Tutorials in Opto-mechanics

"The calculation of focal length using the nodal slide"

Yen-Te Lee

Dec 1, 2008

1. Abstract

First order properties completely describe the mapping from object space to image space. The object-image relationship are well defined by the cardinal points which are front focal point, rear focal point, front principal point, rear principal point, front nodal point and rear nodal point (F, F', P, P', N, N'). Among them, nodal points (N, N') feature an important and useful characteristic of defining the location of unit angular magnification for a focal system. That means a ray passing through one nodal point of the system is mapped to a ray passing through the other nodal point having the same angle with respect to the optical axis. This tutorial explains the properties of nodal points and applies them obtain the focal length of this system.

2. Introduction

An object in space has six degrees of freedom, i.e. three transverse motion plus three rotation. When designing an optical system, it is very important to estimate the allowable tolerances that keep the image quality acceptable due to the six degrees of freedom. However, not all of the tolerances are sensitive to the specific criterion we interest. For example, the tilt of the lens has less sensitivity than the transverse motion of the lens to the line of sight (LOS). And therefore, it is more effective to constrain the sensitive tolerances and loose other ones which are not sensitive especially when there is a cost issue.

In some particular case, the motion does not affect the criterion we interest at all. Nodal point has the property that when we rotate the optical system about that point, the image position does not move. In the following section, we introduce the first order properties of the nodal point and the calculation of effective focal length. Then, we provide the procedures to set up the mechanism of rotation about the nodal point and obtain the effective focal length of the system.

3. Nodal points of a system

3-1. Position of nodal points

To define the location of the nodal points and explore their properties, we use Gaussian equations in this tutorial, which calculate the cardinal points of an optical system with respect to the principal plane.

Consider an optical system as shown in Fig.1. Here the unprimed symbols is used in object space and primed symbols in image space. Ray 1 is the ray emerges from the object space parallel to the optical axis. When mapping to the image space, it will cross the optical axis in rear focal point, F'. Ray 2, the ray emerges from the front focal plane, intercepts with ray 1 in front focal plane. Assume the ray 2 in object space is parallel to the ray 1 in image space. The ray 2 in image space must be parallel to the ray 1 in image space since their conjugate rays cross in the front focal plane. This indicates that the triangles are not only similar, but identical.

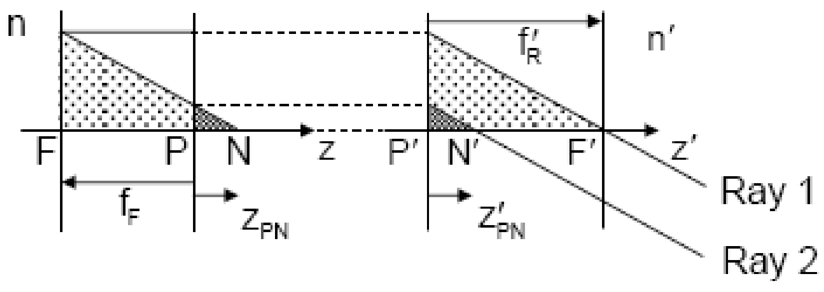


Fig.1. the location of nodal points with respect to the principal points

And therefore, the distance from P to N must be the same as that from P' to N'.

$$Z_{PN} = Z'_{PN}$$

$$Z_{PN} - f_F = f'_R$$

$$Z_{PN} = Z'_{PN} = f_F + f'_R$$

3-2. Magnification

In previous discussion, we obtain the relationship of the location of nodal points. We could also use the distance of nodal points to principal plane to solve the magnification in the plane of nodal points. The way is using the thickness magnification.

$$\frac{Z'_{PN}}{Z_{PN}} = \frac{Z'_N - Z'_P}{Z_N - Z_P} = \left(-\frac{f'_R}{f_F}\right) m_P m_N$$

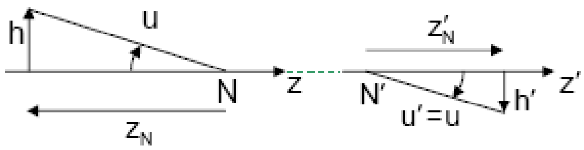
where m_P is the magnification of principal plane.

It is proven that the front and rear principal planes are conjugate planes with magnification equal to 1, i.e. $m_P = 1$. And therefore,

$$\frac{Z'_{PN}}{Z_{PN}} = \left(-\frac{f'_R}{f_F}\right) m_N$$

$$m_N = \left(-\frac{f_F}{f'_R}\right)$$

For an optical system in air, the front focal length is equal to rear focal length with minus sign. So, the magnification of planes of nodal points is unity and the nodal points are coincident with the respective principal planes. If the object and image locations are measured relative to the nodal points, the angle subtended by the object height h as seen from the front nodal point equals the angle subtended by the image height h’ as seen from the rear nodal point. Fig. 2 illustrates the relationship of angular magnification.



$$\frac{h}{-z_N} = \frac{-h'}{z'_N} \qquad m \equiv \frac{h'}{h} = \frac{z'_N}{z_N}$$

Fig.2. angular magnification of nodal points

4. Nodal slide-rotation about the nodal point

From derivation above, the way we correct image rotation due to system rotation is to use the mechanism which rotates about the nodal point. Most of the system is set up in air. In this tutorial, we assume the system is in air so that the nodal points coincide with the principal points. And the use of a nodal slide allows the principal planes and the focal length to be experimentally determined. Nodal slide is the stacks of translation stage and rotation stage, which rotates the system about its rear nodal point. And the image will not move even though the ray bundle forming the image is skewed as shown in Fig. 3.

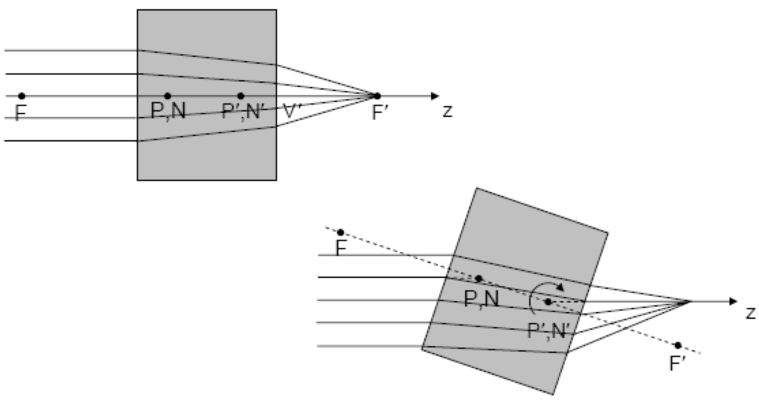


Fig.3. rotation about the rear nodal point of the optical system

The following procedures explain how the nodal slide to be carried out.

1) Mount the optical system on a translation stage and then stack on a rotation stage.

2) Actuate the translation stage until the rear vertex of the optical system coincides with the rotation axis of the rotation stage. With properly positioned, the vertex will not translate when the optical system is rotated.

3) Using a collimated beam emerges to the system and we can determine the rear focal point, F’.

4) Using a microscope (with a micrometer) to measure the distance between the rear vertex V’ and the rear focal point F’. This is by definition the Back Focal Distance (BFD).

5) When we actuate the rotation stage, the image translates because the rotation axis is now coincident with rear vertex of the system. So, we observing the image and reposition the system with the translation stage until the image does not translate when the rotation stage is actuated. And the rear nodal point is now over the rotation axis.

6) The amount the optical system was moved is the separation d’ between the rear vertex and the rear principal plane.

7) Knowing BFD and the distance between rear vertex to rear principal plane d’. The system focal length is therefore found by the relationship shown in Fig. 4.

$$f = f'_R = BFD - d'$$

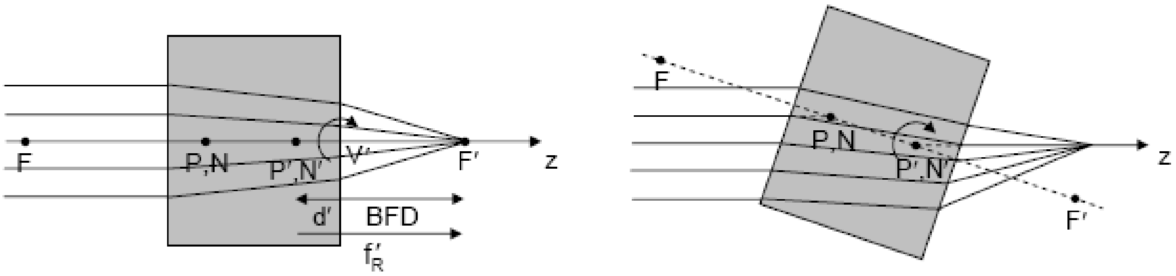


Fig.4. the derivation of focal length

5. Conclusion

The accuracy of the focal length we obtained is determined by the stages we choose. The more accurate stage we use the more accurate result we obtain. The accuracy of calculation degrades with the errors, for example, the roll, pitch and yaw angular errors in translation stage, the axial runout and displacement errors in rotation stage…etc. So, the selection of stages is also an important issue in this application.

6. Reference

[1] John E. Greivenkamp, "Field Guide to Geometrical Optics", SPIE Press, 2004.

[2] Prof. Jim Burge, class notes and lectures of "Introductory opto-mechanicl engineering", Fall, 2008.