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A Comparison of the merits of open-back, symmetric sandwich, and contoured back mirrors as light-weighted optics.

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ABSTRACT

There is a need for reliable mirrors which are both light-weight and stiff. However very little documentation exists that compare different types of light weight mirrors. This lead to this parametric study to compare different mirror types, namely, contoured back mirrors, symmetric sandwich mirrors, and open back mirrors. This paper examines each mirror type as compared in each of several categories:

1) Self weight induced deflection, which is a product of stiffness and weight, for mirrors mounted on their backs and of equivalent thickness.
2) Efficiency of mirrors which are of equivalent weight, where efficiency is a function of self weight induced deflection and mirror thickness.
3) Fabrication constraints including ease of manufacture, ease of mounting, mirror thickness, and quilting or print-through of the mirror face plate.

The results obtained in these categories can be used to design light-weight mirrors with greater confidence in preferred mirror type.

1. INTRODUCTION

In order to compare different mirror types, a case study was performed where diameter of the mirror and mirror material were held constant. A representative flat, 40 inch diameter, Corning 7940 fused silica mirror was investigated and was assumed to have the following physical properties:

$E = 10.1 \times 10^6$ psi
$\nu = 0.17$
$\rho = 0.08 \ \frac{lb}{in3}$
Polishing pressure of the mirror = 0.3 psi.

A polishing pressure of 0.3 psi was chosen as an estimate of the average polishing pressure for mirrors of this material and size.

For each mirror type, the shape, or mirror geometry was optimized as well as the mirror support radius. The contoured back mirrors were optimized according to results presented by Cho.[1] These optimized shapes were then analyzed using the GIFTS finite element program to find their self weight induced deflections. The sandwich and open back mirrors were optimized using equations derived by Mehta.[2] In all cases the mirrors were mounted on their backs and were supported using simple ring supports. Ring supports simplified the analysis and made axisymmetric finite element analysis much less time consuming.

There were a few assumptions made in this analysis which should be noted early. Optimization of the mirror geometry and support radius were made to reduce the mechanical deflection of the mirror rather than the optical deflection of the mirror. Optimum mechanical deflection does not always correlate exactly with optimum optical performance. This is especially true of the double arched contoured back mirror. For these analyses however, it was assumed that the amplitude of the mechanical deflection is a good approximation of the optical performance. It should also be noted that thermal loading of the mirror, while an important and complex design concern, was not considered for this comparative study of mirror shapes.

2. NOMENCLATURE

A = Area of the mirror

a = Mirror cell constant

B = Diameter of cell inscribed circle

D = Flexural rigidity of mirror

d = Mirror diameter

E = Young's modulus

G = Shear modulus $\frac{E}{2(1+\upsilon)}$

h_c = Rib height

h = Total mirror height

I_1 = Energy in central maximum with quilting

I_0 = Energy in central maximum without quilting

m = 1 for open back mirror
 = 2 for sandwich mirror

C_3 = Constant $\left[\frac{5}{6}\right] G t_b$

C_n = Constant $\left[\frac{5}{6}\right]\left[\frac{E t_b}{\nu}\right]$

P = Polishing pressure

r = Variable radius

r_1 = Mirror support radius

r_2 = Mirror outer radius

t_b = Equivalent bending thickness

t_f = Front face sheet thickness

t_w = Rib thickness

W = Mirror weight

δ_c = Quilting amplitude (peak to peak)

η = Rib solidity ratio

λ = Wavelength

ρ = Weight density

ν = Poisson's ratio

ψ = Geometric quilting constant

ω_1 = Deflection in region $0 \leq r \leq r_1$

ω_2 = Deflection in region $r_1 \leq r \leq r_2$

3. CONTOURED BACK MIRRORS

Two types of contoured back mirrors were considered for this study, the single arch mirror and the double arch design.[3] An illustration of these mirror geometries and their configurations can be found in Figure 1. For both mirror types, the geometry of the mirrors were taken directly from the optimum shape study conducted by Cho.[1] The diameter of the mirrors as well as the edge thicknesses were held constant. The only varying factor was the overall height of the mirror. Ring supports were used to support the mirrors and were placed at the optimal locations according to Cho's analysis. GIFTS axisymmetric finite element models were then created for varying mirror heights. These models were subjected to gravity loading in order to find their self weight deflection. Both single arch and double arch mirror types were analyzed at overall heights of four, five, and seven inches. Equivalent weights were also investigated: 175, 200, and 250 lb models were compared. These Gifts finite element models are shown in Figures 2 and 3. The deflected shapes are shown as well as the undeflected model outline. For each model, QA4, four noded isoparametric ring elements, and TA6, six noded linear strain triangle elements were used.

4. OPEN-BACK AND SYMMETRIC SANDWICH MIRRORS

4.1 Optimization

The next step of the study was to analyze open-back and sandwich mirrors in order to compare these configurations to contoured back mirrors. Once again the mirror diameter was constant; Figure 1 illustrates the overall geometry of each mirror type. Determining the self weight induced deflection of these mirror types however, could not easily be determined using finite element analysis. Due to the complicated ribbed cell structure of these mirror types, an analytical approach was used for modelling.

The first step was optimization of the mirror geometry. Many factors can vary in these mirror types, not only overall height but also faceplate thickness t_f, and the rib solidity ratio η. Mehta[2] has developed relationships which optimally distribute the material between the faceplates and the core or rib structure. For a given overall height or weight, the optimum faceplate thickness can be found (for varying rib solidity) which produces a mirror with the greatest possible flexural rigidity. It should be noted at this point that flexural rigidity, while a measure of stiffness is not a measure of self weight induced deflection. Mehta's optimization equations are as follows:

For the optimum open-back section

$$4\left[t_f + \frac{\eta h_c}{2}\right]\left\{\left[1 - \frac{\eta}{2}\right]\left[t_f^3 + \frac{h_c^3}{2}\right] + \frac{(\eta - 1)(t_f + h_c)^3}{2}\right\} - \left[\frac{1}{2}\right]\left\{\left[1 - \frac{\eta}{2}\right]\left[t_f^4 - \frac{\eta h_c^4}{2}\right] + \frac{\eta(t_f + h_c)^4}{2}\right\} = 0 \qquad (1)$$

For the optimum symmetric sandwich section

$$t_f = \frac{W\left[\sqrt{1 - \frac{\eta}{2}} - \sqrt{1 - \eta}\right]}{\rho A\left[2\left\{\sqrt{1 - \frac{\eta}{2}} - \sqrt{(1 - \eta)^3}\right\}\right]} \qquad (2)$$

Where

$$\eta = \frac{(2B + t_w)t_w}{(B + t_w)^2} \qquad (3)$$

Once these parameters are optimized the flexural rigidity of the mirrors as well as the weight can easily be computed.

$$D = \frac{Et_b^3}{12(1 - \nu^2)} \qquad W = \rho A(mt_f + \eta h_c) \tag{4,5}$$

For an open-back mirror

$$t_b^3 = \frac{\left\{\left[1 - \frac{\eta}{2}\right]\left[t_f^4 - \frac{\eta h_c^4}{2}\right] + (t_f + h_c)^4 \frac{\eta}{2}\right\}}{\left[t_f + \frac{\eta h_c}{2}\right]} \tag{6}$$

For a symmetric sandwich mirror

$$t_b^3 = (2t_f + h_c)^3 - \left[1 - \frac{\eta}{2}\right]h_c^3 \tag{7}$$

Figures 4 and 5 illustrate these optimal relationships between faceplate thickness, overall height and rib or core geometry. For this study, open-back and symmetric sandwich mirrors were optimized by varying the rib solidity ratio from 0.1 to 0.8 for heights ranging from four to seven inches and weights ranging from 175 to 250 lbs.

In order to calculate the self weight deflections of these mirrors, only the flexural rigidity, weight, and equivalent bending thickness need to be known. However, when actually designing an open-back or symmetric sandwich mirror there are other design considerations which must be addressed. Cell wall thickness, core cell structure, and quilting or print-through during mirror polishing are also important issues in the design of these mirror types.

The limits on the cell wall thickness are governed by more than one factor. Manufacturing constraints limit the wall thickness to a value required for machining that will not be in great danger of breaking during the manufacturing process. Light-weighting constraints keep the wall thickness from becoming too large and greatly increasing the weight of the mirror. From a study of the expression for the cell solidity ratio η, it can be shown that for any given value of B, the diameter of the cell inscribed circle, the cell wall thickness increases as η increases. For reasonable cell sizes, and cell wall thicknesses at $.0625 \leq t_w \leq 1.250$, a solidity ratio of approximately 0.4 is obtained.[4] This relationship between solidity ratio and cell wall thickness is given in Figure 6.

Choosing the geometry of the cell to be used in an open-back or symmetric sandwich mirror should also be considered. Square, triangular, and hexagonal cells are all commonly used cell geometries. An examination of the peak to peak quilting deflection during mirror polishing of the various cells can aid in choosing the appropriate cell geometry. The expression for mirror quilting as derived by Barnes[5] is dependent upon polishing pressure and faceplate thickness as well as cell geometry.

Peak to peak quilting deflection

$$\delta_c = \psi \left[\frac{E t_f^3}{12(1 - \nu^2)} \right]^{-1} P B^4 \tag{8}$$

$$\psi_{square} = .00126 \quad \psi_{triangle} = .00151 \quad \psi_{hexagonal} = .00111$$

Figures 7, 8, and 9 illustrate the relationship between faceplate thickness and quilting deflection for each of the three cell geometries at a representative polishing pressure of 0.3 psi. Upon examination of these Figures, it becomes obvious that for even modest lightweighting, some significant quilting deflection will occur. For a diffraction limited system, quilting reduces the energy in the central maxima of the diffraction disk. The reduction in energy by quilting is given in Reference 6.

$$\frac{I_1}{I_0} = 1 - \frac{4\pi^2 \left[\frac{\delta_c}{2\lambda}\right]^2}{\left[1 - 2\pi^2 \left[\frac{\delta_c}{2\lambda}\right]^2\right]\left[1 - 4\pi^2 \left[\frac{\delta_c}{2\lambda}\right]^2\right]} \tag{9}$$

The tolerable reduction in energy $\left[\frac{I_1}{I_0}\right]$ will vary for different system requirements, however, so in Figures 7, 8, and 9, the cut-offs are marked for a ten and five percent reduction in energy.

4.2 Deflection

After addressing these issues in design of open-back and symmetric sandwich mirrors, the self weight deflections were calculated for comparison with the contoured back mirrors. An approach used by Selke[7] to find the elastic deflections of a circular mirror on a ring support was used. This approach applies Reissner's theory for the bending of elastic plates which takes into account the transverse shear deformations of the plate. Selke has developed exact, closed form solutions for the deflection of the disk under its own weight, and he has also obtained a solution for the optimum support radius for which the center and edge deflections are equal.

The optimum support radius, such that ω_1 at $r = 0$ equals ω_2 at $r = r_2$ may be found from the following:

$$4r_1^2 \left[2(1 + \nu) \ln\left[\frac{r_1}{r_2}\right] - (3 + \nu)\right] + (7 + 3\nu)r_2^2 - \left[\frac{16D}{C_3}\right] \times (1 + \nu)\left[1 + 2\ln\left[\frac{r_1}{r_2}\right]\right] + \left[\frac{32D(1 + \nu)}{C_n}\right] = 0 \tag{10}$$

The value of r_1 found from this equation is considered the optimal ring support radius. The solutions for the deflections in the two regions $0 \leq r \leq r_1$ and $r_1 \leq r \leq r_2$ are as follows:

$$\omega_1 = \frac{W}{(1+\nu)8\pi D}(r^2 - r_1^2)\left\{\left[\frac{2D(1+\nu)}{r_2^2}\left[\frac{1}{C_3} - \frac{2}{C_n}\right] + (1+\nu)\ln\left[\frac{r_1}{r_2}\right] + \frac{(1+3\nu)}{4}\right] + \frac{1}{8}\left[-\frac{(3-5\nu)r_1^4}{r_2^2} + \frac{4(1-\nu)r^2 r_1^2}{r_2^2} - \frac{(1+\nu)r^4}{r_2^2}\right]\right\} \tag{11}$$

$$\omega_2 = \frac{W}{8\pi D}\left\{\frac{D}{C_3}\left[4\ln\left[\frac{r_1}{r}\right] + \frac{2(r^2-r_1^2)}{r_2^2}\right] + \frac{4D(r_1^2 - r^2)}{C_n\, r_2^2} + r_1^2 \ln\left[\frac{r_2\, r}{r_1^2}\right] + r^2 \ln\left[\frac{r}{r_2}\right] - \frac{(3-5\nu)r_1^4}{8(1+\nu)r_2^2} - \frac{r^4}{8r_2^2} - \frac{(r^2 - r_1^2)(3+\nu)}{4(1+\nu)} + \frac{(r^2\, r_1^2)(1-\nu)}{2r_2^2\,(1+\nu)}\right\} \tag{12}$$

5. MIRROR COMPARISON

Before making a comparison of the merit of the contoured back mirrors to the open-back and sandwich mirrors, the question may be raised of whether the two methods used to calculate deflection are reliable and comparable. In order to answer that question a numerical study was performed to compare the GIFTS finite element method with Selke's equations for maximum mechanical deflection. A solid four inch thick, 40 inch diameter, fused silica mirror was analyzed using both methods. As an independent check, a NASTRAN finite element model of the mirror was also constructed and analyzed for comparison. The calculated maximum self-weight deflections are shown in Figure 10. While there may be variation between the three methods, all are approximations, and as such, some deviation is expected. The difference between the methods is negligible and therefore comparison between deflections calculated using GIFTS and using Selke are assumed valid.

With this confidence in the ability to compare mirror types, the results of different mirror configurations in several different categories: height vs. weight, height vs. deflection, weight vs. deflection, and weight vs. efficiency are shown in Figures 11-14. The mirror efficiency is defined as the total mirror height divided by the mechanical deflection; a high efficiency is desirable. It should be noted that the open-back and sandwich mirrors chosen for comparison with the contoured back mirrors all had a small rib solidity ratio. The lower the solidity ratio, the greater the stiffness. Limits were placed on the lowest acceptable solidity ratio in each case by physical constraints, i.e. faceplate thickness, and overall thickness. For this study it was assumed that the faceplate could be no thinner then .0625 inches and that the overall height was not to exceed 10.0 inches.

Some important conclusions can be drawn by examining these graphs. The sandwich mirror performed best and the single arch the worst in all comparisons except weight vs. height. When equivalent weight mirrors are compared, the double arch is a strong competitor, but when equivalent thickness mirrors are compared, the open-back and solid out perform it. It is interesting to note how closely the open-back and solid mirrors correlate. The open-back is only slightly better then the solid, and when equivalent heights are compared, there is a transition point between which mirror type has the least deflection. Referring to Figure 4, the flexural rigidity of an open-back mirror never exceeds that of an equivalent height solid mirror. This fact may make this transition seem unbelievable. However, simply evaluating mirrors on the basis of flexural rigidity is not sufficient, shear effects must also be considered. Due to these effects an open-back mirror can indeed out perform a simple solid mirror of equivalent height under certain circumstances.

6. FABRICATION AND MOUNTING CONSTRAINTS

In evaluating the performance of different mirror configurations, issues apart from weight and stiffness must be considered. Three additional issues are ease of fabrication, cost of fabrication, and ease of mounting. Each of the mirror configurations studied have definite advantages and disadvantages in each of these specific areas.

Solid mirrors are not only simple to model and analyze but they are also the easiest and cheapest of all the mirror shapes to produce. The simple geometry of a right circular cylinder aids in mounting solid mirrors. It is therefore not surprising that the overwhelming majority of mirrors produced are simple solid configurations.

The contoured back mirrors are slightly more difficult to fabricate and mount then the simple solid. The first configuration studied was the single arch. It is relatively easy to produce, a single cut is required to generate the shape from a solid; alternatively, the mirror may be cast. Unfortunately, the thin edge and cantilever form of the single arch complicate optical fabrication driving the cost of production up. Ease of mounting however, may compensate for the cost of production; a simple center hub is used to mount this configuration. Although the single arch has relatively poor stiffness to weight, the extremely low mass of the simple center support mounting may make the combination of center support and mirror very competitive in weight for mirrors below 0.5m diameter. The second contoured back configuration, the double arch, is slighty more difficult to produce and mount. Two cuts are required to produce the back contour, making fabrication more complicated than fabrication for the single arch. Compensating for this complexity, is the overall greater stiffness of the mirror which reduces fabrication cost. Since sockets placed in the back of the mirror are required to mount the double arch, it is a more difficult mirror shape to mount.

Open-back and sandwich mirrors are the most difficult of all the mirror configurations examined to fabricate and mount. Open-back mirrors are produced by machining or casting. Machining is used for low thermal expansion materials that can not be cast, and is a relatively expensive and high risk operation. Breakage rates of one mirror in two or three machined are not unusual. Quilting is a complication for these mirror types increasing fabrication cost. Low mirror stiffness, combined with the thin structural sections of this type of mirror make the open-back configuration difficult to mount. Finally, the sandwich mirror is quite difficult to produce. For high coefficient of expansion materials, such as the borosilicate glasses, either casting or blow molding are often used. For fused silica, an expensive fritting process is used to produce sandwich mirrors. Beryllium sandwich mirrors are made using an expensive and difficult hot isostatic pressing (HIP) process. Like the open-back mirror, quilting increases the cost of producing this mirror shape. The sandwich mirror is perhaps the most difficult of all mirror shapes to mount, due to the exceptionally thin structural sections used in this type of mirror.

7. CONCLUSION

It is apparent that choosing the appropriate mirror type for an application is not always a clear cut task. Figure 15 is a summary chart of fabrication and mounting factors as well as stiffness and weight factors. Caution must be exercised in use of this chart, in that different factors will usually not have the the same weighing. In a space system, for example, weight is usually much more important than cost. This chart can, however, be a useful tool in making an informed decision on preferred mirror type.

8. ACKNOWLEDGEMENTS

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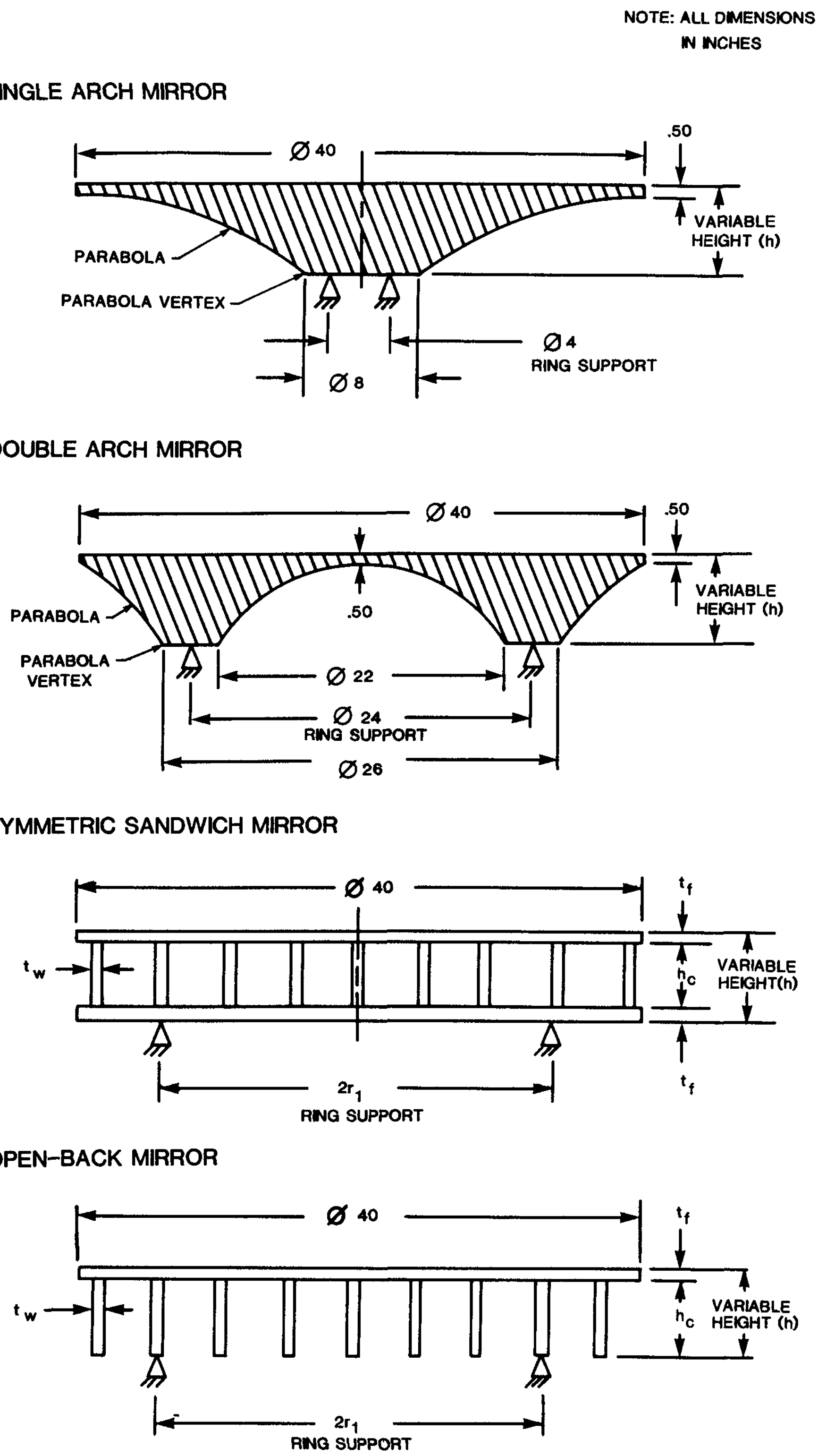

Fig. 1. Mirror geometry for single arch, double arch, symmetric sandwich, and open-back mirrors.

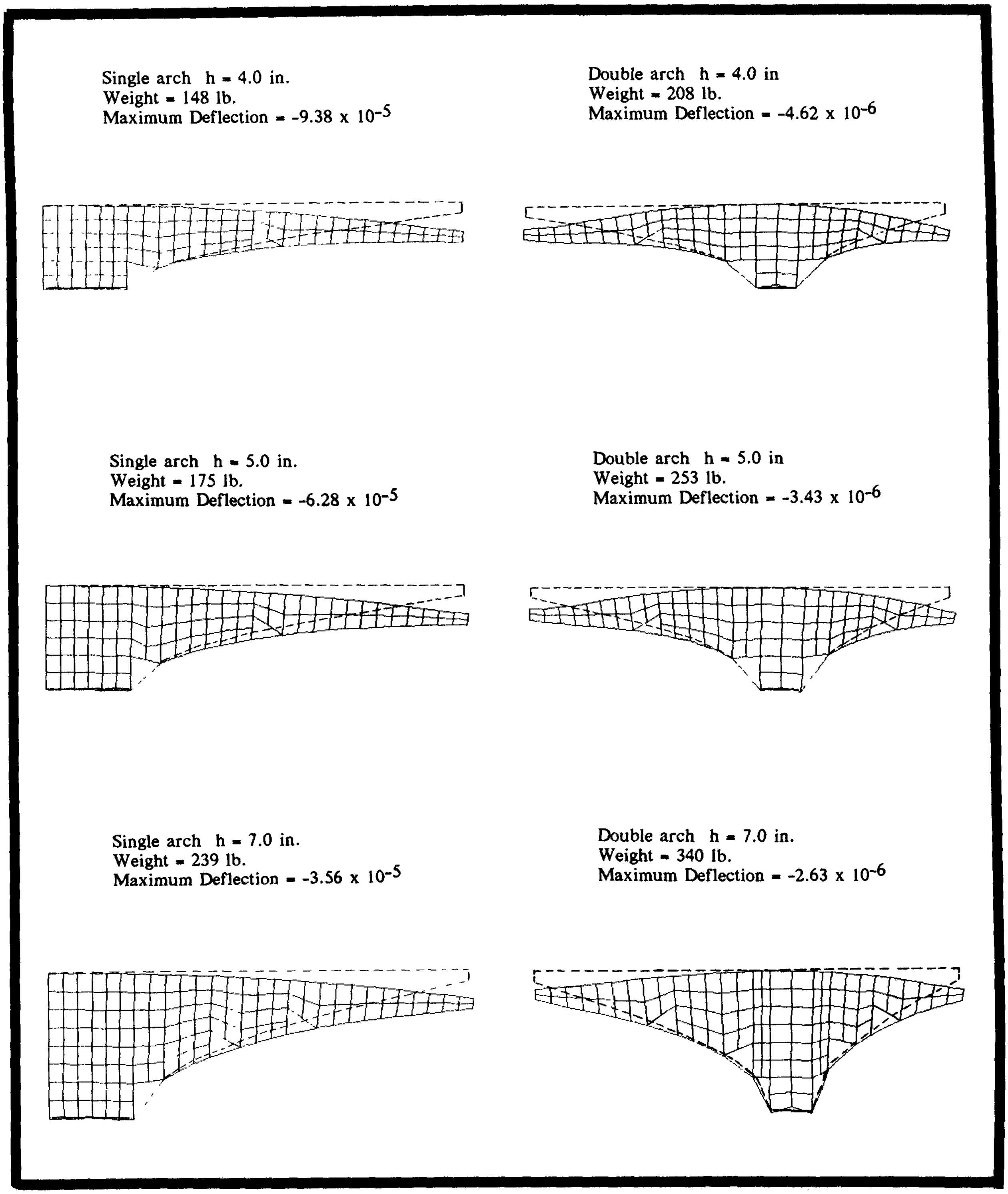

Fig. 2. Gifts finite element models for contoured back mirrors of equivalent height.

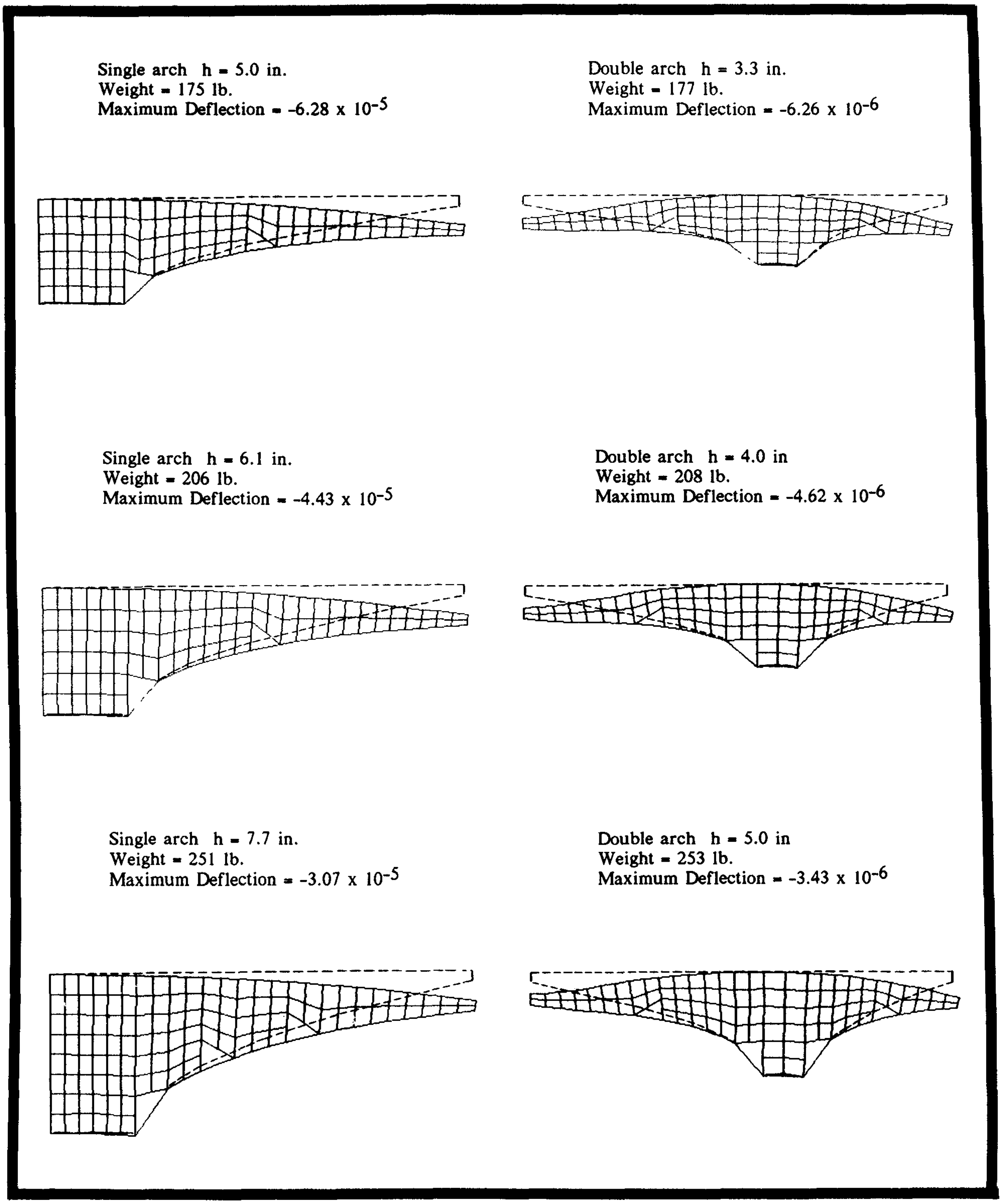

Fig. 3. Gifts finite element models for contoured back mirrors of equivalent weight.

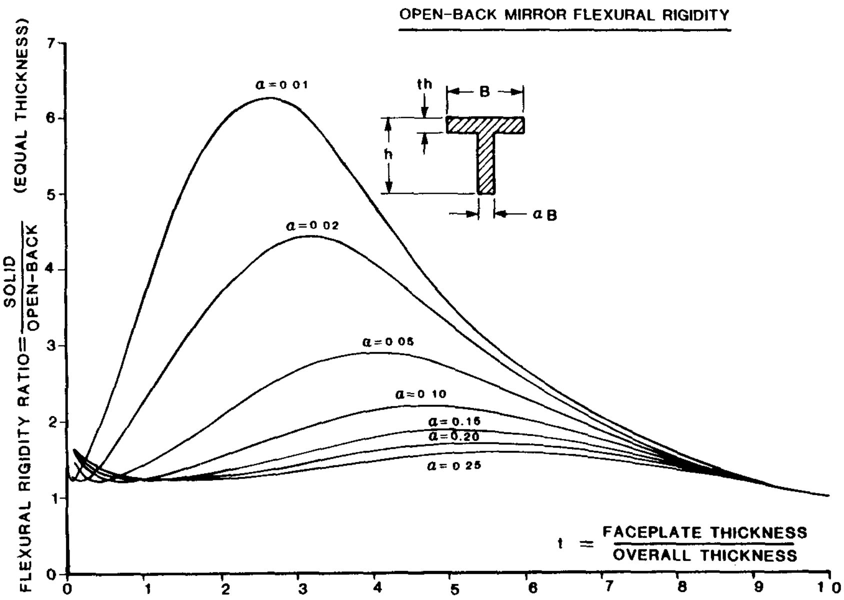

Fig. 4. Open-back mirror flexural rigidity vs. faceplate thickness.

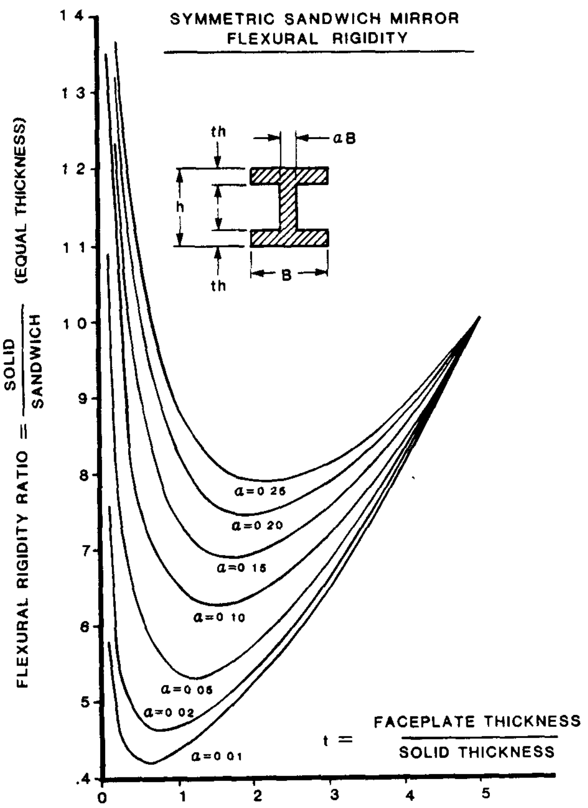

Fig. 5. Symmetric sandwich mirror flexural rigidity vs. faceplate thickness.

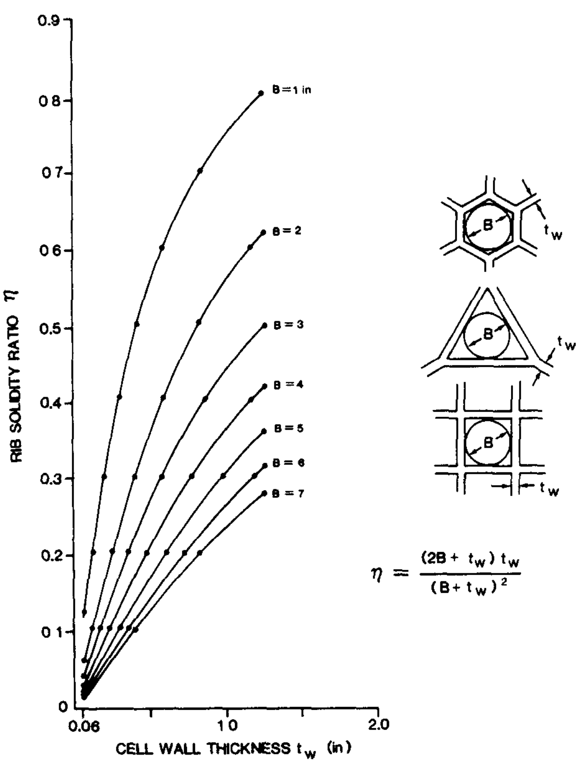

Fig. 6. Cell wall thickness vs. rib solidity ratio for cellular core mirrors.

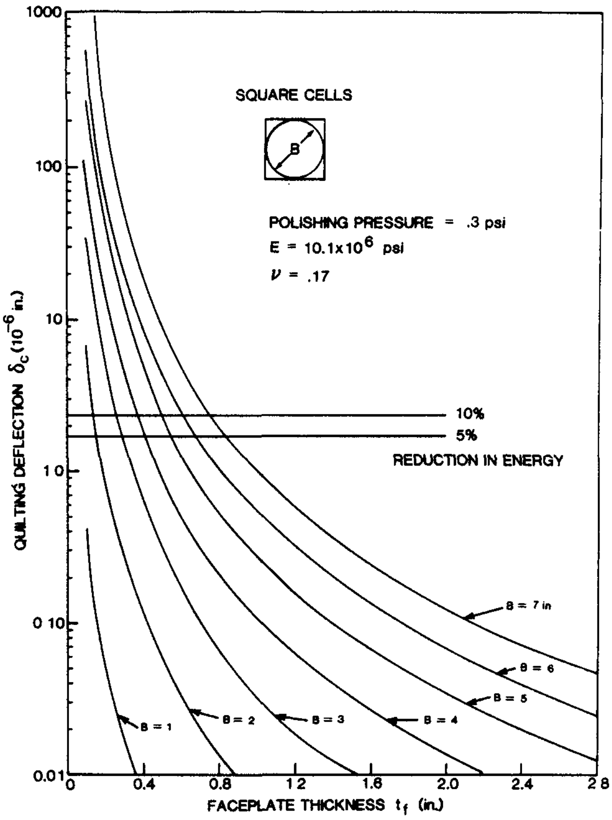

Fig. 7. Faceplate thickness vs. quilting deflection for square cell geometry.

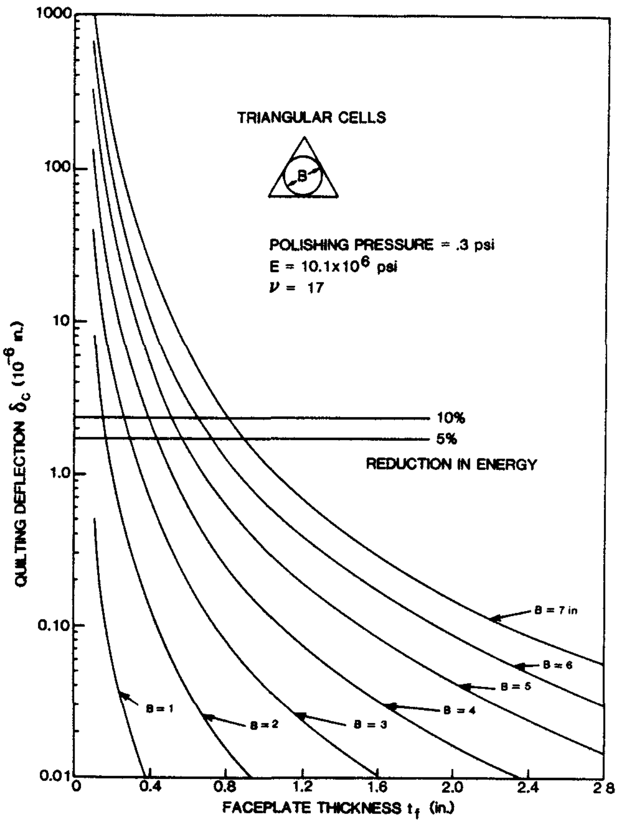

Fig. 8. Faceplate thickness vs. quilting deflection for triangualr cell geometry.

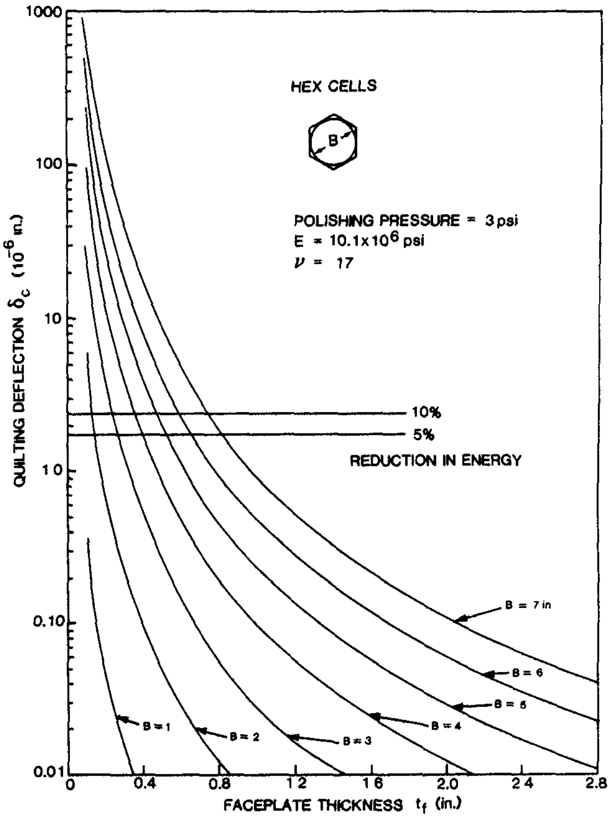

Fig. 9. Faceplate thickness vs. quilting deflection for hexagonal cell geometry.

Method	Support Radius	Element Type	Max. Deflection (in)
GIFTS	13.6	QA4	2.97×10^{-6}
Selke	13.55	---	2.95×10^{-6}
NASTRAN	13.6	TRIARG	2.95×10^{-6}

Fig. 10. Calculated self weight induced deflections for a 40 in. diameter, 4 in. thick, solid fused silica mirror.

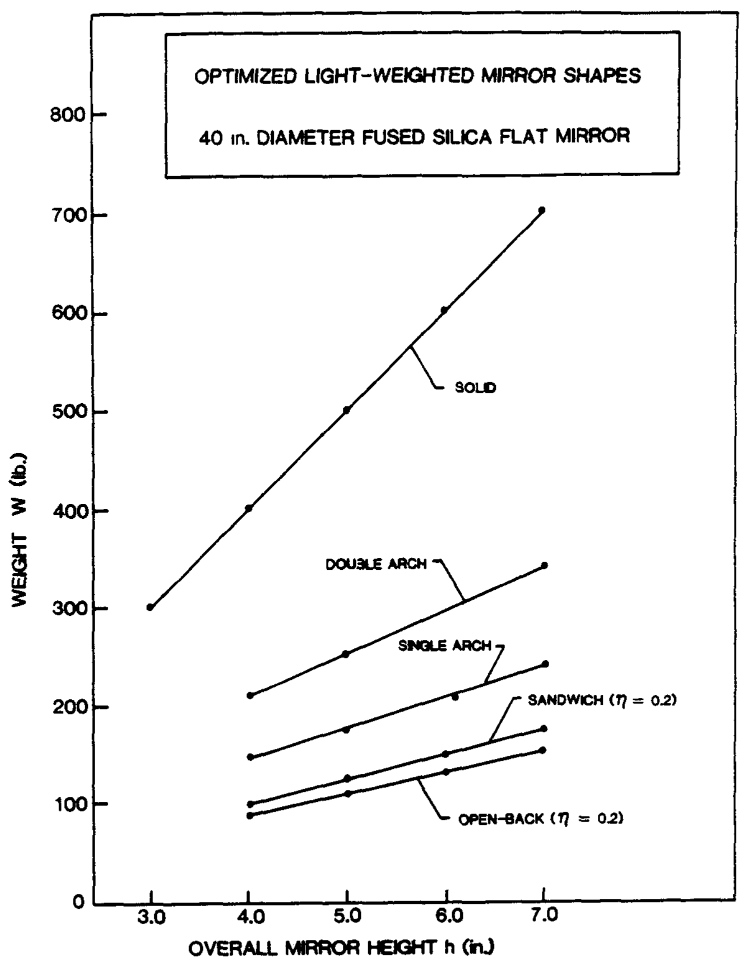

Fig. 11. Overall mirror height vs. mirror weight.

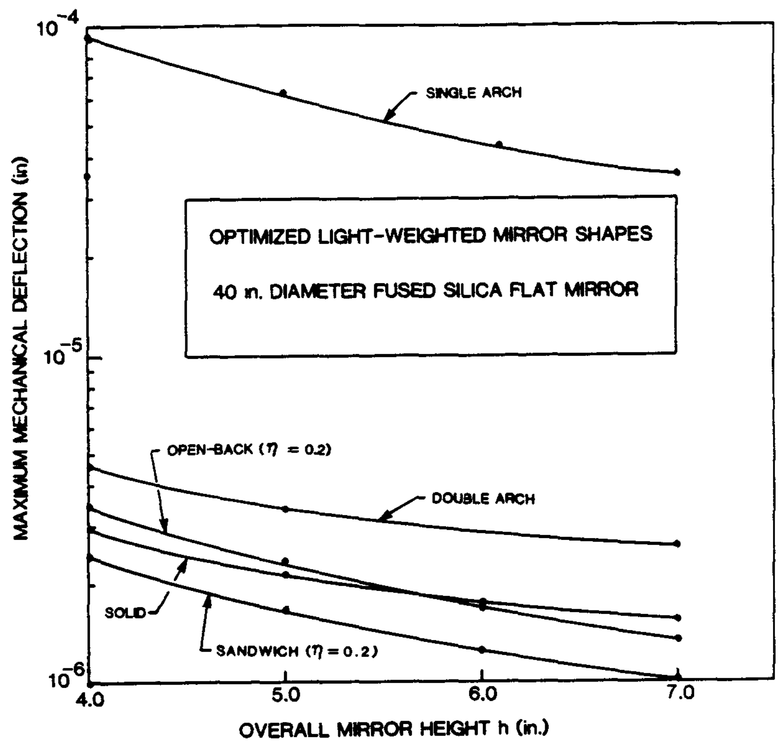

Fig. 12. Overall mirror height vs. mechanical deflection.

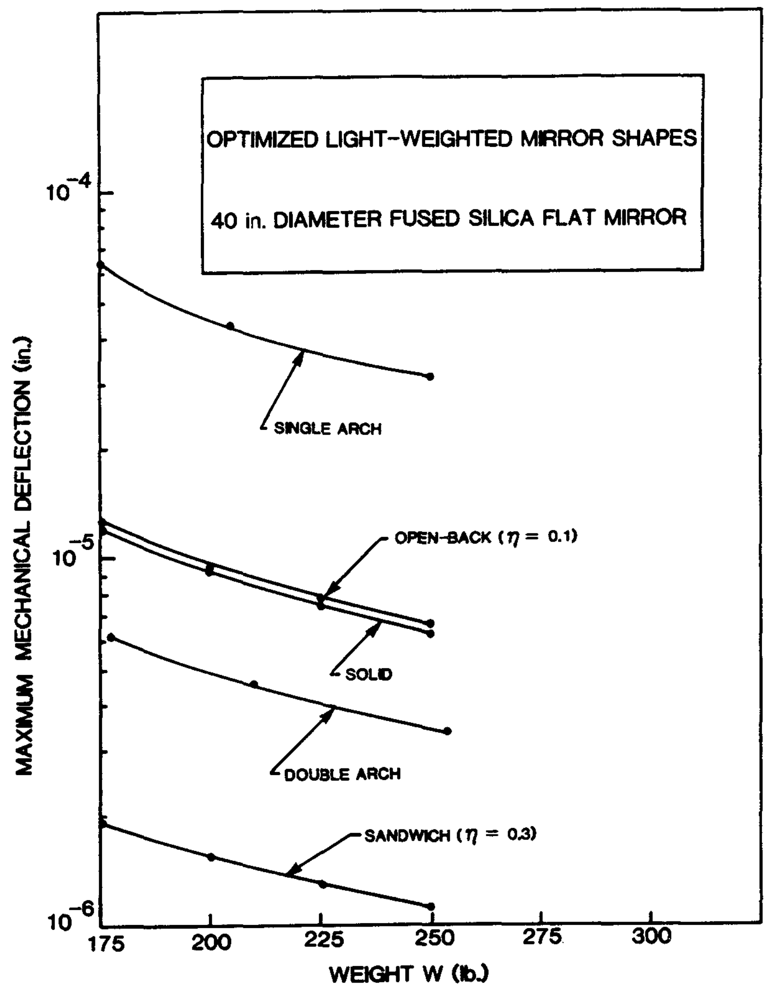

Fig. 13. Mirror weight vs. mechanical deflection.

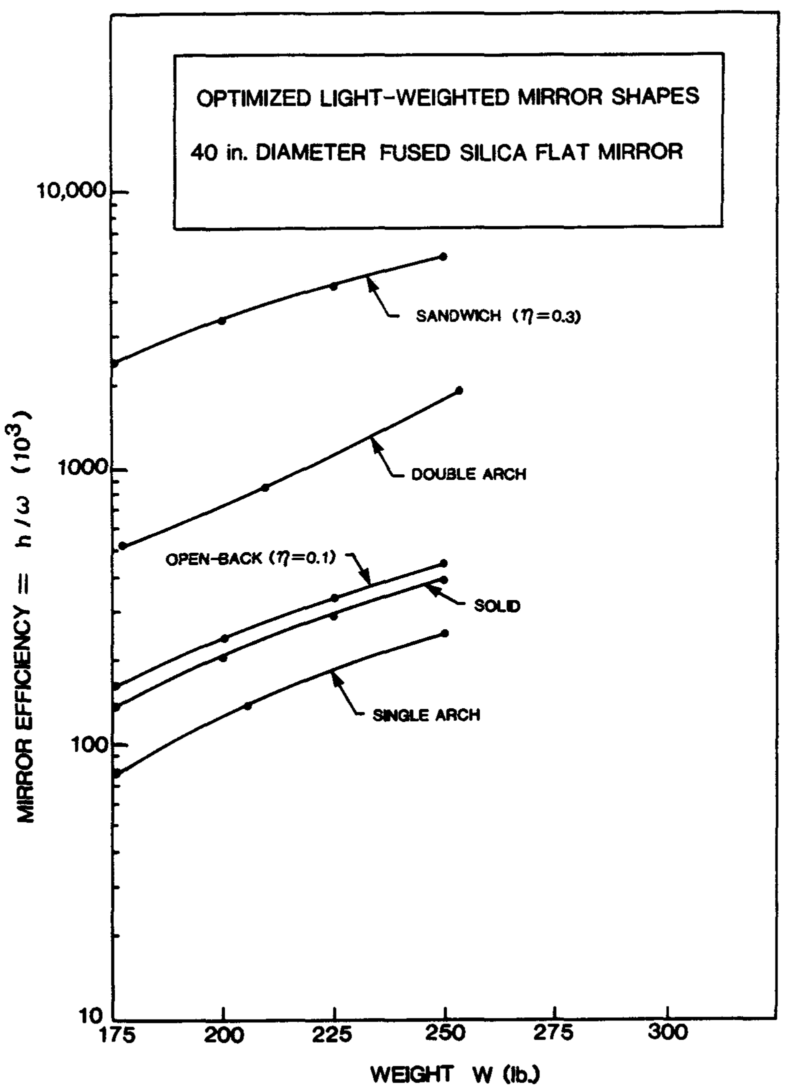

Fig. 14. Mirror weight vs. mirror efficiency.

Mirror Type	Weight vs. Height	Weight vs. Deflection	Height vs. Deflection	Weight vs. Efficiency	Ease & Speed of Fabrication	Cost of Fabrication	Ease of Mounting
Single Arch	3	5	5	5	2	3	2
Double Arch	4	2	4	2	3	2	3
Open-back	1	4	2	3	5	4	4
Symmetric Sandwich	2	1	1	1	5	5	5
Solid	5	3	3	4	1	1	1

1 = Best performance

5 = Worst performance

Fig. 15. Mirror performance summary chart.