

TIE-29: Refractive Index and Dispersion

0. Introduction

The most important property of optical glass is the refractive index and its dispersion behavior.

This technical information gives an overview of the following topics:

- Dispersion
 - o Principal dispersion (page 2)
 - o Secondary spectrum (page 3)
 - o Sellmeier dispersion equation (page 4)
- Temperature dependence of refractive index (page 6)
- Influence of the fine annealing process on the refractive index and Abbe number (page 9)
- Tolerances (page 12)
- Refractive index measurement (page 13)

1. Refractive Index

If light enters a non-absorbing homogeneous material reflection and refraction occurs at the boundary surface. The refractive index n is given by the ratio of the velocity of light in vacuum c to that of the medium v

$$n = \frac{c}{v} \quad (1-1)$$

The refractive index data given in the glass catalogue are measured relative to the refractive index measured in air. The refractive index of air is very close to 1.

Practically speaking the refractive index is a measure for the strength of deflection occurring at the boundary surface due to the refraction of the light beam. The equation describing the amount of deflection is called Snell's law:

$$n_1 \cdot \sin(a_1) = n_2 \cdot \sin(a_2) \quad (1-2)$$

The refractive index is a function of the wavelength. The most common characteristic quantity for characterization of an optical glass is the refractive index n in the middle range of the visible spectrum. This principal refractive index is usually denoted as n_d – the refractive index at the wavelength 587.56 nm or in many cases as n_e at the wavelength 546.07 nm.

2. Wavelength Dependence of Refractive Index: Dispersion

The dispersion is a measure of the change of the refractive index with wavelength. Dispersion can be explained by applying the electromagnetic theory to the molecular structure of matter. If an electromagnetic wave impinges on an atom or a molecule the bound charges vibrate at the frequency of the incident wave.

The bound charges have resonance frequency at a certain wavelength. A plot of the refractive index as a function of the wavelength for fused silica can be seen in Figure 2-1. It can be seen that in the main spectral transmission region the refractive index increases towards shorter wavelength. Additionally the dotted line shows the absorption coefficient as a function of the wavelength.

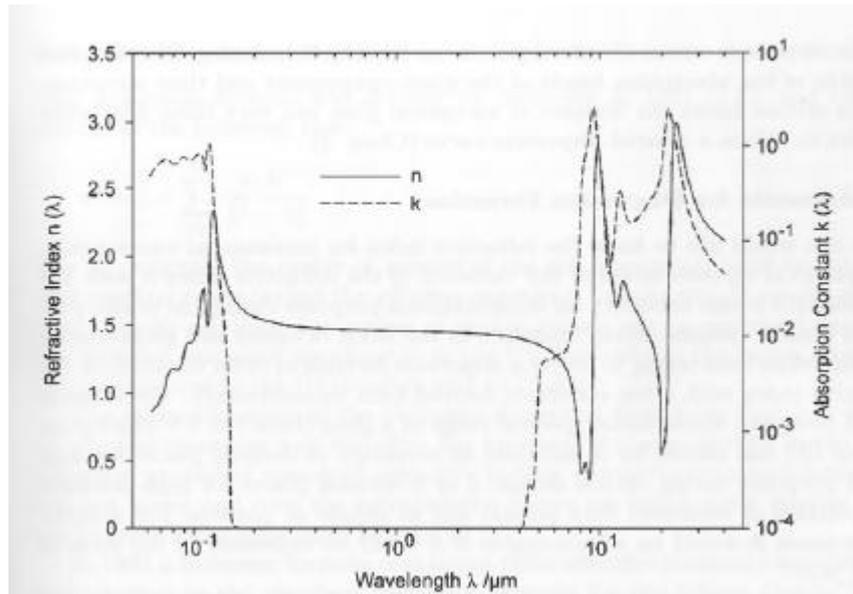


Figure 2-1: Measured optical constants of fused silica (SiO₂ glass) [1]

2.1 Principal Dispersion

The difference ($n_F - n_C$) is called the principal dispersion. n_F and n_C are the refractive indices at the 486.13 nm and 656.27 nm wavelengths.

The most common characterization of the dispersion of optical glasses is the Abbe number. The Abbe number is defined as

$$n_d = (n_d - 1)/(n_F - n_C) \tag{2.1-1}$$

Sometimes the Abbe number is defined according to the e line as

$$n_e = (n_e - 1)/(n_F - n_C) \tag{2.1-2}.$$

Traditionally optical glasses in the range of $v_d > 50$ are called crown glasses, the other ones as flint glasses.

Glasses having a low refractive index in general also have a low dispersion behaviour e.g. a high Abbe number. Glasses having a high refractive index have a high dispersion behaviour and a low Abbe number.

2.2. Secondary Spectrum

The characterization of optical glass through refractive index and Abbe number alone is insufficient for high quality optical systems. A more accurate description of the glass properties is achievable with the aid of the relative partial dispersions.

The relative partial dispersion $P_{x,y}$ for the wavelengths x and y is defined by the equation:

$$P_{x,y} = (n_x - n_y) / (n_F - n_C) \quad (2.2-1)$$

As Abbe demonstrated, the following linear relationship will approximately apply to the majority of glasses, the so-called "normal glasses"

$$P_{x,y} \approx a_{xy} + b_{xy} \cdot n_d \quad (2.2-2)$$

a_{xy} and b_{xy} are specific constants for the given relative partial dispersion.

In order to correct the secondary spectrum (i.e. color correction for more than two wavelengths) glasses are required which do not conform to this rule. Therefore glass types having deviating partial dispersion from Abbe's empirical rule are especially interesting.

As a measure of the deviation of the partial dispersion from Abbe's rule the ordinate difference ΔP is introduced. Instead of relation (2.2-2) the following generally valid equation is used:

$$P_{x,y} = a_{xy} + b_{xy} \cdot n_d + \Delta P_{x,y} \quad (2.2-3)$$

The term $\Delta P_{x,y}$ therefore quantitatively describes a dispersion behavior that deviates from that of the "normal glasses."

The deviations $\Delta P_{x,y}$ from the "normal lines" are listed for the following five relative partial dispersions for each glass type in the data sheets.

$$\begin{aligned} P_{C,t} &= (n_C - n_t) / (n_F - n_C) \\ P_{C,s} &= (n_C - n_s) / (n_F - n_C) \\ P_{F,e} &= (n_F - n_e) / (n_F - n_C) \\ P_{g,F} &= (n_g - n_F) / (n_F - n_C) \\ P_{i,g} &= (n_i - n_g) / (n_F - n_C) \end{aligned} \quad (2.2-4)$$

The position of the normal lines is determined based on value pairs of the glass types K7 and F2. The explicit formulas for the deviations $\Delta P_{x,y}$ of the above-mentioned five relative partial dispersions are:

$$\begin{aligned}
 \Delta P_{C,t} &= (n_C - n_t) / (n_F - n_C) - (0.5450 + 0.004743 \cdot n_d) \\
 \Delta P_{C,s} &= (n_C - n_s) / (n_F - n_C) - (0.4029 + 0.002331 \cdot n_d) \\
 \Delta P_{F,e} &= (n_F - n_e) / (n_F - n_C) - (0.4884 - 0.000526 \cdot n_d) \\
 \Delta P_{g,F} &= (n_g - n_F) / (n_F - n_C) - (0.6438 - 0.001682 \cdot n_d) \\
 \Delta P_{i,g} &= (n_i - n_g) / (n_F - n_C) - (1.7241 - 0.008382 \cdot n_d)
 \end{aligned}
 \tag{2.2-5}$$

Figure 2.2-1 shows the $P_{g,F}$ versus the Abbe number v_d diagram.

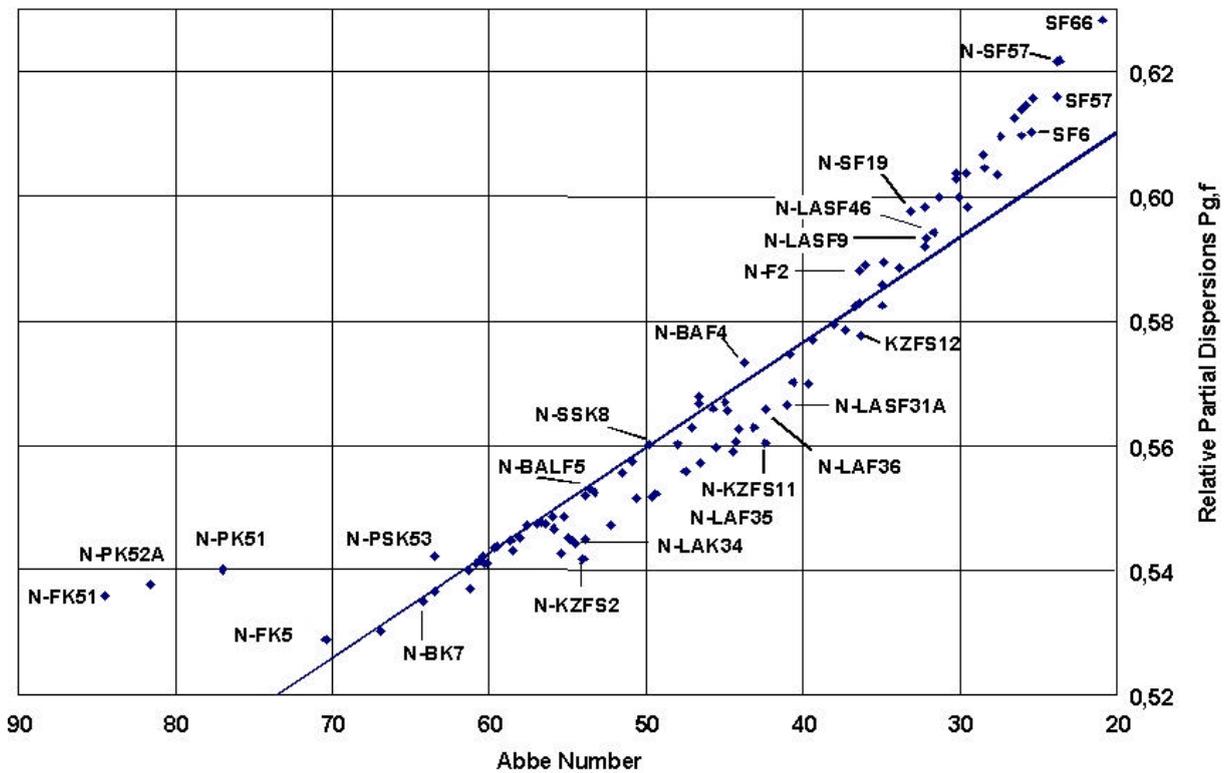


Figure 2.2-1: $P_{g,F}$ as a function of the Abbe number for Schott's optical glass sortiment. Additionally the normal line is given.

The relative partial dispersions listed in the catalog were calculated from refractive indices to 6 decimal places. The dispersion formula (2.3-1) can be used to interpolate additional unlisted refractive indices and relative partial dispersions (see chapter 2.3).

2.3. Sellmeier Dispersion Equation

The Sellmeier Equation is especially suitable for the progression of refractive index in the wavelength range from the UV through the visible to the IR area (to 2.3 μm). It is derived from the classical dispersion theory and allows the description of the progression of refractive index over the total transmission region with one set of data and to calculate accurate intermediate values.

$$n^2(I) - 1 = \frac{B_1 \cdot I^2}{(I^2 - C_1)} + \frac{B_2 \cdot I^2}{(I^2 - C_2)} + \frac{B_3 \cdot I^2}{(I^2 - C_3)} \quad (2.3-1)$$

The determination of the coefficients was performed for all glass types on the basis of precision measurements by fitting the dispersion equation to the measurement values. The coefficients are listed in the data sheets.

The dispersion equation is only valid within the spectral region in which refractive indices are listed in the data sheets of each glass. Interpolation is possible within these limits. The wavelengths used in the equation have to be inserted in μm with the same number of digits as listed in Table 2.3-1. For practical purposes Equation 2.3-1 applies to refractive indices in air at room temperature. The achievable precision of this calculation is generally better than $1 \cdot 10^{-5}$ in the visible spectral range. The coefficients of the dispersion equation can be reported for individual glass parts upon request. This requires a precision measurement for the entire spectral region, provided the glass has sufficient transmission.

Wavelength [nm]	Designation	Spectral Line Used	Element
2325.42		infrared mercury line	Hg
1970.09		infrared mercury line	Hg
1529.582		infrared mercury line	Hg
1060.0		neodymium glass laser	Nd
1013.98	t	infrared mercury line	Hg
852.11	s	infrared cesium line	Cs
706.5188	r	red helium line	He
656.2725	C	red hydrogen line	H
643.8469	C'	red cadmium line	Cd
632.8		helium-neon-gas-laser	He-Ne
589.2938	D	yellow sodium line	Na
		(center of the double line)	
587.5618	d	yellow helium line	He
546.074	e	green mercury line	Hg
486.1327	F	blue hydrogen line	H
479.9914	F'	blue cadmium line	Cd
435.8343	g	blue mercury line	Hg
404.6561	h	violet mercury line	Hg
365.0146	i	ultraviolet mercury line	Hg
334.1478		ultraviolet mercury line	Hg
312.5663		ultraviolet mercury line	Hg
296.7278		ultraviolet mercury line	Hg
280.4		ultraviolet mercury line	Hg
248.3		ultraviolet mercury line	Hg

Table 2.3-1: Wavelengths for a selection of frequently used spectral lines

3. Temperature Dependence of Refractive Index

The refractive indices of the glasses are not only dependent on wavelength, but also upon temperature. The relationship of refractive index change to temperature change is called the temperature coefficient of refractive index. This can be a positive or a negative value. The data sheets contain information on the temperature coefficients of refractive index for several temperature ranges and wavelengths. The temperature coefficients of the relative refractive indices $\Delta n_{rel}/\Delta T$ apply for an air pressure of $0.10133 \cdot 10^6$ Pa. The coefficients of the absolute refractive indices dn_{abs}/dT apply for vacuum.

The temperature coefficients of the absolute refractive indices can be calculated for other temperatures and wavelengths values with the aid of equation (3-1).

$$\frac{dn_{abs}(I, T)}{dT} = \frac{n^2(I, T_0) - 1}{2 \cdot n(I, T_0)} \cdot (D_0 + 2 \cdot D_1 \cdot \Delta T + 3 \cdot D_2 \cdot \Delta T^2 + \frac{E_0 + 2 \cdot E_1 \cdot \Delta T}{I^2 - I_{TK}^2}) \quad (3-1)$$

Definitions:

T_0	Reference temperature (20°C)
T	Temperature (in °C)
ΔT	Temperature difference versus T_0
λ	Wavelength of the electromagnetic wave in a vacuum (in μm)
D_0, D_1, D_2, E_0, E_1 and λ_{TK} :	constants depending on glass type

This equation is valid for a temperature range from -40°C to +80°C and wavelengths between 0.6438 μm and 0.4358 μm . The constants of the dispersion formula are also calculated from the measurement data and listed on the test certificate.

The temperature coefficients in the data sheets are guideline values. Upon request, measurements can be performed on individual melts in the temperature range from -100°C to +140°C and in the wavelength range from 0.3650 μm to 1.014 μm with a precision better than $\pm 5 \cdot 10^{-7}/\text{K}$. The accuracy at the limits of the measurement range is somewhat less than in the middle of this interval.

The temperature coefficients of the relative refractive indices $\Delta n_{rel}/\Delta T$ and the values for Δn_{abs} can be calculated with the help of the equations listed in Technical Information TI Nr. 19 (available upon request).

Figures 3-1 to 3-4 show the absolute temperature coefficient of refractive index for different glasses, temperatures and wavelengths.

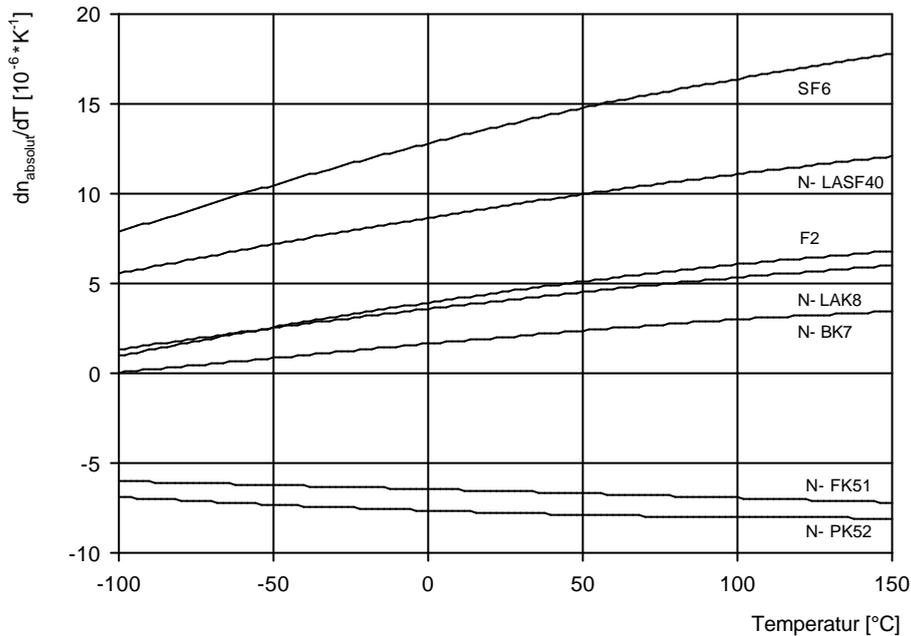


Figure 3-1: Temperature coefficient of the absolute refractive index of several optical glasses in relationship to temperature at the 435.8 nm wavelength.

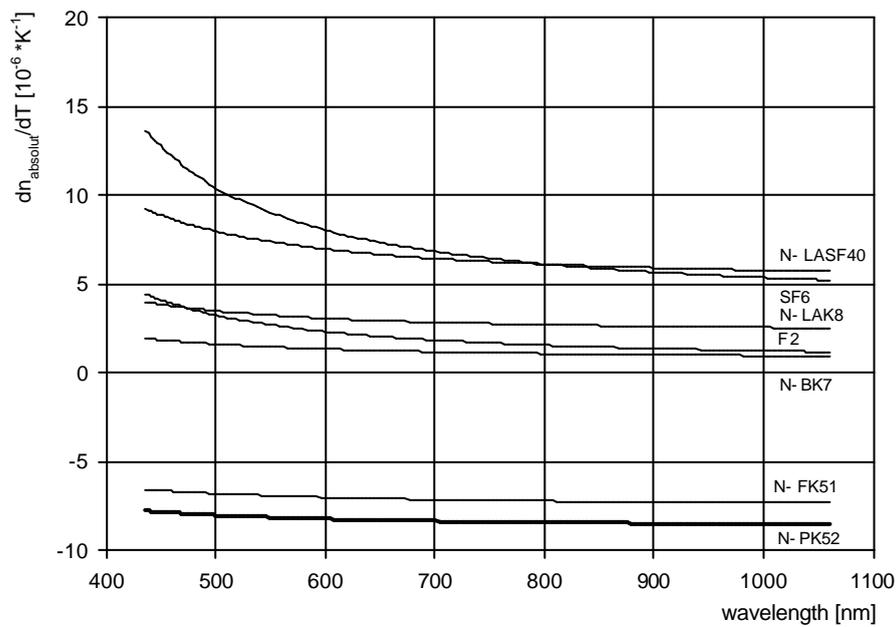


Figure 3-2: Temperature coefficient of the absolute refractive index of several optical glasses in relationship to the wavelength at 20°C.

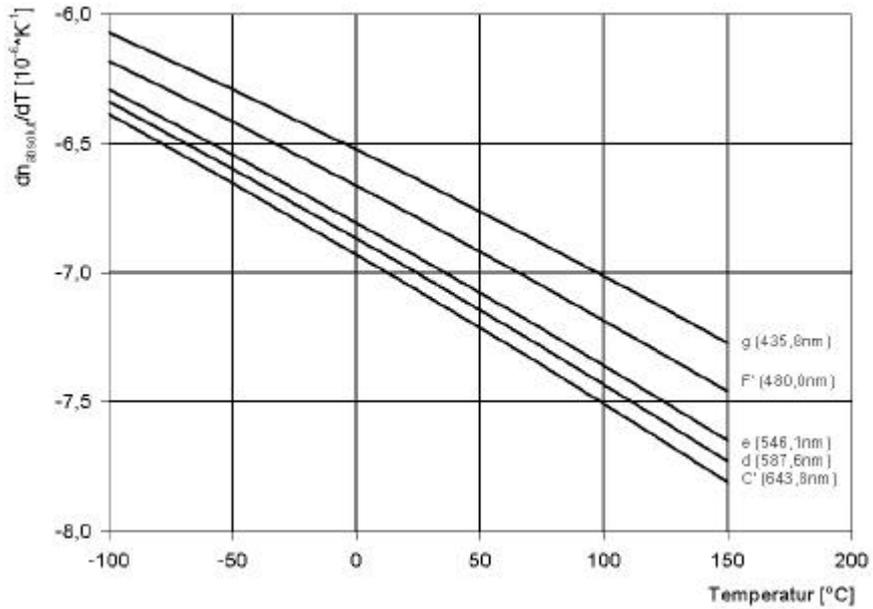


Figure 3-3: Temperature coefficient of the absolute refractive index in relation to the temperature for several wavelengths in the visible spectral range for glass type N-FK51.

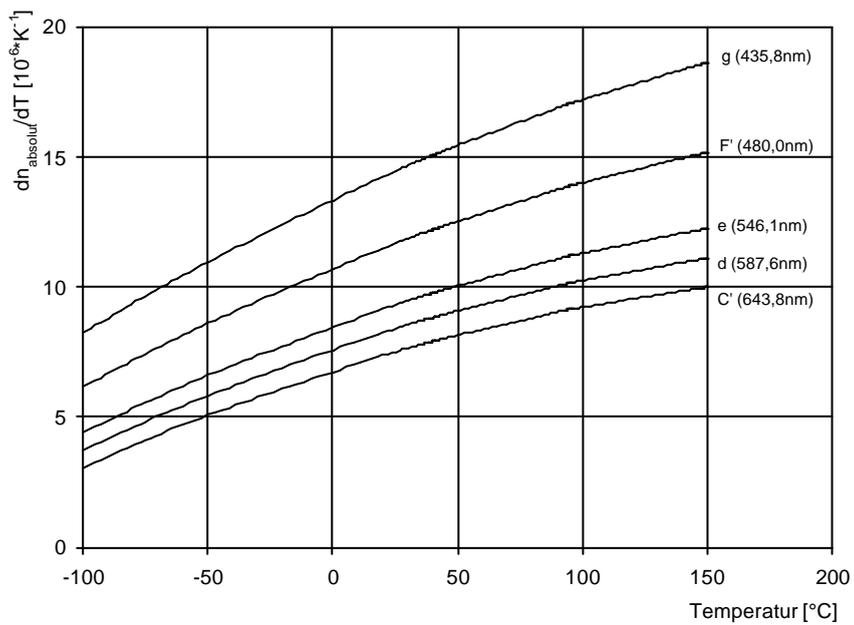


Figure 3-4: Temperature coefficient of the absolute refractive index in relation to the temperature for several wavelengths in the visible spectral range for glass type SF 6.

4. Influence of the Fine Annealing Process on the Refractive Index and Abbe number

The optical data for a glass type are chiefly determined by the chemical composition and thermal treatment of the melt. The annealing rate in the transformation range of the glass can be used to influence the refractive index within certain limits (depending on the glass type and the allowable stress birefringence). Basically slower annealing rates yield higher refractive indices. In practice, the following formula has proven itself.

$$n_d(h_x) = n_d(h_0) + m_{nd} \cdot \log(h_x / h_0) \tag{4-1}$$

h_0 Original annealing rate

h_x New annealing rate

m_{nd} ? Annealing coefficient for the refractive index depending on the glass type

The refractive index dependence on annealing rate is graphically shown in Figure 4-1.

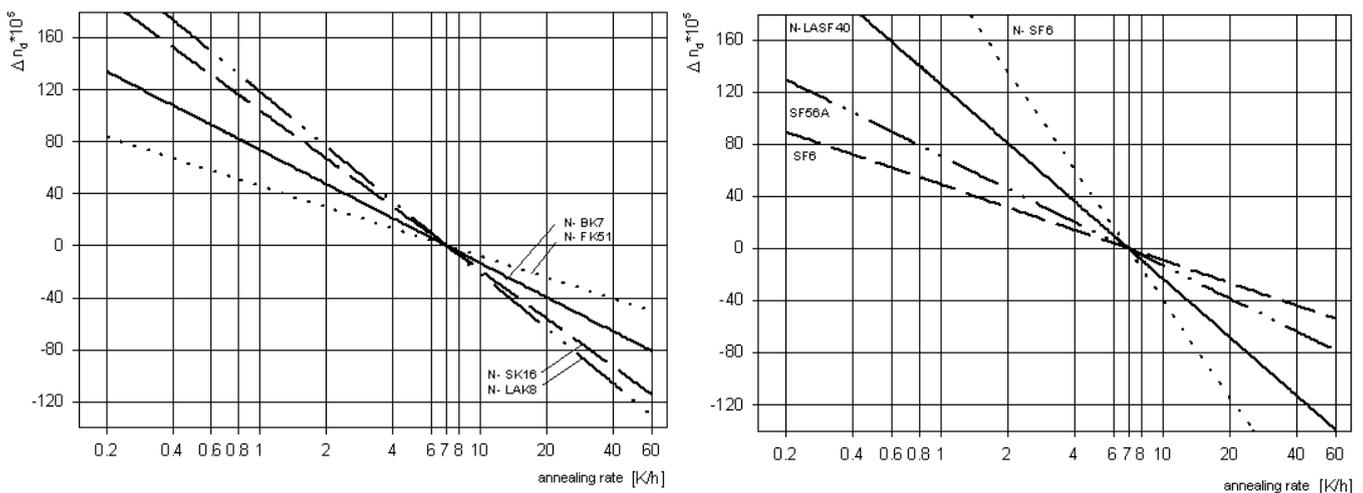


Figure 4-1: Dependence of refractive index on the annealing rate for several glass types. Reference annealing rate is 7 K/h

An analogous formula applies to the Abbe number.

$$n_d(h_x) = n_d(h_0) + m_{nd} \cdot \log(h_x / h_0) \tag{4-2}$$

m_{vd} ? Annealing coefficient for the Abbe number depending on the glass type

The annealing coefficient m_{vd} can be calculated with sufficient accuracy with the following equation:

$$m_{nd} = (m_{nd} - n_d(h_0) \cdot m_{nF-nC}) / (n_F - n_C) \tag{4-3}$$

The coefficient m_{nF-nC} has to be determined experimentally.

Figure 4-2 shows that individual glass types vary greatly in their dependence of the Abbe number on the annealing rate. In general also the Abbe number increases with decreasing annealing rate. High index lead free glass types like N-SF6 show anomalous behavior. Anomalous behaviour means that the Abbe number decreases with decreasing annealing rate.

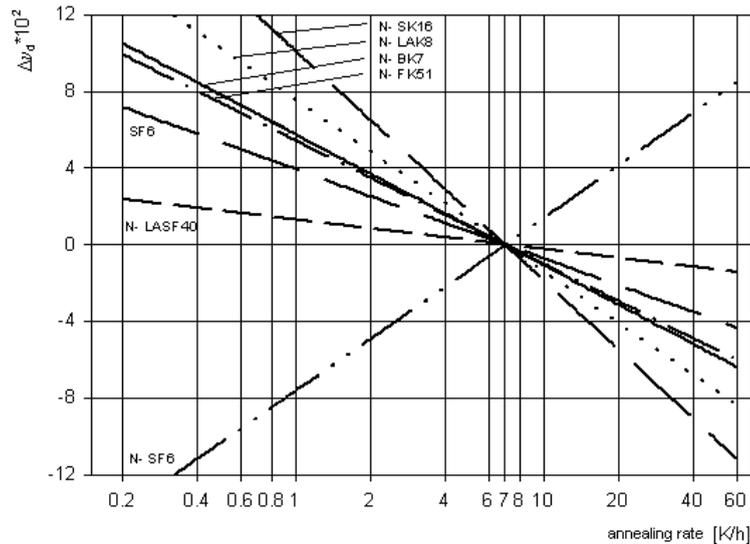


Figure 4-2: Abbe number as a function of the annealing rate for several glass types. Reference annealing rate is 7 K/h

Values for Annealing coefficients of some optical glasses are shown in Table 4-1. We will provide the values for the annealing coefficients of our glasses upon request.

	m_{nd}	m_{nF-nc}	m_{hd}
N-BK7	-0.00087	-0.000005	-0.0682
N-FK51	-0.00054	-0.000002	-0.0644
SF 6	-0.00058	+0.000035	-0.0464
N-SF6	-0.0025	-0.000212	0.0904

Table 4-1: Annealing coefficients for several selected glass types

The annealing rate can be used to adjust the refractive index and Abbe number to the desired tolerance range.

In practice the annealing rate influences the refractive index and the Abbe number simultaneously. Figure 4-3 shows a diagram of the Abbe number versus the refractive index for N-BK7. The rectangular boxes indicate the tolerance limits (steps) for the refractive index and the Abbe number. For example the largest box with a dotted frame indicates the tolerance borders for step 3 in refractive index and step 4 in Abbe number. The smallest box indicates step 1 in refractive index and Abbe number. In the center of the frames is the nominal catalog value.

After melting the optical glass is cooled down at a high annealing rate. To control the refractive index during the melting process samples are taken directly from the melt after each casting. These samples are cooled down very fast together with a reference sample of the same glass. The reference sample has a known refractive index at an annealing rate of 2°C/h. By measuring the change in refractive index of the reference sample the refractive index of the sample can be measured with moderate accuracy in the range of $\pm 10^{-4}$.

The annealing rate dependence of the Abbe number and refractive index of each glass is represented by a line in the diagram having a slope that is characteristic for the glass type. For a given melt the position of the line in the diagram is given by the initial refractive index / Abbe number measurement for a cooling rate of 2°C/h as a fix-point together with the glass typical slope. The refractive index and Abbe number for a given glass part can be adjusted by a fine annealing step along this characteristic line.

Glass for cold processing has to be fine annealed to reduce internal stresses. During this fine annealing the annealing rate is in general lower than 2°C/h. The initial refractive index has to be adjusted during melting in such a way that the desired tolerances can be reached during fine annealing. The initial refractive index of N-BK7 for example is in general lower than the target value.

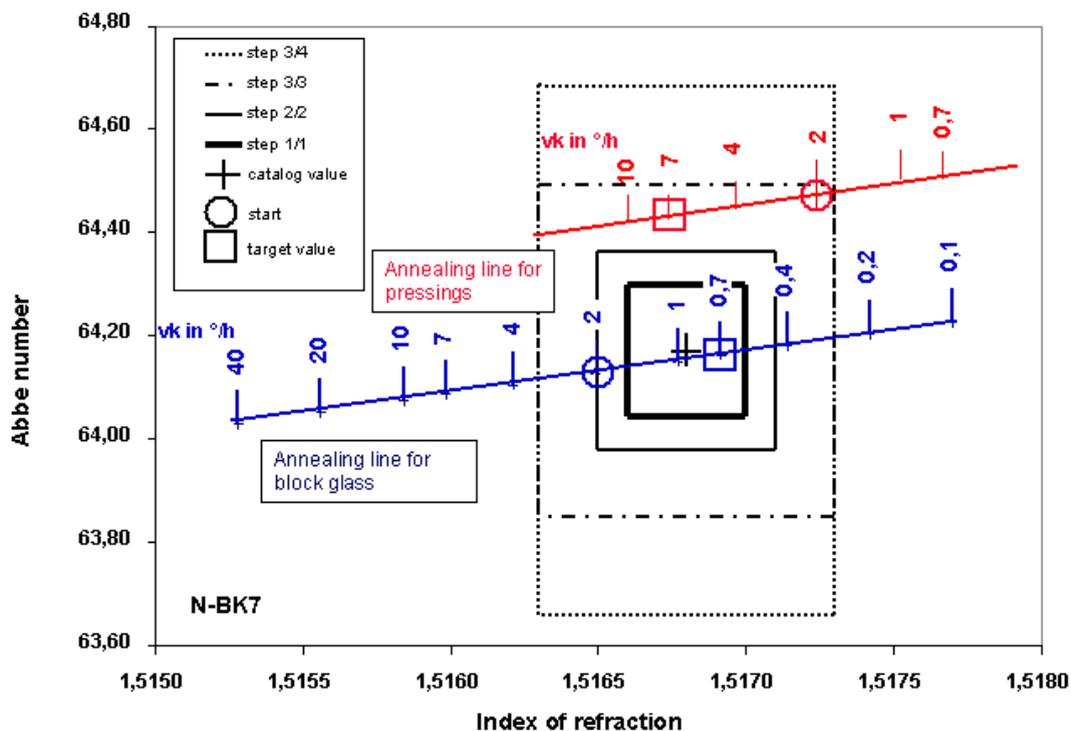


Figure 4-3: The influence of the annealing rate on the refractive index and Abbe number of N-BK7 for different initial refractive indices.

Glass for hot processing i.e reheat pressing is subjected to much more rapid annealing. The heat treatment processes used by the customer in general use annealing rates much higher than 2°C/h. Therefore for N-BK7 pressings for example the initial refractive index needs to be higher than the target value. For a better visualization in figure 43 the annealing line for pressings was shifted to higher Abbe numbers. In general it is also possible to achieve step 1/1 for pressings after hot processing. We deliver an annealing schedule for each batch of glass for hot processing purpose. This annealing schedule contains the initial refractive index at 2°C/h and the limit annealing rates to stay within the tolerances.

5. Tolerances

The refractive indices, which are listed to 5 decimal places in the data sheets, represent values for a melt with nominal n_d-v_d position for the glass type in question. The refractive index data are exact to five decimal places (for $\lambda > 2 \mu\text{m}$: $\pm 2 \cdot 10^{-5}$). The accuracy of the data is less in wavelength regions with limited transmission. All data apply to room temperature and normal air pressure ($0.10133 \cdot 10^6 \text{ Pa}$).

Defining tolerances for the refractive index of a glass the customer has to distinguish between the refractive index tolerance, the tolerance of refractive index variation within a lot and the refractive index homogeneity (figure 5-1).

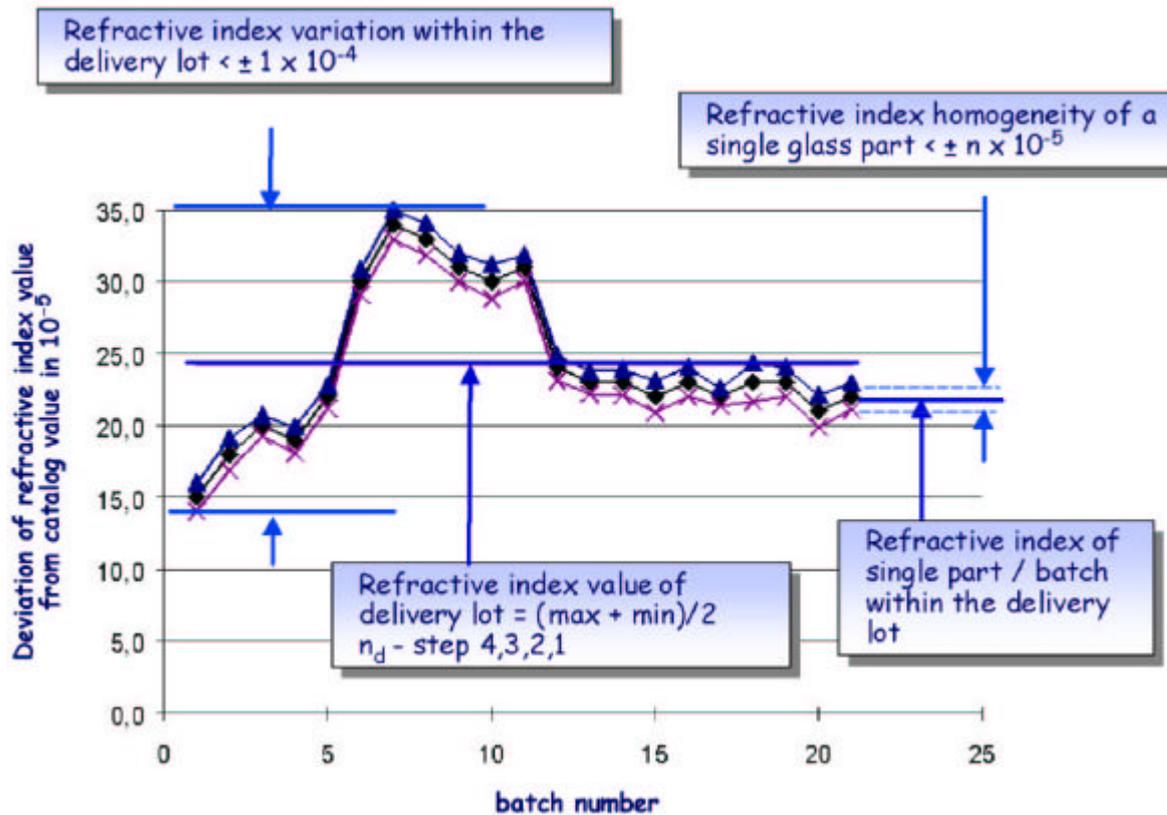


Figure 5-1: Refractive index variation from within a production sequence.

All deliveries of fine annealed block glass and fabricated glass are made in lots of single batches. The batch may be a single block or some few strip sections. More information on the new lot id system can be found in [3].

The refractive index and Abbe number tolerance is the maximum allowed deviation of a single part within the delivery lot from nominal values given in the data sheets of the catalog. The refractive index of the delivery lot given in the standard test certificates is given by the following formulae:

$$n_{lot} = (n_{max} + n_{min}) / 2 \tag{5-1}$$

n_{max} is the maximum and n_{min} the minimum refractive index within the lot.

The refractive index variation from part to part within a lot is always smaller than $\pm 1 \cdot 10^{-4}$. The refractive index homogeneity within a single part is better than $\pm 2 \cdot 10^{-5}$ in general [4]. A short summary of the refractive index tolerance, variation and homogeneity grades can be found in table 5-1. More information is given in the optical glass catalogue [5].

Tolerance	Grade	Refractive Index [$\cdot 10^{-5}$]	Abbe Number
Absolute	Step 4	--	$\pm 0.8\%$
	Step 3	± 50	$\pm 0.5\%$
	Step 2	± 30	$\pm 0.3\%$
	Step 1	± 20	$\pm 0.2\%$
Variation	SN	± 10	--
	S0	± 5	--
	S1	± 2	--
Homogeneity	H1	± 2	--
	H2	± 0.5	--
	H3	± 0.2	--
	H4	± 0.1	--
	H5	± 0.05	--

Table 5-1: Refractive Index Tolerances

6. Refractive Index Measurement

For refractive index measurement two different measurement setups are used: the v-block refractometer (figure 6-2) and the spectral goniometer. Figure 6-1 shows the principle of the v-Block measurement. The samples are shaped in a nearly square shape. One sample is about 20x20x5 mm small. The sample will be placed in a v shaped block prism. The refractive index of this prism is known very precisely. The refraction of an incoming light beam depends on the refractive index difference between the sample and the v-block-prism. The advantage of this method is that up to 10 samples can be glued together into one v-block stack. Therefore many samples can be measured in a very short time. The relative measurement accuracy is very high therefore differences in refractive index within one v-block stack can be measured very accurately. Standard measurement temperature is 22°C.

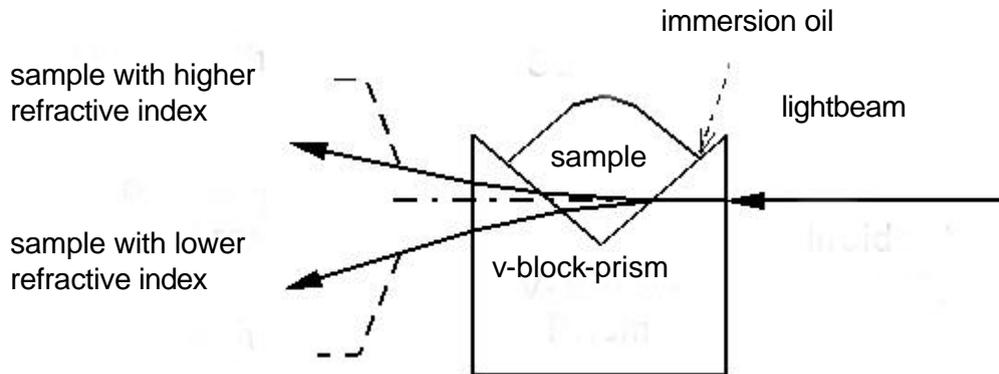


Figure 6-1: Refractive index variation from within a production sequence.

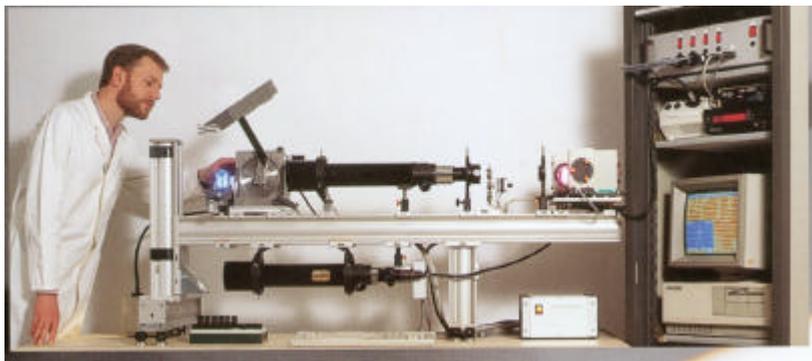


Figure 6-2: V-block refractometer.

The spectral goniometric method is based on the measurement of the angle of minimum refraction in a prism shaped sample. This is the most accurate absolute refractive index measurement method. In our laboratory we have an automated spectral goniometer with high accuracy and the ability to measure in the infrared and UV region (figure 6-3).

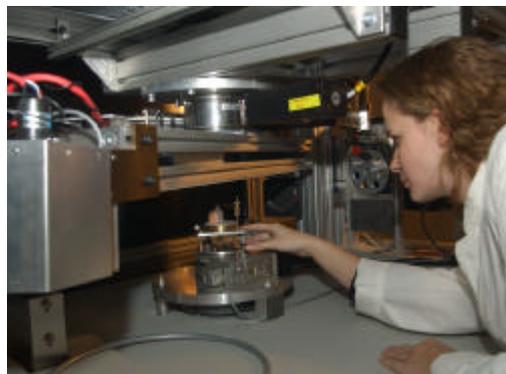


Figure 6-3: Automated spectral goniometer.

With the automated spectral goniometer, the Ultraviolet to infrared Refractive Index measurement System (URIS), the refractive index of optical glasses can be measured to an accuracy of $\pm 4 \cdot 10^{-6}$. The measurement accuracy for the dispersion ($n_F - n_C$) is $\pm 2 \cdot 10^{-6}$. These measurement accuracies can be achieved independent of the glass type and over the complete wavelength range from 185 nm to 2325 nm. The measurement is based on the minimum angle of deviation principle. The samples are prism shaped with dimensions of about 35 x 35 x 25 mm³. The standard measurements temperature is 22°C. The temperature can be varied between 18 to 28°C on request. The standard measurement atmosphere is air. On special request also nitrogen is possible.

Table 6-1 shows a summary of the refractive index measurements available at Schott.

Measurement	Measurement accuracy		Wavelengths	Method
	Refractive index	Dispersion		
V-block standard	$\pm 30 \cdot 10^{-6}$	$\pm 20 \cdot 10^{-6}$	g, F', F, e, d, C', C	v-block refractometer
V-block enhanced	$\pm 20 \cdot 10^{-6}$	$\pm 10 \cdot 10^{-6}$	l, h, g, F', F, e, d, C', C, r, t	
Precision spectrometer	$\pm 4 \cdot 10^{-6}$	$\pm 2 \cdot 10^{-6}$	185 nm – 2325 nm	URIS automatic spectral goniometer

Table 6-1: Absolute refractive index measurement accuracies

The temperature coefficient of refractive index is measured using the automated spectral goniometer and a temperature controlled climate chamber with a temperature range from -100°C up to +140°C. The temperature coefficient can be measured with an accuracy of $\pm 0.5 \cdot 10^{-6} \text{ K}^{-1}$.

7. Literature

- [1] The properties of optical glass; H. Bach & N. Neuroth (Editors), Springer Verlag 1998
- [2] SCHOTT Technical Information No. 19 (available upon request).
- [3] SCHOTT Technical Note No. 4: Test report for delivery lots
- [4] SCHOTT Technical Information No. 26: Homogeneity of optical glass
- [5] SCHOTT Optical Glass Pocket Catalogue

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