Global Effect of Mirror Mounted on Aluminum with Adhesive

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Goal:

This project is more of a research project, which means it is a specific type of design project. The goal is to look at the effect of temperature changes on a mirror bound to an aluminum plate.

Theory:

When temperature changes occur, different materials expand at different rates. This differential expansion can induce stresses in materials when they are bonded together. Aluminum and glass have very different coefficients of thermal expansion. If these two materials could be bonded together and measured before and after a thermal change, there would be a change in the glass surface.

In mirrors, moments cause changes in the surface, and angular deflections in a mirror cause large errors. In this experiment I will use a thin glass mirror bonded to an aluminum block. The changes in thermal expansion will cause a differential expansion between the block and the mirror. The induced moment on the glass will be investigated. For different adhesive configurations there should be different results. In this analysis I will look at a 3-pt configuration and a ring configuration, and compare the two results.

Design:

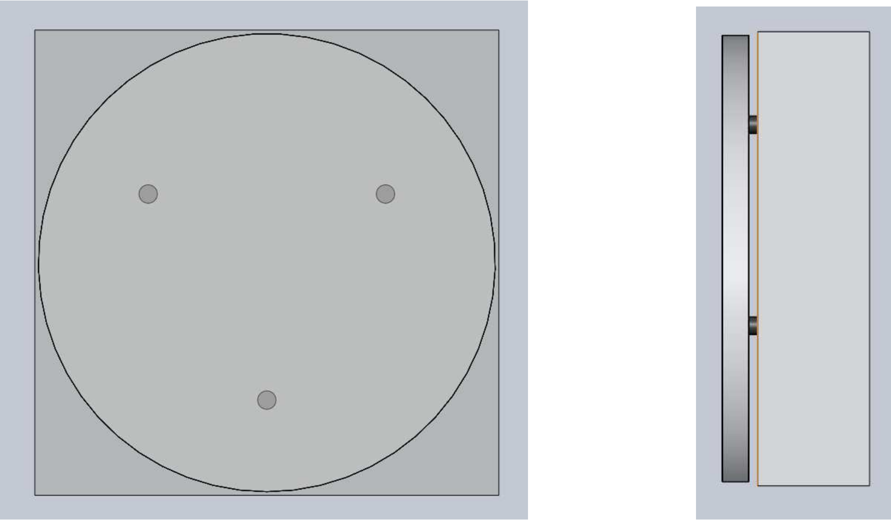
This design will also be implemented in an experiment, so the aluminum plate and mirror have already been chosen. Their dimensions are:
Aluminum plates: 2"x 2"x 0.5", Mirror: 50mm diameter, 3mm thick

The adhesive being used is 3M 2216 epoxy because it is not very compliant in shear and should induce a large moment in the mirror that could be measured.

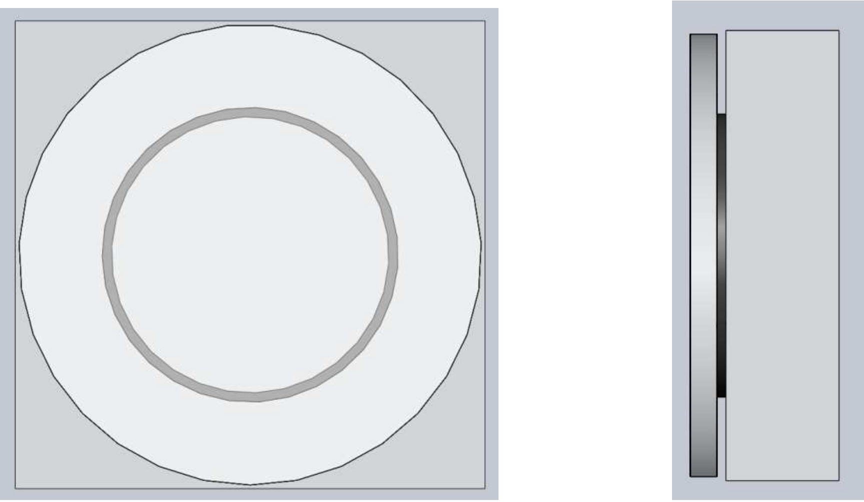
An initial normalized radius of 60% was selected because this is near the minimum radial position for 3-point self-weight deflection, which is 68%. The adhesive thickness suggested by Yoder is 0.004" +/- 0.001", and the material data sheet references to 0.005" bond thickness, so that is what I have chosen. The bond width was chosen to be 2mm, or 0.00787" by a rule of thumb to look at the global effect of the adhesive and not the localized effect. A parametric analysis was done to compare the results from varying radii of adhesive.

The two configurations shown below were the ones analyzed:

3 point configuration:



Ring configuration:



Materials:

Adhesive properties of 3M 2216 Epoxy:
Using the data sheet from Yoder's book, and the equation shown below, the different properties of 2216 epoxy were found.

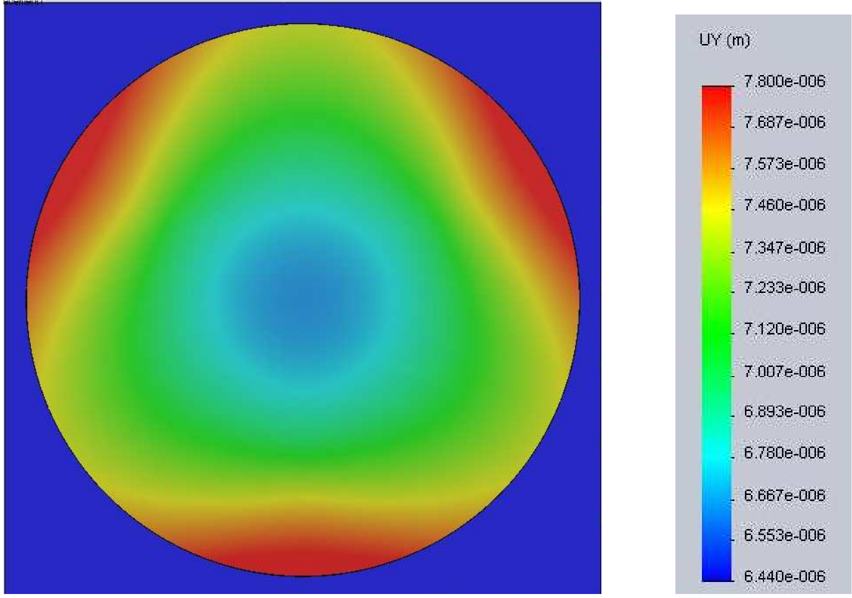
Young's Modulus	689 MPa
Poisson Ratio	0.43
Bulk Modulus	241 MPa

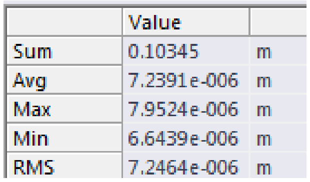
$$G = \frac{E}{2(1+\upsilon)}$$

FEA results:

Analysis of temperature change from 20C to 40C.
The data below show the surface figure, RMS deflection, and peak to peak deflection from COSMOSworks. Both configurations are shown with an initial radius of 60%, or 15mm radius.

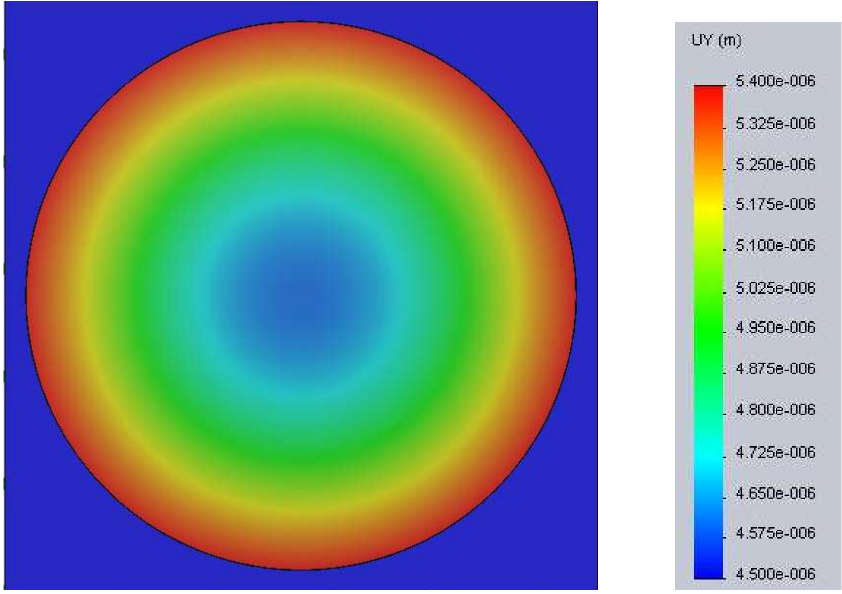
3-point configuration:

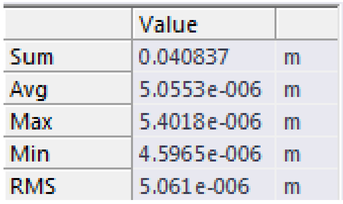




	Value	
Sum	0.10345	m
Avg	7.2391e-006	m
Max	7.9524e-006	m
Min	6.6439e-006	m
RMS	7.2464e-006	m

Ring:

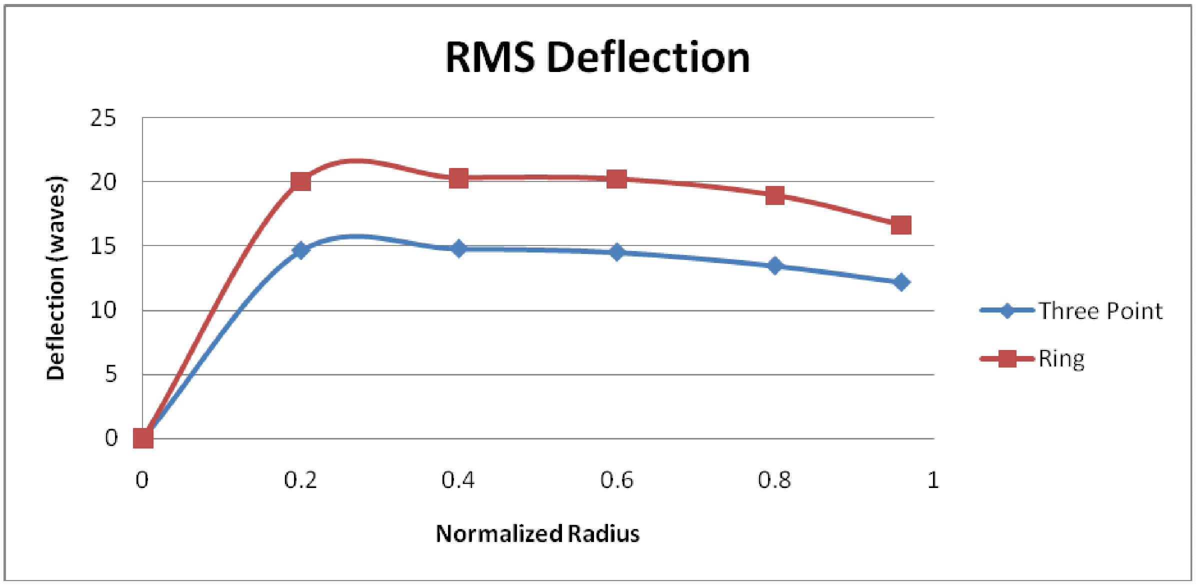




	Value	
Sum	0.040837	m
Avg	5.0553e-006	m
Max	5.4018e-006	m
Min	4.5965e-006	m
RMS	5.061e-006	m

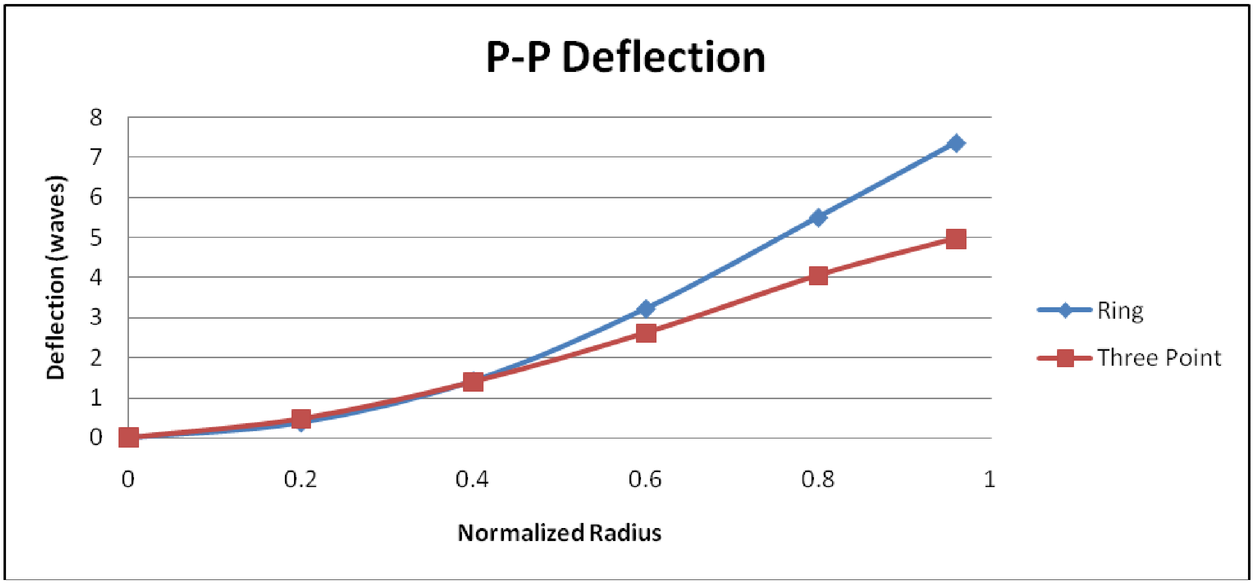
FEA result comparison:

The data given by COSMOSworks for both configurations are shown below:



The Ring configuration has an increased RMS deflection for all radii. The difference between the two seems to remain constant. The other important thing to note is that the RMS deflections seem to follow the same rate of change of slowly decreasing as the radius increases.

While RMS deflection is a good comparison, the Roark formulas can only give the peak to peak deflection, so the COSMOSworks peak to peak deflection is shown below.



The table below shows the combined differences of the two configurations referenced from the ring to the three point:

Radius	P-P Deflection		RMS Deflection	
0.0	0.0000	0.0%	0.0000	0.0%
0.2	-0.0954	-20%	5.4602	37%
0.4	0.0256	2%	5.5696	38%
0.6	0.6048	23%	5.7512	40%
0.8	1.4524	36%	5.5354	41%
0.96	2.3934	48%	4.5036	37%

Difference of Ring to Three Point

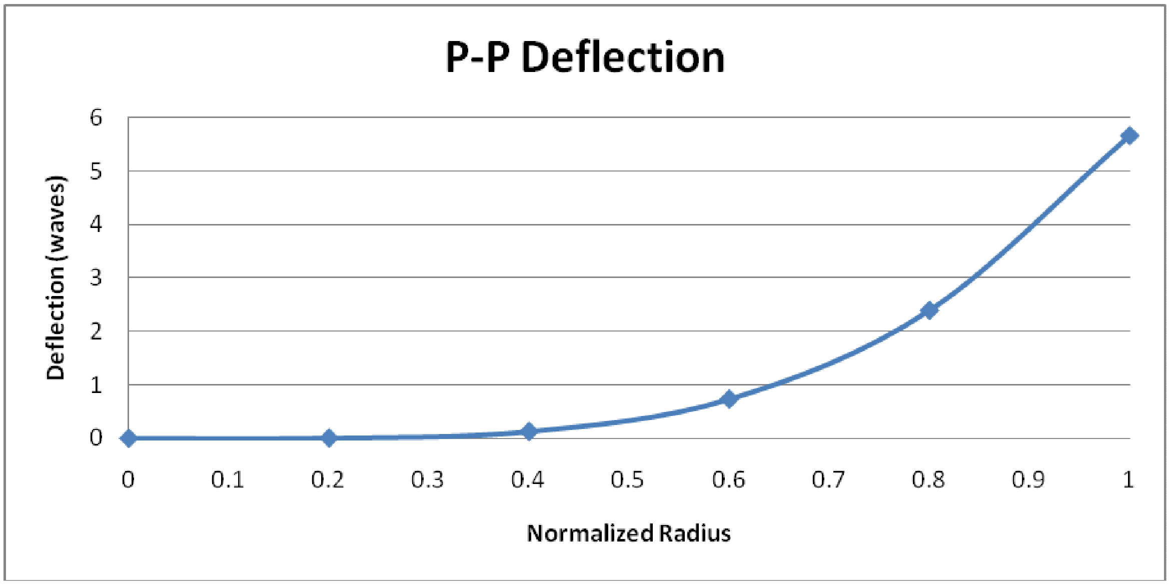
The two results seem to be comparable to each other, and there is an expected increase between the ring and three point. It would make sense that at a larger radius there would be a larger moment on the glass with the ring than the three point. At the maximum radius, there would be a larger moment applied because more of the adhesive is applying a moment to the glass.

One interesting thing is that the peak to peak difference of the ring is smaller than the three point for the 20% radius. This could be due to a higher order component in the ring configuration rather than the three point, which would cause it to be lower at small radii and then increase at a faster rate for larger radii.

While peak to peak information is helpful, RMS deflection is more defined for wavefront distortions. The difference in RMS deflection is nearly constant over all the radii. At larger radii the RMS deflection decreases for both configurations. One reason for this could be that with a larger radius, the moment arm remains the same, and the force pulling on the mirror increases, so there is a larger axial force. This larger axial force could actually flatten out the mirror, which would decrease the RMS deflection. Whatever the reason, both configurations experience this decrease in RMS deflection, and the ring remains proportionally larger by the same amount.

Roark calculation:

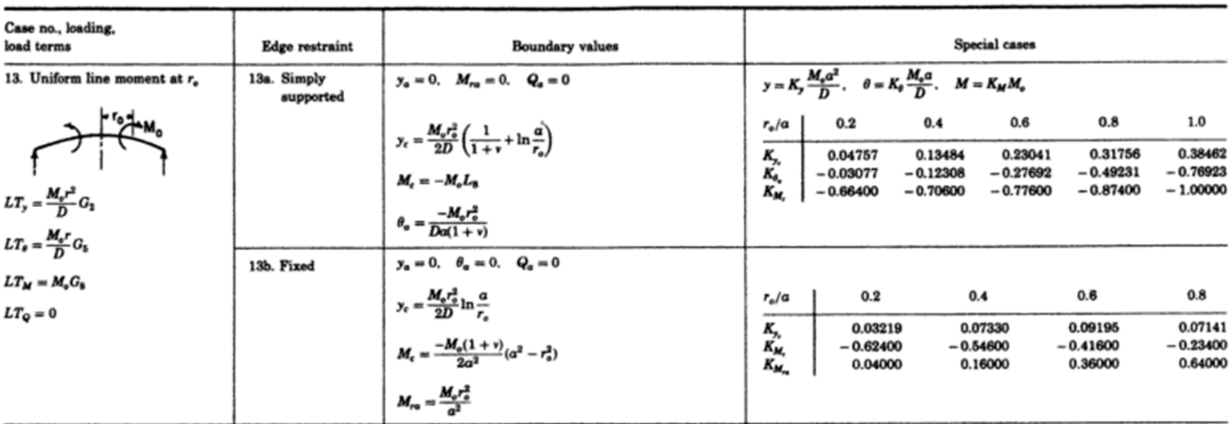
I also used a formula from Roark's formulas for Stress and Strain to calculate analytically what the deflection of the ring configuration should be.



The radius used was 15mm of a 25mm ring, so the peak-to-peak deflection was calculated to be 0.73 waves or 0.365µm.

The Roark formula was calculated using Table 24 – Formulas for flat circular plates of constant thickness. Case 13 of a uniform line moment fits my analysis the most.

Here is a page from Roark showing the equations for Case 13.



Case no., loading, load terms	Edge restraint	Boundary values	Special cases
13. Uniform line moment at r_o $LT_y = \frac{M_o r^2}{D} G_3$ $LT_\theta = \frac{M_o r}{D} G_6$ $LT_M = M_o G_9$ $LT_Q = 0$	13a. Simply supported	$y_a = 0, \quad M_{ra} = 0, \quad Q_a = 0$ $y_c = \frac{M_o r_o^2}{2D}\left(\frac{1}{1+\nu} + \ln\frac{a}{r_o}\right)$ $M_c = -M_o L_8$ $\theta_o = \frac{-M_o r_o^2}{Da(1+\nu)}$	$y = K_y \frac{M_o a^2}{D}, \quad \theta = K_\theta \frac{M_o a}{D}, \quad M = K_M M_o$
	13b. Fixed	$y_a = 0, \quad \theta_a = 0, \quad Q_a = 0$ $y_c = \frac{M_o r_o^2}{2D} \ln\frac{a}{r_o}$ $M_c = \frac{-M_o(1+\nu)}{2a^2}(a^2 - r_o^2)$ $M_{ro} = \frac{M_o r_o^2}{a^2}$	

r_o/a	0.2	0.4	0.6	0.8	1.0
K_{y_c}	0.04757	0.13484	0.23041	0.31756	0.38462
K_{θ_o}	−0.03077	−0.12308	−0.27692	−0.49231	−0.76923
K_{M_c}	−0.66400	−0.70600	−0.77600	−0.87400	−1.00000

r_o/a	0.2	0.4	0.6	0.8
K_{y_c}	0.03219	0.07330	0.09195	0.07141
K_{M_c}	−0.62400	−0.54600	−0.41600	−0.23400
$K_{M_{ro}}$	0.04000	0.16000	0.36000	0.64000

The method I used for calculating the line moment was from Professor Burge's notes on adhesives. The following equation for Shear stress, τ, was taken directly from the notes:

$$\tau = \frac{Ga}{2t_{2216}}\left(\alpha_{al} - \alpha_g\right)\Delta T$$

The Roark formula requires a line moment, M, to put into the equations. The way I calculated M was to multiply the shear stress of the adhesive with the bond width and the moment arm of the shear force, which I calculated as half the thickness of the glass. This is shown in the equation below:

$$M = \tau * w * \left(\frac{t_g}{2}\right)$$

The units on M are in-lbs/in because it is used in the equation as a circumferential moment.

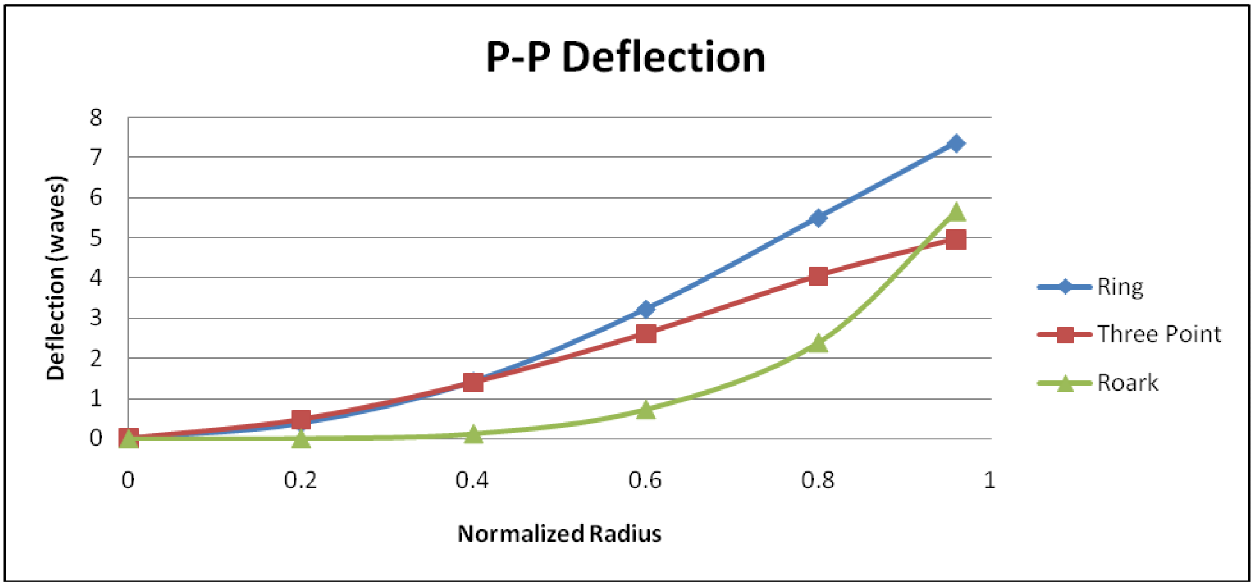
I used the following material properties and experiment parameters:

Symbol	Parameter	Value	
α	CTE-bk7	7.00E-06	/C
α	CTE-Al	2.30E-05	/C
E	Young mod-2216	9.95E+04	psi
υ	Poisson-2216	0.43	
t	thickness-2216	0.005	in
ΔT	delta temp	20	C
w	bond width-2216	0.0787	in
E	Young mod-bk7	1.19E+07	psi
υ	Poisson-bk7	0.206	
t	thickness-bk7	0.118	in

These numbers were found in Yoder's book: *Opto-Mechanical System Design, Third Edition*. One problem arose with the poisson ratio of the 2216 epoxy. Poisson ratios are fairly difficult to measure for materials, but extremely difficult to measure when the value approaches 0.5 because the poisson effect starts to dominate, making measurements difficult. Otherwise the parameters were well defined.

Result comparison:

The Roark and COSMOSworks deflections are very similar. There is a higher power term with the Roark formula than the COSMOSworks measurements which makes the Roark result increase slowly at first and then increase very rapidly towards the maximum radius value. Overall the results are comparable.



Lessons Learned:

One of the most significant lessons that I learned was that it is important to do an intuition check on an answer that comes out of several equations or a software program. There are many different things that go into work that results in one answer. A difficulty is that I don't have an intuition about a circumferential moment of different numbers that come as intermediate steps, so it is difficult to troubleshoot problems and find where the errors are.

Roark's formula's for Stress and Strain is very useful and applicable but fairly complex. It is difficult to find a case that closely matches the real situation because certain assumptions need to be made, and it can be difficult to choose these assumptions correctly. Also, I initially did the problem using metric units thinking that it would still work out, but some constants I used were derived for SI units, not metric. I also found that it is much easier to use a computer program to do such a complex calculation because it allows for easy manipulation of different steps and automatically adjusts the end result.

When an entire software analysis is done to get an output of one or two numbers, double check the final result because my COSMOSworks result was actually correct, but I was anticipating an incorrect result and I misread an exponent of negative six, corresponding to microns, as negative nine, corresponding to nanometers. This caused me considerable difficulties as I was troubleshooting my software analysis to find an error that was not present.

Conclusion:

The results were different that I was expecting before doing the analysis. I would have expected the results to be smaller than the micron range. The results seem to make sense because they are so similar, I successfully troubleshooted the analysis several times, and other people have verified the results in this range.

The Roark formula is similar to the COSMOSworks results, which was expected. The two results did coincide at the center and edges with some differences for radii in the middle. While not surprising, it was exciting to see that two very different methods of calculation resulted in similar results.

One of the major problems with the comparisons between Roark and COSMOSworks is that the peak to peak distortion was the only numbers that were possible to relate. I would be able to make a more accurate comparison if I had a way to calculate the RMS deflection from the Roark equation.