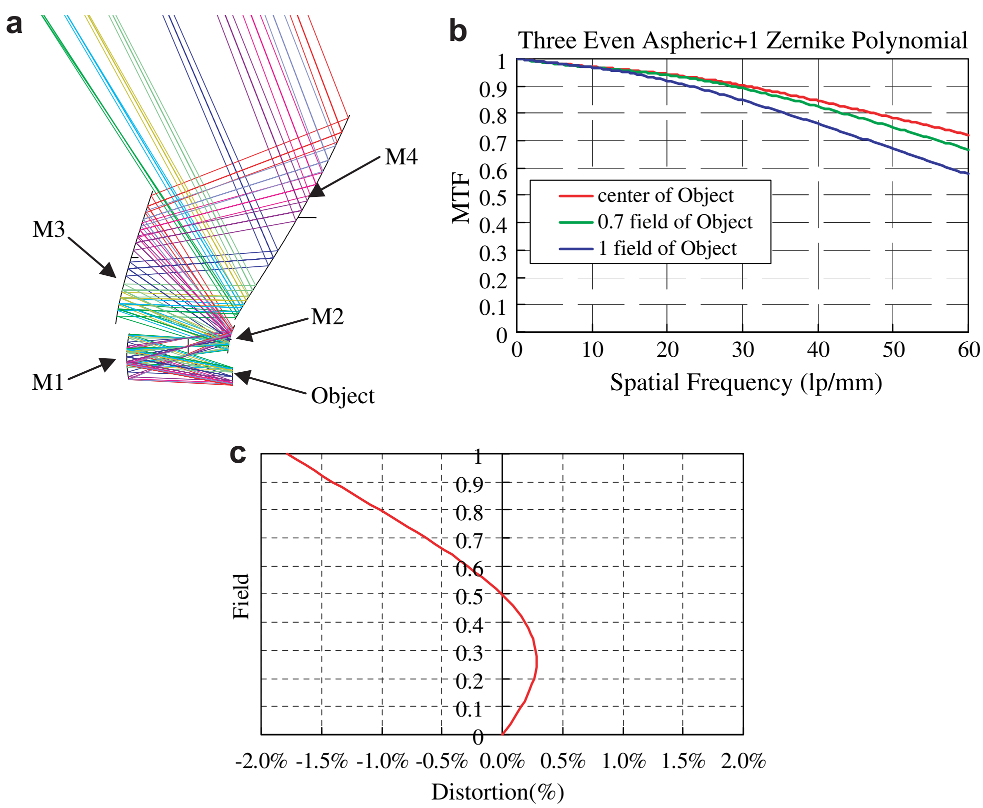

Fig. 4. Layout and optical performance of projection lens with three even aspheric mirrors + 1 Zernike polynomial surface.

60 lp/mm to 57% at 60 lp/mm. Fig. 4c shows the distortion result.

3.2. Reflective projection lens design with three Zernike polynomial surfaces

To achieve simpler structure and higher performance, three Zernike polynomial reflective surfaces are used to replace four even aspheric surfaces; Fig. 5 shows the layout of reflective projection lens with three Zernike polynomial free form surfaces, the chief ray condition is telecentric in object space. Light flux starting from object plane is reflected by the three Zernike polynomial reflecting mirrors M1, M2, M3, and then projected on the image surface. In this design, the first mirror surface M1 is concave to reduce

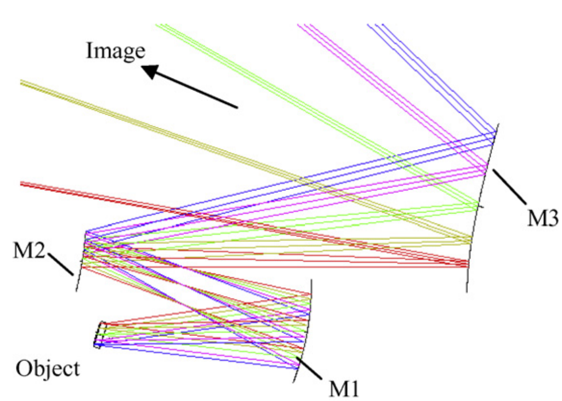

Fig. 5. Layout of design with three Zernike polynomial reflective surfaces.

the size of the second mirror, a stop aperture is arranged between the first and the second mirror to achieves high contrast ratio, the second mirror surface M2 and the third mirror surface are used to achieve a shorter projection distance and correct major aberrations such as spherical aberration and coma aberration, the third mirror M3 is also convex to expand and project the image and the aspheric coefficients of this mirror are used to correct distortion. Table 2 shows the structure data of projection lens with Zernike polynomial mirrors.

In Table 2, C and K are defined by Eq. (1), the distances of the object, image, and mirrors from the origin of the surface coordinate system are positive when located to the right of surface. The object's size is 0.42 in. and projection magnification is 143. Table 3 is the Zernike polynomial coefficients of Zernike polynomial mirrors of the above designs.

The MTF result of design is shown in Fig. 6a. Because the use of Zernike polynomial surfaces, under the condition of F-number = 2.5 and angle of view $2a = 130°$, the modulation transfer function is over 60% at 100 lp/mm, and distortion is less than 2%. Fig. 6b shows the distortion result.

Table 2
The structure data of reflective lens with three Zernike polynomial mirrors

Optical part	Position (mm)			Tilt (°)	C	K
	x	y	z			
Object	0	0	0	12	0	0
M1	110.04	3.15	0	4.15	−124.34	0.004477
Stop	40.72	20.31	0	0	0	0
M2	−10.04	31.48	0	2.43	−161.3	0.1280
M3	182.46	60.15	0	8.06	312.15	0.04182
Image	−240	305.37	0	0	0	0

Moreover, this design with three Zernike polynomial surfaces has simpler structure, lighter weight, lower cost and higher performance.

Comparisons of ray aberration between three designs are shown in Fig. 7.

Design-A means the design of four even aspheric mirrors; Design-B means the design of three even aspheric mirrors + 1 Zernike polynomial surface; Design-C means the design of three Zernike polynomial surfaces.

Fig. 7a and b shows the tangential ray aberration and the sagittal ray aberration, respectively. In Fig. 7a, the maximum tangential ray aberration of Design-A, Design-B, and Design-C is 919, 737, and 148 μm. In Fig. 7b, the maximum sagittal ray aberration of Design-A, Design-B, and Design-C is 450, 276, and 54 μm, respectively. Fig. 7 shows that the design of three Zernike polynomial surfaces has the best performance, and the performance of three even aspheric mirrors + 1 Zernike polynomial surface is better than four even aspheric mirrors. It's proved that the Zernike polynomial surface can improve the optical performance of projection lens.

4. Conclusion

In this paper, wide view angle off-axis reflective projection lens are designed by using Zernike polynomials mirrors. The detail description and performance of Zernike

Table 3
Zernike polynomial parameters of Zernike polynomial mirrors

Item	Zernike polynomial $Ei(x,y)$	Mirror M1, A_i	Mirror M2, A_i	Mirror M3, A_i	Mirror M4, A_i
1	X^1Y^0	0	0	0	−0.142273
2	X^0Y^1	−0.2795	−0.085219195	−0.51823812	0.037121
3	X^2Y^0	−0.000127172	−0.000189189	0.000980361	−1.37316 E−03
4	X^1Y^1	0	0	0	7.376399 E−05
5	X^0Y^2	−0.000294935	0.000605635	0.002832199	−6.68487 E−04
6	X^3Y^0	0	0	0	3.23782 E−07
7	X^2Y^1	−9.04 E−06	−2.98 E−05	−1.14 E−05	−1.25753 E−06
8	X^1Y^2	0	0	0	1.522594 E−06
9	X^0Y^3	−9.76 E−06	−2.81 E−05	−1.70 E−05	−1.88175 E−07
10	X^4Y^0	5.25 E−08	1.11 E−09	−9.84 E−08	−4.93058 E−10
11	X^3Y^1	0	0	0	−4.69877 E−10
12	X^2Y^2	1.17 E−07	−1.75 E−07	−1.33 E−07	−1.49279 E−08
13	X^1Y^3	0	0	0	−8.09031 E−10
14	X^0Y^4	3.74 E−08	4.07 E−07	−6.76 E−08	1.231028 E−10
15	X^5Y^0	0	0	0	2.547224 E−10
16	X^4Y^1	−1.09 E−09	−3.38 E−08	2.81 E−09	9.484691 E−11
17	X^3Y^2	0	0	0	−2.09355 E−11
18	X^2Y^3	−1.23 E−09	−5.89 E−08	1.46 E−09	8.296171 E−11
19	X^1Y^4	0	0	0	3.961791 E−12
20	X^0Y^5	−7.08 E−10	−5.01 E−08	1.10 E−09	3.506619 E−13
21	X^6Y^0	1.77 E−12	1.77 E−10	1.45 E−12	9.928709 E−14
22	X^5Y^1	0	0	0	−8.67005 E−14
23	X^4Y^2	−1.20 E−11	2.48 E−09	−1.73 E−11	−9.95596 E−14
24	X^3Y^3	0	0	0	−3.83591 E−13
25	X^2Y^4	−6.33 E−12	2.36 E−09	1.70 E−11	1.678566 E−14
26	X^1Y^5	0	0	0	2.279295 E−14
27	X^0Y^6	−2.31 E−12	1.19 E−09	−2.82 E−13	8.855778 E−15

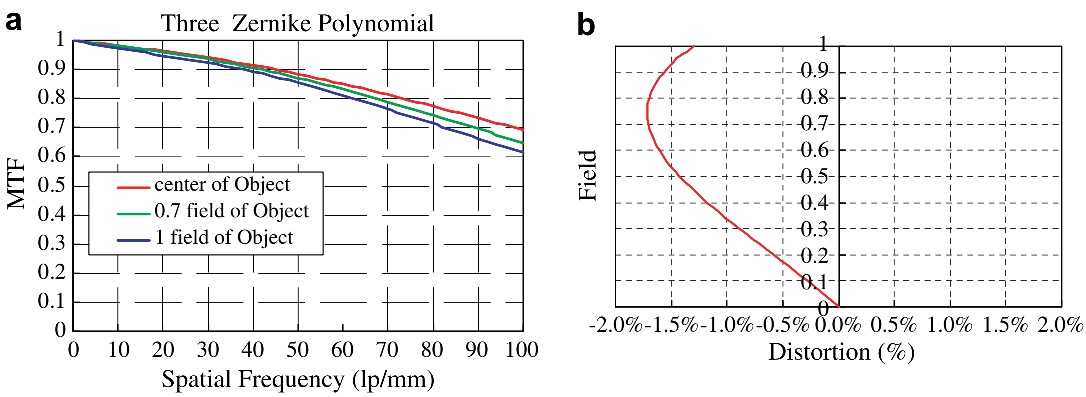

Fig. 6. MTF and optical performance of three free form reflective lens.

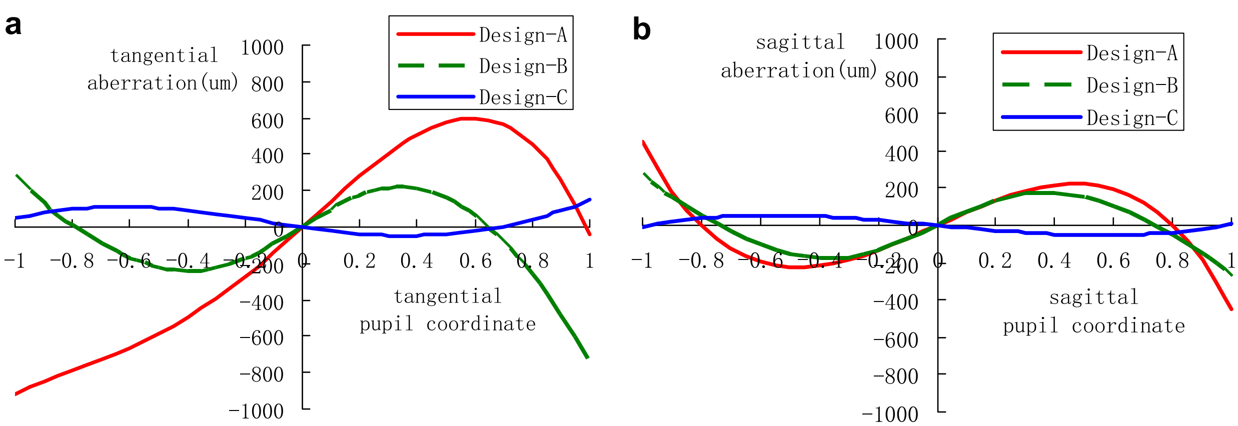

Fig. 7. Comparisons of ray aberration between three designs.

polynomials surfaces are analyzed. Wave front aberration is used to design the Zernike polynomials reflective lens. Two types of reflective projection lens are designed with Zernike polynomials mirrors, one layout is three aspheric mirrors + 1 Zernike polynomial mirror, another is three Zernike polynomials mirrors, and lenses are telecentric in object space. Projection lens's optical performance are analyzed, under the condition of F-number = 2.5 and angle of view $2a = 130°$, both of projection lens's MTF (Modulation Transfer Function) are over 60% at 60 lp/mm on magnification side, moreover, the MTF of three Zernike polynomials mirrors is over 60% at 100 lp/mm and distortion is less than 2%.

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