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A Parametric Approach to Mirror Natural Frequency Calculations

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Abstract

A hybrid analytical/graphical method is presented to calculate the fundamental natural frequency of rectangular mirrors mounted at 3 points. A NASTRAN assisted parametric approach was used to calculate the characteristic roots of the plate vibration equation for mirrors with aspect ratios ranging from 1.0 x 1.0 to 10.0 x 1.0. Also considered were simply supported boundary conditions at three mirror corner points or at two corner points on one edge and one point along the opposite edge. Experimental varification within 6.0% was achieved for the extreme case tested with approximately a ±2.0% average experimental error overall.

Introduction

When designing an optical imaging system, it is important to know that the dynamic response of the individual components is sufficiently small to minimize the aerial image motion. Since typical imaging systems contain as many as six mirrors between the platen and photoreceptor, the fundemental natural frequency of these mirrors should be calculated as the first step in designing a dynamically acceptable system. The first natural frequency of the mirrors should be designed to avoid excitation frequencies produced by motors, fans and other continually running sources, yet be high enough so the transient response produced by scan system motion, solenoid actuation and other impulse excitations is small or has time to diminish prior to imaging.

Reviewing the open literature shows a significant amount of work that has been published dealing with the vibration of plates. Leissa[1] reviewed 473 papers in the area of plate vibrations and summarized the solutions necessary for an engineer to determine natural frequencies of common shaped plates with various boundary conditions. Out of the 161 papers devoted to the vibration of rectangular plates, not one showed how to calculate the case of a 3 point mount. Most of the closed form solutions were for continuous boundary conditions along an entire edge(s) and all the graphical solutions for discrete boundary conditions were for point mounts at each of the 4 corners. In the case of mounting high aspect ratio mirrors (ie. length-to-width ratio of 5 to 1 or greater) at 3 corners, some of the forementioned solutions are very good approximations as well as the simply supported beam solution given by Timoshenko and Young[2]. However, there are many imaging systems which use low aspect ratio mirrors and not all mirror mounting configurations are at corner points. As discussed by Nowak[3], sometimes it is desirable to mount a mirror at 2 corner points on the same edge and the third point along the opposite edge. A significant increase in the natural frequency would result from this mounting condition as opposed to mounting at 3 corners.

Parametric study approach using NASTRAN

From plate vibration theory we have the following equations which can be used to calculate the bending natural frequency of of any rectangular plate with any type of boundary conditions:

$$f_i = \frac{1}{2\pi} \cdot \frac{\lambda_i}{a^2} \sqrt{\frac{D}{\rho^*}} \quad , \quad D = \frac{E\,t^3}{12(1-\nu^2)} \qquad (1)$$

where f_i is the natural frequency of the ith mode in hertz,
λ_i is the characteristic root indicative of boundary conditions, natural frequency and modeshape,
a is the length of the long edge in inches,
E is the modulus of elasticity in $lbs./inch^2$,
t is the plate thickness in inches,
ν is Poisson's ratio,
ρ^* is the mass density per unit area in $lbs.\ sec.^2/inch^3$ (ie. $\rho \times t$).

Reviewing the above equation, the only unknown variable preventing the calculation of f_i is λ_i. Using NASTRAN a series of finite element models covering aspect ratios (a/b) from 1.0 to 10.0 and 3 point boundary conditions (c/a) from 0.0 to 0.5, were generated and are depicted below.

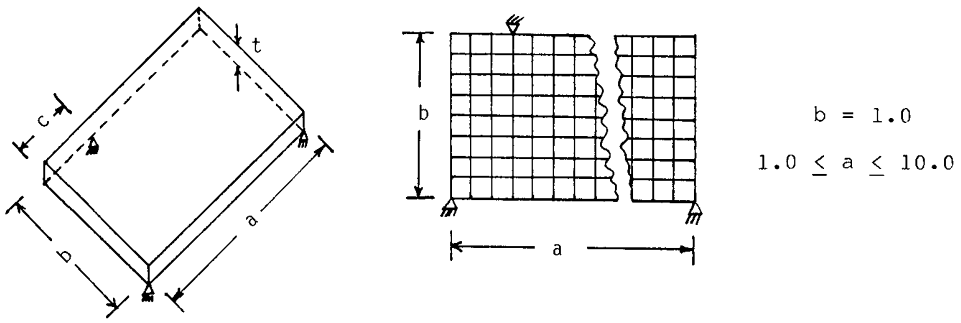


Figure 1. Schematic of mirror and finite element geometry.

All finite element models were generated using square QUAD4 plate elements with an aspect ratio of 0.125 (ie. 8 elements per every 1.0 length of plate edge). Since mirrors can be considered "thin" plates, the shear flexibility effects where neglected and only bending stiffness was used in the calculation. From the resulting natural frequencies, f_i, which are produced as NASTRAN output, the forementioned plate vibration equation can be solved for λ_i as shown below.

$$\lambda_i = 2\pi \; f_i \; a^2 \sqrt{\frac{\rho^*}{D}} \tag{2}$$

Using a systematic calculation procedure for various aspect ratios and 3 point boundary conditions, a series of c/a curves for λ_1 vs. a/b are plotted in figure 2. With this data, the fundamental natural frequency of any mirror mounted at 3 points can be calculated.

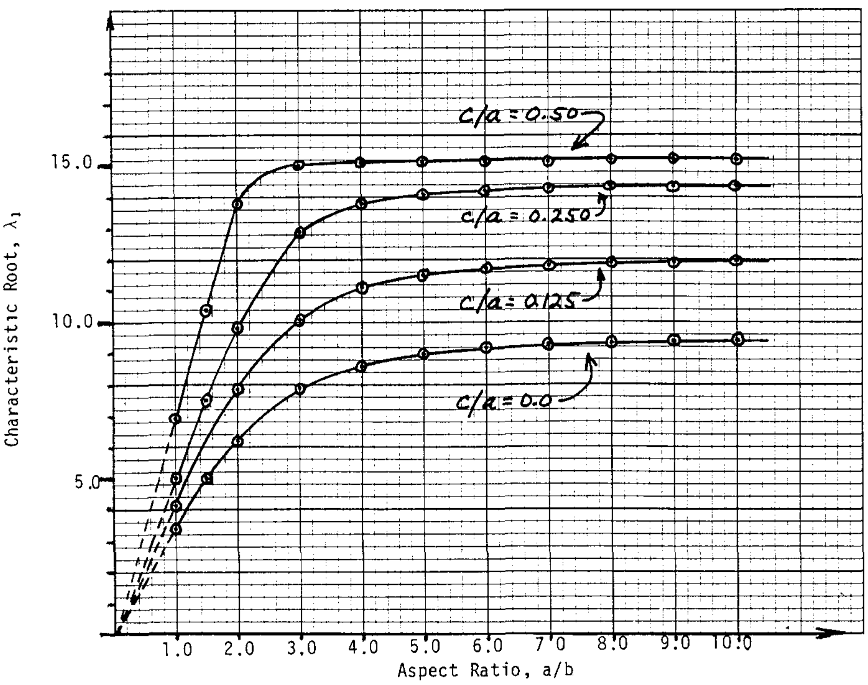
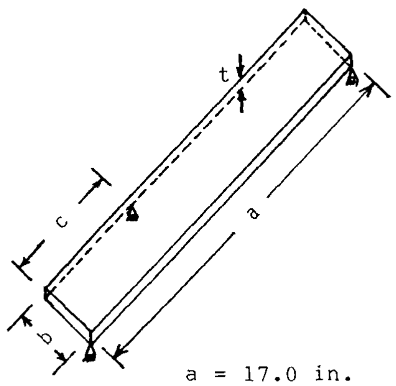


Figure 2.

Sample calculations

The two general classes of mirrors used in xerographic processors are high aspect ratio mirrors characteristic of scanning systems, and low aspect ratio mirrors typically used as imaging mirrors in flash systems. Figure 3 and 4 show sample calculations and test verifications for each mirror class. The material properties used in these calculations are typical of soda lime glass and the exact values given should be used. While it may be apparent that the variables in the plate vibration equation can be changed to allow the use of any material, the parametric calculation of λ_i is coupled with Poisson's ratio, ν , which is used in the NASTRAN calculations. This value was fixed at 0.24 in the NASTRAN models and any other values inserted into the plate vibration equation will result in a small calculation error as described by Timoshenko and Krieger[4]. All other structural properties are independent of λ_i and could be changed to approximate plate first natural frequencies using other materials.



a = 17.0 in.
b = 2.0 in.
t = 0.394 in.

Soda Lime Glass Properties

$E = 10.0 \times 10^6$ lbs./in.^{2}
$\nu = 0.24$
$\rho = 2.303 \times 10^{-4}$ lbs.sec^2/in.^{4}

Using equation (1),
solve for f_1 in terms of λ_1 ...

$$f_1 = 13.445\ \lambda_1$$

Case 1	Case 2
a/b = 8.5	a/b = 8.5
c = 0.0 in.	c = 8.5 in.
c/a = 0.0	c/a = 0.5

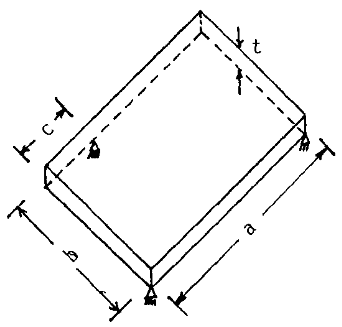
From figure 2 ...

$\lambda_1 = 9.35$	$\lambda_1 = 15.20$
f_1 = 125.7 hz.	f_1 = 204.4 hz.

Test results yield ...

f_1 = 127.1 hz.	f_1 = 192.3 hz.

Figure 3. High aspect ratio mirrors.



a = 13.5 in.
b = 8.50 in.
t = 0.230 in.

Soda Lime Glass Properties

$$f_1 = 12.446\ \lambda_1$$

Case 1	Case 2
a/b = 1.59	a/b = 1.59
c = 3.375 in.	c = 6.75 in.
c/a = 0.25	c/a = 0.5

From figure 2 ...

$\lambda_1 = 8.00$	$\lambda_1 = 11.10$
f_1 = 99.6 hz.	f_1 = 138.2 hz.

Test results yield ...

f_1' = 99.0 hz.	f_1 = 136.1 hz.

Figure 4. Low aspect ratio mirrors.

The accuracy of the method is consistent with any graphical approach used to condense large amounts of tabular data and is dependent on how well figure 2 can be read. Expected analytical accuracy should be within $\pm 2.0\%$ of a finite element or finite differance calculation on the exact mirror size and location of the three point support. When compairing these calculations against experimental results, this type of accuracy should not be expected. As was tested in the lab, results for a mirror resting on 3 rigid points free of moment constraint achieved an experimental error of $\pm 6.0\%$. If the mirror is then mounted in a carriage or on the frame, the actual first natural frequency could be less than calculated due to support flexibility or larger due to over contraint at the support points.

Conclusions

The hybrid analytical/graphical method presented is a good analysis tool for preliminary dynamic design of optical systems. Extensions of the NASTRAN parametric approach should encompass semi-rigid and over constrained mirror boundary conditions, and support of platens, lenses and other optical components.

References

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