Synopsis of a published paper

"Fundamentals of establishing an optical tolerance budget"

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-Abstract

This report gives a synopsis of a published paper [1], entitled "Fundamentals of establishing an optical tolerance budget" by Warren J. Smith. This paper explained how we deal with the combination of tolerances for multi-element system in statistical aspects with an example illustrated. Then, the author introduces a simple and more useful way called the square Root of the Sum of the Squares, which is often abbreviated R.S.S. A more realistic example is used to explain how we determine the error budget knowing the total budget and the residual optical design error by the use of R.S.S. rule.

-Introduction

When designing an optical system, we have to be given the required performance specifications. Usually, this information could be provided by the customer or by the experiences and data base of the designer. And the design is normally somewhat better than the performance specifications require, in order to allow for the degradation from the fabrication tolerances. Therefore, the error budget for fabrication is determined when knowing the residual error of the design.

Usually, an optical system is complex and consists of numbers of element. The performance of the optical system subjects to the fabrication tolerance variation include surface radius and figure, element thickness, airspace, refractive index, chromatic dispersion, surface tilt, etc. Some of them are very sensitive and the others have little effects to the performance. So, the balance of individual tolerance becomes tricky. An excellent design should maintain the tolerances to assure the performance of the optical system and keep the cost lower at the same time.

In this paper, the author considers the effects of fabrication errors to the performance requirement based on wavefront deformation (Optical Path Difference, or OPD). The resultant fabrication error budget is determined by calculating a “change table”, that is, a table of the partial differentials of the aberrations with respect to the change of the fabrication tolerances mentioned above. The individual effects of the tolerances are combined by taking the square root of the sum of the squares of the effects. This is compared with the performance requirement, and the trial budget is then adjusted to achieve the goal.

-Analysis (statistical aspect)

Nothing is perfectly fabricated, i.e. the fabrication of element must include the tolerances which come from the precision of the machinery and the control of the machine. For example, the dimension of the specific characteristic can be express as 0.1 unit ±0.005 unit, where 0.1 unit is the “nominal” dimension of this characteristic and ±0.005 unit are the upper and lower limit tolerances allowable. In statistical aspect, however, the combination of multi-element is not as simple as just adding the nominal dimensions and tolerances individually. Because the more elements are combined the less uniform of the probability distributes. A straightforward example is illustrated by the author. Assume a production line which is simply stacks of disks. Each disk is equally likely to be made to any thickness within the tolerance range of 0.1 inch ±0.005 inch. This is called a uniform or rectangular probability distribution, and is plotted in Fig.1. So, the probability of a disk being fabricated in the thickness range between, say, 0.095 inch and 0.096 inch is one in ten.

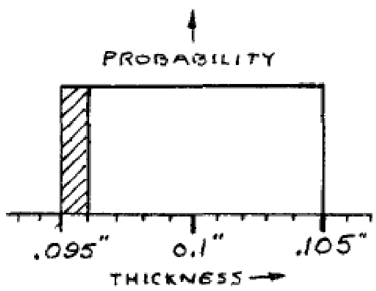


Fig. 1 - Uniform probability distribution

When stack of two disks are consider, the probability distribution is no longer uniform anymore as shown in Fig. 2. The more elements are being assembled, the greater the probability will be near their nominal value as plotted in Fig. 3. The importance of this statistical example is that if some percentage, say X%, of the individual pieces making up an assembly fall within some fraction of their tolerance ranges, then X% of the assemblies will fall within $(1/N)^{1/2}$ of the range sum, where N is the number of pieces making up the assembly.

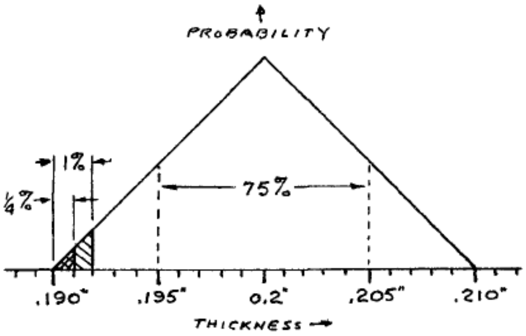


Fig. 2 - The probability distribution for an assembly of two disks, each of which has a uniform probability distribution.

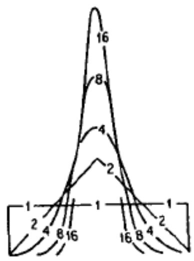


Fig. 3 - The more components there are in an assembly, the greater the probability that the characteristics will be near their nominal value.

-Analysis (using R.S.S. rule)

The other rule, which is easier, gives the same results as statistics is calculating the R.S.S. of individual tolerance. The expression takes the form

$$T = \sqrt{\sum t_i^{\,2}}$$

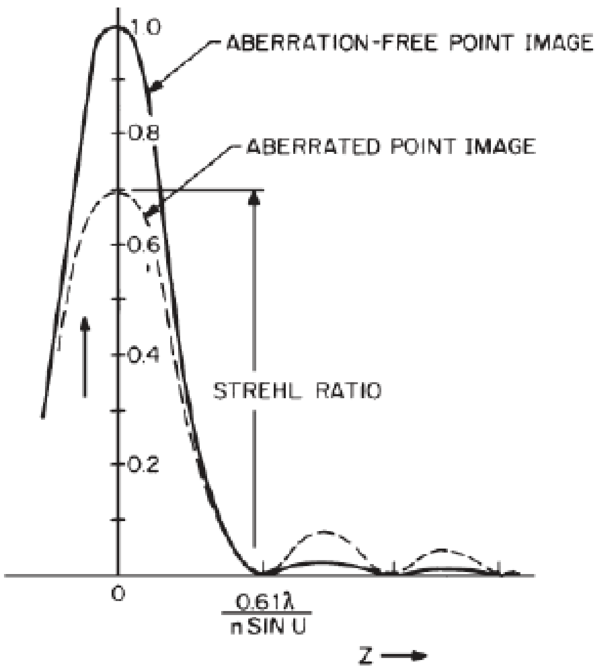
where t_i are the effects of the individual tolerances and T is the maximum value that the combination of all the effects will produce.

The author used a numerical example helps to make the concept of this paper easily understand. Table 1 is the design of the optics.

TABLE 1 - 14mm NA 0.42 LASER RECORDING LENS

	Radius	Thickness	Glass	Clear Aperture
0	Object	76.539		
1	+50.366	2.80	SF 11	11.65
2	-39.045	.4353	Edge Contact at	11.62
3	-19.836	2.0	SF 11	11.62
4	-34.36	0.2		11.90
5	+17.42	2.65	SF 11	11.81
6	+79.15	11.84		11.22
7	+ 7.08	2.24	SF 11	5.24
8	+15.665	3.182		4.13
9	Plano	2.032	Acrylic	
10	Plano			

The specification for total error budget is that the Strehl ratio must be no less than 0.75 of the perfect design. From the lecture note [2], The Strehl ratio is defined as



$$SR \cong e^{-\sigma^2} \cong 1-\sigma^2$$

Where σ is RMS wavefront error in ***radians***

$$SR \cong e^{-(2\pi W_{rms}/\lambda)^2} \cong 1-(2\pi W_{rms}/\lambda)^2$$

Where W_{rms} is RMS wavefront error in μm (assuming λ in μm)

This specification of a Strehl ratio of 0.75 corresponds to an OPD of 0.288 waves of peak-to-valley wavefront distribution [2]. From the nominal design value of optics, the residual OPD is 0.23 waves at the edge of the field. By using the R.S.S. rule, the fabrication error budget and an error tree in Fig. 4. are therefore obtained

$$Total \quad error \quad budget = \sqrt{(residual \quad of \quad design)^2 + (fabrication \quad error \quad budget)^2}$$

$$\Rightarrow fabrication \quad error \quad budget = 0.173\, waves$$

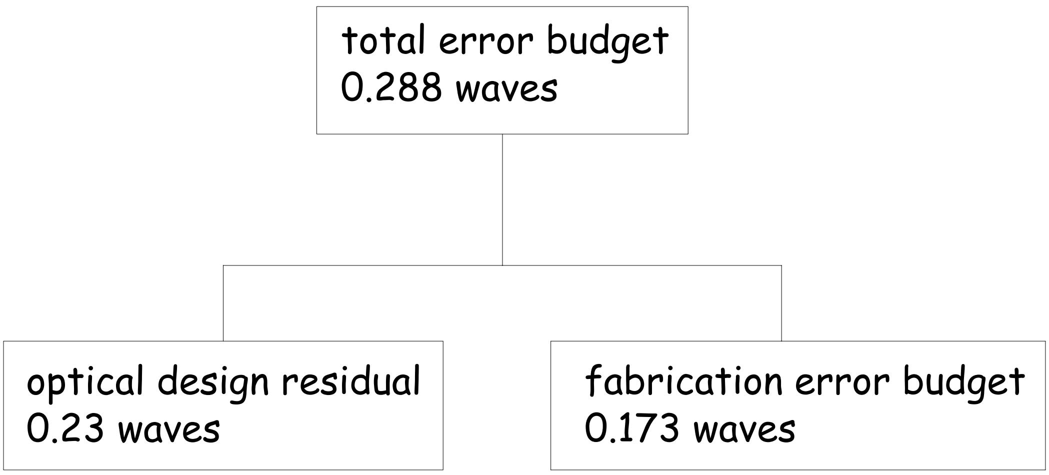


Fig. 4 Error tree

Thus the task is to determine a tolerance budget no more than 0.173 waves of OPD. This tolerance analysis could be done either using the tolerance analysis concept [2] or using the tolerance analysis function provided by commercial optical software, for example, Zemax, Code V. The author also provides a result of his tolerance analysis for fabrication as showing below in Table 2., which meets the fabrication error budget.

Tolerance			P-V RSS OPD
Radius test plate fit	:	1 Ring	.010 λ
Surface regularity	:	1/5 Ring	.156
Index variation	:	.001	.011
Surface tilt	:	.0002 Radians	.055
Thickness tolerance	T1:	0.10	.002
	T2:	0.02	.004
	T3:	0.05	.009
	T4:	0.04	.005
	T5:	0.05	.009
	T6:	0.03	.009
	T7:	0.07	.004
		RSS Total	.167 λ

Table 2. a numerical example of tolerance analysis using R.S.S. rule

-Conclusion

Doing the tolerance analysis is important than just finishing the "paper" design of optics because the design has to be made. Furthermore, the determination of the target, error budget, is prior to the tolerance analysis. This usually based on the estimation of image quality, the precision machinery control limit and even environmental effects to the material. And R.S.S. rule plays an important and effective role in setting the error budget that makes the design comes true.

-Reference

[1] Warren J. Smith, "Fundamentals of establishing an optical tolerance budget", *Proc. of SPIE* Vol. 0531.

[2] Prof. Jim Burge, class notes and lectures of "Introductory opto-mechanicl engineering", Fall, 2008.