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Engineering with Lightweight Mirrors

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Introduction

The purpose of this paper is to help determine whether or not a lightweight mirror is an appropriate engineering solution in place of a more traditional, solid mirror. The relatively recent manufacture of borosilicate lightweight mirrors both by casting[1] and by the Gas Fusion process[2] make this study timely. Because lightweight borosilicate mirror substrates tend to be substantially less expensive than the equivalent lightweight substrates made of very low expansion materials, a review of the properties of lightweight mirror structures in general will make it possible to determine if the cost savings of the borosilicate substrates are justified when balanced against the performance of the very low expansion mirror substrate materials.

We begin by reviewing the manufacturing process for lightweight, sandwich type, mirror substrates. The discussion is general and applies equally to all closed back lightweight structures whether made by machining out a solid blank or fusing the structure together out of sheets and tubes. The mechanical properties of these sandwich type, lightweight structures are then discussed. Since the mechanical properties of all glass and glass ceramic mirror materials are essentially the same, the mechanical characteristics of similar structures will be the same regardless of the material used.

We give criteria for specifying faceplate thickness to avoid "print through" of the rib structure during polishing and a method of estimating the gravity deflection of lightweight mirrors. The simple formulas given are probably adequate for mirrors up to 1.5 to 2 meters in diameter. For larger mirrors, nothing can substitute for a thorough engineering analysis, but these criterion give an approach to see if further analysis of a given design is even warranted.

The principle difference between borosilicate glasses (Pyrex, E6, Duran and Tempax are all commercially available examples) and their more expensive counterparts is the coefficient of thermal expansion (CTE). The borosilicates have a CTE about 6 times greater than fused silica and 100 times greater than Corning ULE or Schott Zerodur at room temperatures. Depending on the application, this large difference in CTE may or may not make a large functional difference in optical system performance.

The other thermal difference that will be stressed is the time it takes a mirror blank to return to thermal equilibrium once it has been perturbed. This thermal time constant affects performance in two ways, how fast the mirror figure returns to normal and how long the mirror acts as a thermal source to the surrounding air and disturbs seeing. The thin face sheets that are practical with machined lightweights and the Gas Fusion process have relatively low thermal time constants.

Lightweight mirror blank fabrication

The lightweight mirror blanks discussed here are made of two thin facesheets separated by a core structure made up of plate like ribs or tubing stood on end as in Fig. 1. We will not describe open back, cast mirror blanks with thick ribs because these are not as stiff as the same thickness sandwich type blanks.

As a specific example, we will describe Hextek Gas Fusion mirror blanks made up of a tubular core structure. The results we derive for faceplate and mirror thickness are also applicable to any sandwich type, lightweight mirror as long as the faceplates and core are monolithic, that is, thoroughly fused together or machined out of a solid so the whole structure behaves as one. In the Hextek process, a backsheet with 1/4" holes drilled in the center of each cell is placed on a manifold in a high temperature furnace. A length of circular glass tubing the height of the core is placed concentrically about each of the drilled holes and a faceplate is placed over the tubing just as in Fig. 1a.

The whole structure is then heated until the glass starts to soften. At this point, air pressure is introduced through the holes in the backplate to just balance the weight of the faceplate so the whole structure does not sag as the tubes fuse to the facesheets. Once a good fusion bond is obtained, the air pressure is increased to blow the tubes out into their neighbors to form a completely monolithic core and integral side wall as illustrated in Fig. 1b.

After annealing and cooling, the finished mirror blanks come out of the furnace looking like those pictured in Fig. 2. If it is desired to have a curved mirror such as a telescope primary, the blank is turned upside down on a convex refractory mold and reheated until it slumps to the shape of the mold. Convex slumpings for secondaries are made by an analogous process. This process is gentle enough that none of the original rib and faceplate geometry is lost. The larger blank in Fig. 2 is 40" in diameter and was slumped to an f/0.5 curve.

Determination of faceplate thickness

The first step in designing a lightweight mirror blank is to determine the required faceplate thickness. In a structured blank, the faceplate will flex between the rib supports during polishing. This leads to an effect called "quilting" or print through of the rib structure in the polished mirror surface. All structured blanks will suffer this problem to some degree and it is a matter of setting an acceptable level for the scattering loss due to residual quilting.

From an approximation to the Strehl ratio, the fractional amount of light lost from the central maximum of the image, f, is proportional to the square of the rms figure error, σ [3]. Thus we can write

$$f = e^{-\left(\frac{2\pi\sigma}{\lambda}\right)^2} \tag{1}$$

for small figure errors. The deflection of the faceplate between the ribs due to the polishing pressure is given by plate bending theory for the case of a plate with clamped edges[4].

The net result is a formula for the faceplate thickness in terms of the polishing load, q in psi, the rib spacing or cell size, a in inches, the wavelength of the light being observed, λ in inches (about 20 microinches in the mid-visible), the Young's modulus of the glass, E in psi (about 9×10^6 for all glasses) and the fractional loss of light due to quilting, f, where 0.01 means a 1% loss. The faceplate thickness, t, is

$$t = \left(\frac{.035\,q\,a^4}{\lambda E\, f^{1/2}}\right)^{1/3} \tag{2}$$

This reduces to

$$t = \left(\frac{.0002\,q\,a^4}{f^{1/2}}\right)^{1/3} \tag{3}$$

when we substitute the typical values of E and lambda given above. To fill in other typical values, the cell size in a Hextek mirror is about 2.5 inches, a typical upper limit on polishing pressure is 1 psi and a 1% light loss is reasonable. This would give a faceplate thickness of 0.423". For facedown CP polishing of flats, the load is typically 0.1 to 0.2 psi. Figure 3 shows this relationship for a variety of cell sizes and polishing pressures assuming a 1% light loss in all cases.

Determination of the mirror thickness

The overall mirror thickness determines the mirror's resistance to bending under its own weight when mounted in the telescope. To get diffraction limited images, a criterion consistent with allowing a 1% loss due to quilting, we require that the mirror deflect no more than $\lambda/8$ under its own weight. To accomplish this as efficiently as possible, a ring of 6 equally spaced mounting pads are placed at 7/10th's the radius of the mirror. This placement of mounting pads is about 25 times as efficient as holding the mirror either by the center or by the edge[5].

In addition, if these mounting points attach to the mirror in the plane of the center of gravity (roughly in the middle of the core structure), the mirror can be pointed to the horizon with no deflection at all. This mid-plane mounting method where the mounting bosses are fused directly into the core structure[6] serves as both the axial and radial support system and is completely kinematic so the mount will not introduce any distortions in the mirror.

Assuming a ring of 6 supports, we can go on to calculate the mirror deflection as a function of its thickness. Here we use plate bending theory again, this time using the case for a plate simply supported at its edge[7] and then divide the deflection by 25 because of the location of the supports at the 7/10ths zone. The load, q, is now the areal density of the blank itself and is represented by a fraction, f_1, that depends on the structure of the sandwich section as shown in Fig. 4. This fraction is just the ratio of the weight of the lightweight structure to that of a solid of the same thickness. Thus

$$f_1 = 2\beta + \gamma(1-2\beta) \tag{4}$$

represents the fractional degree of lightweighting for any core geometry.

In a lightweight blank, the effective stiffness is less than that of a solid because of the material removed as the result of the lightweighting. If we let the fraction, f_2, represent the loss of stiffness of the lightweight mirror blank, we can divide the deflection equation by this fraction to account for the loss of stiffness. Just as in the case of f_1, the ratio f_2 depends on the parameters of the rib structure and faceplate thickness. Making these substitutions, we find the deflection, d, is given by

$$d = \frac{2.7\times 10^{-10}\,\rho\, h\, f_1\, \phi^4}{h^3 f_2} \tag{5}$$

where we have adjusted the constant to include E, the 7/10ths zone mounting on 6 points and the use of ϕ to represent the mirror diameter. The density, ρ, of all glass mirror materials is about 0.08 psi.

Notice that if we let f_1 and f_2 be 1.0, we have the deflection equation for a solid blank under its own weight for these same mounting conditions. At this point, we would like to solve for f_2 so that we could find the thickness, h, for the mirror. Unfortunately, f_2 is also a weak function of h so things get messy. However, for a wide range of reasonable values of core structural parameters, a lightweight mirror will only deflect 40 to 50% of what a solid blank will deflect when similarly supported. On the other hand, our deflection equation is based solely on thin plate theory and we have not taken shear deflections of the core into account (also messy).

If we had been more rigorous, we would have found that the shear deflection around the mounting points was about equal to the thin plate deflection. Thus the easy way to handle the situation is to let these two effects cancel each other and simply calculate the deflection based on a solid blank and take the weight savings of the lightweight. The thickness, h, of the solid using this mounting scheme of 6 points on the 7/10th zone that gives diffraction limited imaging in the visible is just

$$h = 2.92 \times 10^{-3} \phi^2. \tag{6}$$

This relation is plotted in Fig. 5 and clearly shows the error in the common rule of a 6:1 diameter to thickness ratio for telescope mirrors. The world is not so simple.

Now that the thickness, h, has been determined, the structural parameter, β, can be found from the faceplate thickness using Eq. 3 as $\beta = t/h$. A typical value for γ in lightweight mirrors is 0.04 so now f_1 can be calculated from Eq. 4. Since the weight, W, of a solid mirror is

$$W = \frac{\pi \rho h \phi^2}{4} \tag{7}$$

where we take the density of glass as 0.08 #/cu. in., the weight, W_{LW}, of a lightweight mirror will be just

$$W_{LW} = W f_1. \tag{8}$$

Since f_1 is typically in the range of 0.2 to 0.3, the lightweight mirror has the same structural performance in the telescope as a solid while weighing only 20 to 30% as much. This weight savings flows down as a cost savings in the cell and mount.

Mirror thermal response

There are two aspects to mirror thermal response. The usual one concerns the length of time it takes following a thermal perturbation until the mirror returns to the same figure as before the perturbation. In this case, a fast thermal response means a shorter "down" time due to thermally induced figure errors. Of course this type of effect can be minimized by using a very low coefficient of expansion glass so that thermal effects do not cause consequential figure errors.

The other aspect of thermal response is that the mirror should rapidly come into thermal equilibrium with the air surrounding it so that the mirror does not act as a thermal source and produce poor seeing due to heated air moving across the mirror surface. In this case, there are no material solutions to the problem other than keeping the thermal response time short.

Since all the thermal mass in a lightweight mirror is in the faceplates, it is the thickness of the faceplates that govern the thermal response. Without going into heat transfer theory, let us just say that the thermal response time of a mirror is directing proportional to the thickness of its faceplates or in the case of a solid (or solid meniscus), to the thickness of the solid. As we have already shown, the only limit on faceplate thickness is the expected polishing loads. The faceplate thickness has nothing to do with the stiffness of a sandwich structure so a lightweight mirror will have, as an extremely conservative estimate, a thermal response time less than a solid by the lightweighting ratio, f_1. This assumes no heat transfer to the back sides of the facesheets through the core.

A more realistic thermal response time would be about 2/3rds the lightweighting ratio so that we can say that a lightweight mirror will return to equilibrium in about 15% of the time it takes a solid to do the same. Since the response time of a 5" thick solid is about 3 hours, the advantage of a lightweight mirror is clear.

Conclusions

We have shown that the first step in specifying a lightweight mirror blank is to determine the faceplate thickness based on expected polishing loads and rib spacing. Following this, the overall blank thickness is determined by calculating the deflection of a solid blank under its own weight. This is a good approximation to the deflection of a lightweight blank when the effects of the lightweighting and shear deflections are taken into account. From these two calculations, the weight savings of using a lightweight blank can be determined. These weight savings produce proportionate cost savings in the moving mass of the telescope mount as well as proportionate improvements in the thermal response time of the mirror.

References

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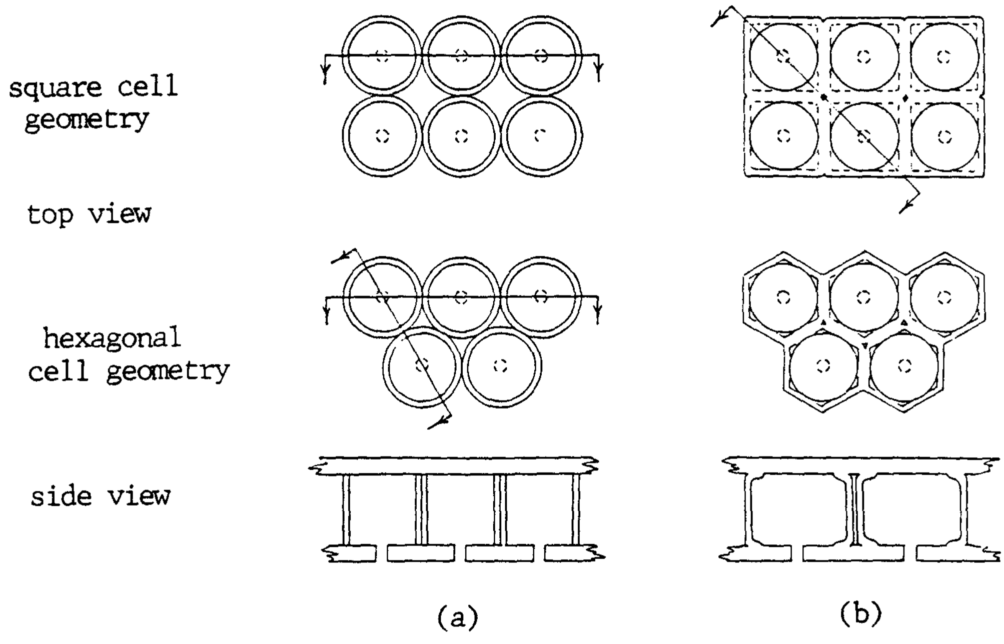

Fig. 1 Lightweight, sandwich structure mirror blank geometry showing facesheets separated by circular tubing. a) Tubing geometry prior to fusion. b) Blank geometry folllowing fusion and pneumatic expansion of tubing.

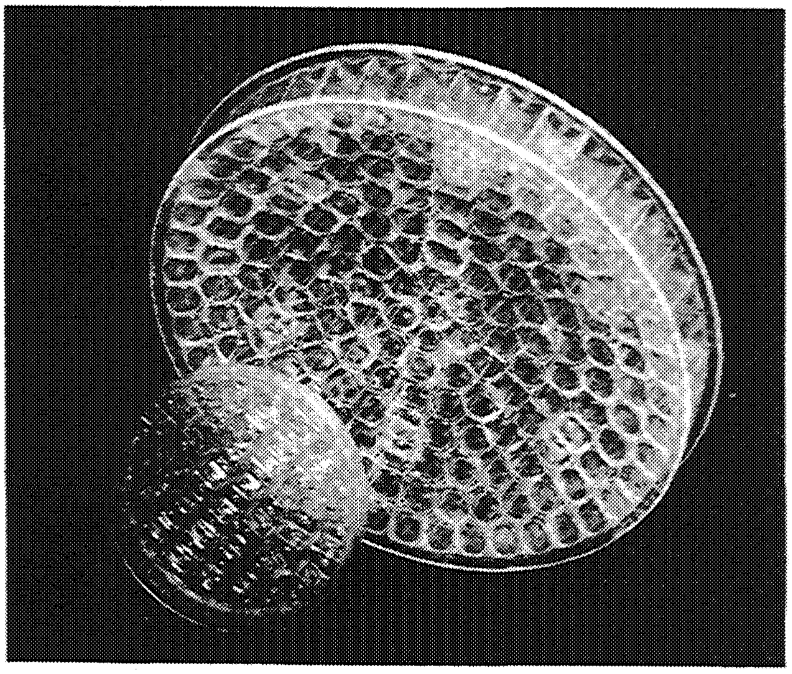

Fig. 2 Lightweight, Gas Fusion mirror blanks after fusion and annealing. The 18" blank is plano while the 40" blank has been reheated and slumped to an f/0.5 curve.

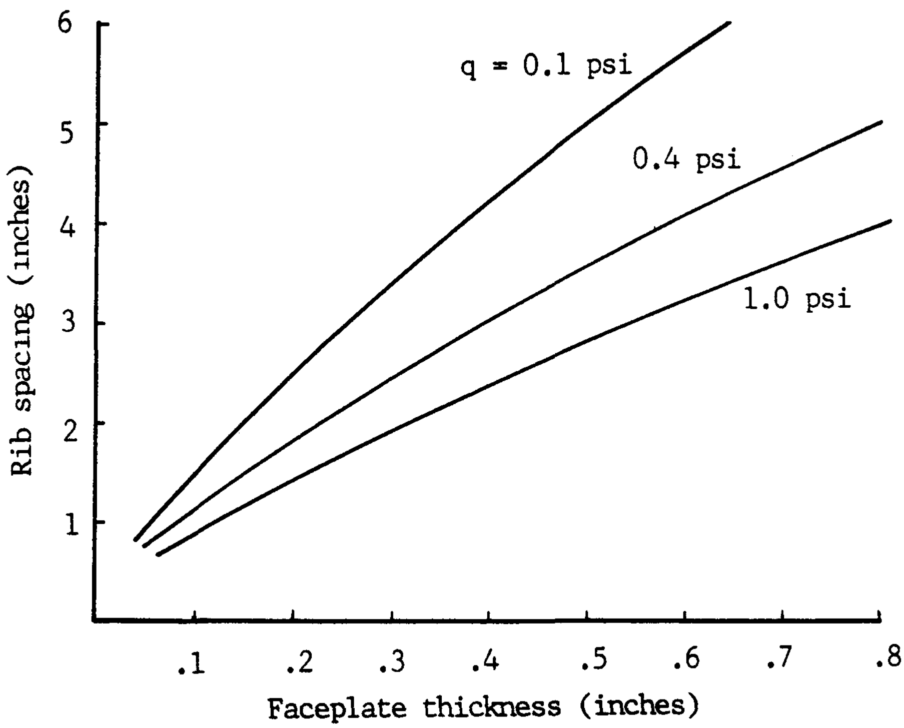

Fig. 3 Lightweight mirror blank minimum faceplate thickness as a function of cell size and polishing loads assuming visible light and a 1% allowable light loss in the image core.

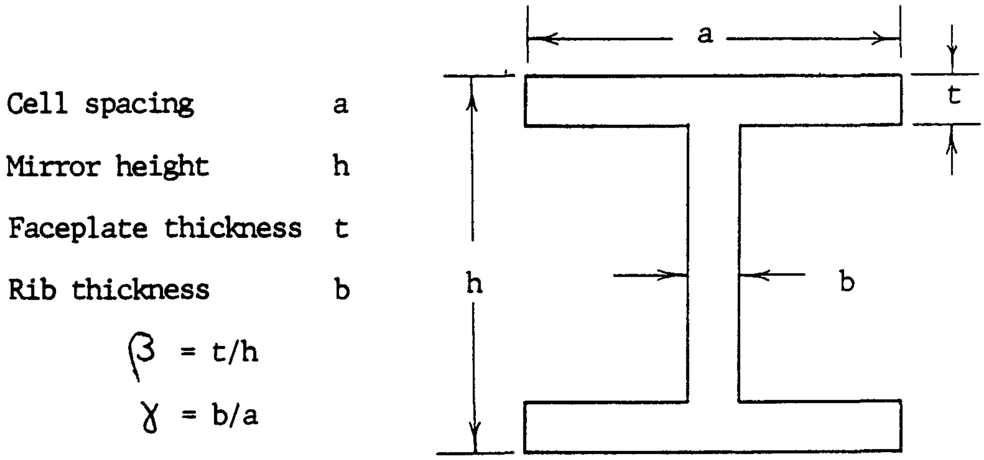

Fig. 4 Parameters of the repeating cell geometry in a light-weight mirror blank.

Fig. 5 Deflection of a solid mirror blank under its own weight as supported on a ring of 6 points located at the 7/10ths zone.