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Optimum shapes for lightweighted mirrors

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Abstract

Two types of monolithic lightweight mirrors with arched backs are discussed: the center-supported single arch and the ring-supported double arch. The theoretical deformations of a 20-in.-diameter double arch mirror are compared with the actual deformations. A mirror of this size weighs about 50% less than an equivalent conventional mirror. The double arch design may be scaled up to 144 in. where the mirror weighs less than 40% of the equivalent conventional mirror. Further weight savings are possible due to the reduced size and simplicity of the support required by the double arch mirror design.

Introduction

Conventional telescope mirrors are right-circular cylinders with a diameter-to-thickness ratio of 6:1. A lightweight mirror may be defined as one in which the weight of the mirror and support system is less than that of a conventional mirror and support for equivalent optical deflections. The cost of a terrestrial telescope scales as a power law with the weight of the structure; therefore, reducing the weight of the primary mirror has merit. For space applications weight reduction is mandatory.

The most widely used lightweight mirror concept consists of two thin plates bonded by a cellular core. The structural efficiency of this approach is good, but the cost is high, and a complex support is needed. Very thin plate or membrane mirrors are another possibility, but they require expensive and sophisticated support systems.

If the support and mirror are considered as a system, it is possible to trade weight and complexity of the support against stiffness in a monolithic mirror that is shaped so as to minimize self-weight-induced deflection. This paper will explore the use of monolithic mirrors with simple supports.

Shape development

The optical deflections of a mirror may be divided into radial and azimuthal components. For an axisymmetric mirror and support system under self-weight loading with gravity acting along the optical axis, the azimuthal deflection may be ignored. Insight into the optimum shape of the mirror can then be gained by considering the bending properties of a two-dimensional cross section of the mirror and support.

This two-dimensional cross section is analogous to a beam in bending. A special case of this analogy is a center-supported mirror with a thick center and thin edge. The back has a parabolic taper, the vertex of the parabola coincident with the edge of the mirror. The shape generator may be considered as a cantilever beam of uniform self-weight-induced stress. This mirror will be called the single arch mirror (see Figure 1).

Another analogous case is a beam on two simple supports. To minimize the total deflection the supports are adjusted to a position intermediate between the end and center of the beam. It is then possible to further reduce deflection by arching the sections of the beam between supports. If the shape so developed is rotated about its centerline, it serves as a generator of the double arch mirror shape.

Single arch mirror

A number of single arch mirrors have been fabricated, in diameters up to 60 in.,[1] and in a variety of materials, including fused silica, Cervit, and aluminum. Typically the weight of such a mirror is about 50% of the equivalent 6:1 ratio of a conventional mirror. Since the size of the single arch mirror support is small and centrally located, the conventional mirror cell and support system may be eliminated, making weight comparisons with conventional mirrors even more favorable. The single arch mirror shape may be cast or generated, leading to a relatively low-cost mirror (as compared with cellular lightweight mirrors).

The aberration limiting the size of the single arch mirror is astigmatism induced when the mirror is in the vertical position. For a 50-in.-diameter single arch, the surface deforms about 24 × 10^{-6} in. (Ref. 2). Despite its advantages, the single arch is not a good candidate for a terrestrial telescope requiring a good surface figure or for larger sizes.

Double arch mirror

The disadvantage of the single arch mirror, coupled with reasoning along the lines of the beam analogy, leads to the double arch shape. The mirror is supported on a ring (or three points) positioned midway between the mirror edge and center. The generator for the back shape is roughly that of a pair of cantilever parabolic arches placed back to back and swept along the support ring. The tradeoff is increased mirror stiffness against a larger support.

Some prior work has been performed on the double arch shape.[3,4] Recently the Optical Sciences Center at the University of Arizona constructed and tested a 20-in.-diameter double arch mirror for NASA Ames.[5] The mirror weighed approximately 40 lb, had a 3.0-in. thickness, 0.50-in. minimum thickness, and a 5.0-in.-diameter center hole. The support radius was 6.8 in., and a 1.0-in. square mounting ring was generated integral with the mirror. The mirror material was Corning Code 7940 fused silica. (See Figures 2 and 3.)

The double arch was compared against a single arch of the same size fabricated at the same time. The double arch proved to have only 0.25 the radial deformation of the single arch when both mirrors were horizontal. When placed on a three-point support, the double arch exhibited an azimuthal deformation due to sag between the supports that was twice that of the single arch. Both mirrors had approximately the same weight.

Given the small radial deformation of the double arch, the mirror was extensively studied using finite element analysis methods in hopes of improving the azimuthal deformation properties.

The finite element analysis made use of plate bending theory; the program used was SAP IV. The mirror model had 60 quadrilateral elements and 61 nodes. Triangular elements led to virtually the same results as the quadrilateral elements. To simplify the analysis, the mirror had a flat optical surface and lacked a center hole. The diameter of the mirror model was 20 in., the maximum thickness was 3.0 in., and the minimum thickness was 0.5 in. During optimization the position of the support radius was changed, a new back contour to fit this radius was generated, and the deflections of the resulting shape were determined. A three-point support was assumed, since this would lead to worst-case deformations. Deflections were determined for both the horizontal and vertical positions of the mirror. The figure

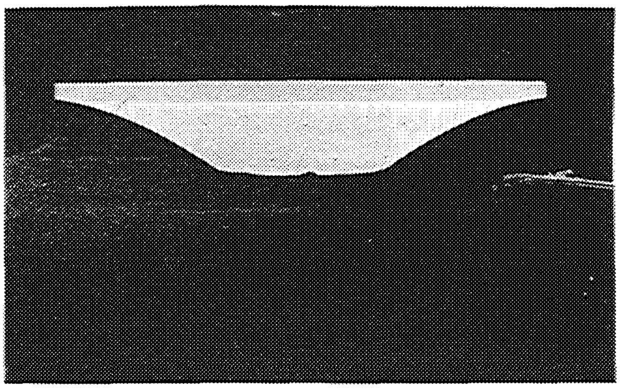

Figure 1. 20-in.-diameter single-arch lightweight mirror.

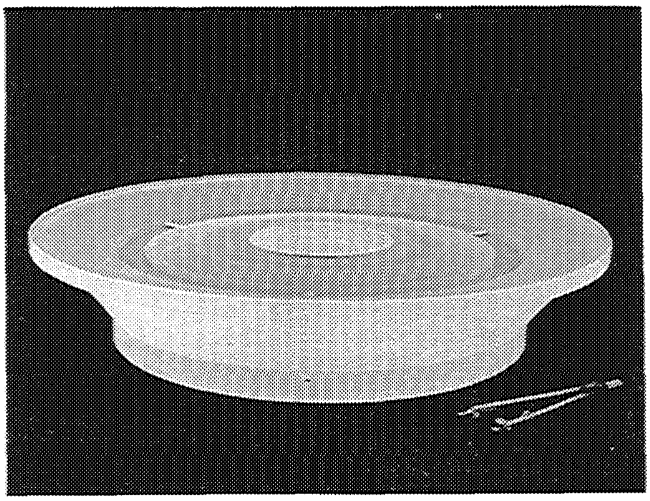

Figure 2. 20-in.-diameter double-arch lightweight mirror.

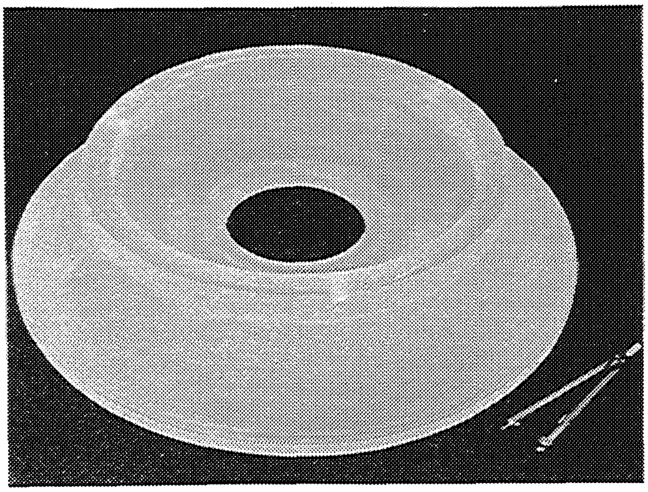

Figure 3. Rear surface of 20-in.-diameter double arch lightweight mirror.

of merit for optimization was minimum deflection in both positions. As a check of the model, the deflections of the constructed double arch mirror were computed and were found to lie within the error bounds of the optical tests.

The resulting optimized mirror is shown in Figure 4. If t is the thickness of the mirror at radius R, the equations of the back surface are

$$t = 0.2778R^2 - 5.556R + 28.28 \qquad 10.0 > R > 7.0$$

$$t = 3.00 \qquad 7.0 \geq R \geq 6.0$$

$$t = 0.1291R^2 - 0.4132R + 0.8306 \qquad 6.0 > R > 1.6$$

$$t = 0.50 \qquad 1.6 \geq R \geq 0.0$$

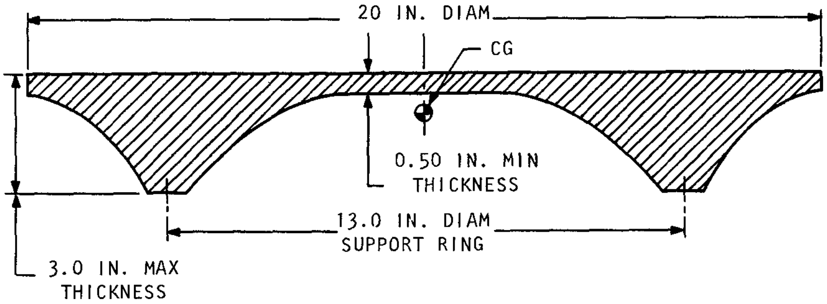

Figure 4. Double arch mirror cross section.

The three support points are equally spaced on a support ring that is 0.65 the diameter of the mirror. The support points act through the center of gravity of the mirror, which is located a distance of 1.008 in. from the flat optical surface. If fabricated from fused silica, the mirror weight is approximately 42 lb.

Figure 5 shows the deformation of the mirror in the horizontal position. The contour interval is 0.432×10^{-6} in., and the RMS deviation is 2.016×10^{-6} in. Figure 6 shows the deformation of the mirror in the vertical position. The contour interval is 0.482×10^{-7} in., and the RMS deviation is 0.1158×10^{-6} in. (The contour interval difference is an artifact of the computer program.) In Figure 6 one support is near the top, while the other two are symmetrically located near the bottom.

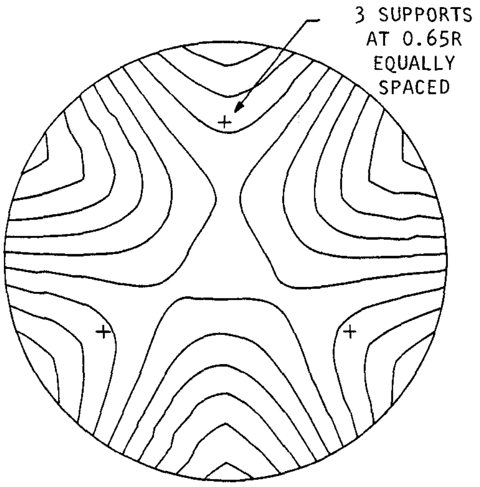

Figure 5. 20-in.-diameter double arch mirror. Optical deformations in horizontal position. Contour interval 0.432×10^{-6} in. RMS deviation 2.016×10^{-6} in.

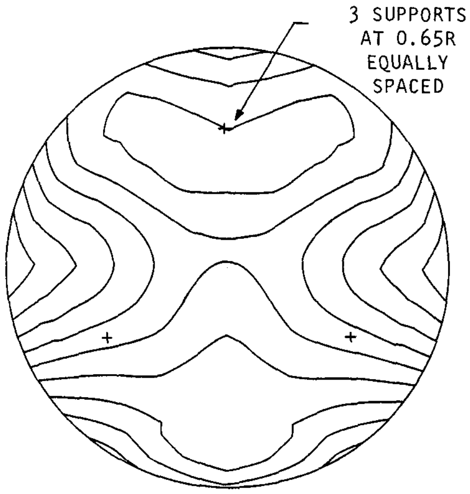

Figure 6. 20-in.-diameter double arch mirror. Optical deformations in vertical position. Contour interval 0.482×10^{-7} in. RMS deviation 0.1158×10^{-6} in.

The limiting aberration for this optimized mirror shape is still azimuthal deformation when the mirror is supported on three points. Even with three support points the aberrations in the vertical position are so small as to be negligible.

Support system

The performance of the double arch mirror shape is based on using a ring support. In the horizontal position the mirror is carried by a ring support; as the mirror is tipped toward vertical, three support points acting through the center of gravity begin to pick up the load. When the mirror reaches the vertical position, these support points carry the entire load.

Assuming a maximum RMS tolerance of 6×10^{-6} in., the mirror support system will vary in configuration with the diameter of the double arch mirror. For a diameter of less than 36 in., three support points are adequate. For minimum deflection in the vertical position the supports must penetrate the back of the mirror and pick up the load at the center of gravity. Accuracy of location of these support points in depth is important. In the 20-in.-diameter model, an error in location of 0.10 in. from the center of gravity caused a factor of 3 figure change when the mirror was in the vertical position. The figure of the mirror in the vertical position can also be improved by a factor of 2 by replacing "stiff" supports with flexures. These support flexures should be compliant radially but stiff in the axial and azimuthal directions; the directions of compliance should be oriented 120° apart.

For mirrors of diameter up to 144 in. a combination support should be used. An airbag ring support located at 0.65R and controlled by a gravity regulator provides axial support. Three classical counterweighted supports acting on the center of gravity are placed in sockets located at 0.65R and provide radial support. Virtually the entire back of the mirror is exposed, and the outer 30% of the diameter need not have any mirror cell around it at all.

The weight of the double arch mirror is typically about 50% of the weight of a conventional solid mirror. When the weight of the support is included, the double arch appears more favorable, since the support is relatively simple and light in comparison with that required for a solid mirror. Mirrors larger than the 20-in.-diameter model can be reoptimized at lighter weights by taking advantage of the relative reduction in the minimum thickness permissible. For a 144-in.-diameter mirror, a minimum thickness of 0.75 in. would reduce the weight to about 38% of that of an equivalent solid mirror.

The relatively thick sections of the double arch design suggest that for thermal reasons a low coefficient of expansion material be used. Since an enveloping mirror cell is not required, thermal equilibrium times for the double arch should be better than those of a conventional solid mirror.

Conclusion

Assuming a maximum permissible RMS tolerance of 6×10^{-6} in., the single arch mirror weighs about 50% of an equivalent solid up to a diameter of 24 in. The single arch is relatively cheap to construct and uses a simple center support. Where a better figure is required, or for larger sizes, the double arch is superior in performance to the single arch. The weight of a double arch will vary from about 50% to under 40% of an equivalent solid as the diameter is increased from 20 in. to 144 in. Further weight reduction for the double arch as compared to a solid is possible due to the reduced size of the support.

The double arch shape may be generated from a solid mirror or preferably cast. Either fabrication technique offers considerable economies over the cost of a cellular mirror of equal size. It is possible to extend the double arch concept to non-circular mirrors, and one such mirror has been studied. Fabrication of a non-circular double arch mirror has yet to be attempted.

Although the arched mirror, either single arch or double arch, does not offer as great a weight reduction as other lightweight mirror technologies, the low relative cost and simplicity of the design make it a serious contender for many applications.

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Acknowledgment

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