Athermal bonded mounts:
Incorporating aspect ratio into a closed-form solution

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ABSTRACT

Several approaches have been used to calculate a closed-form solution for the athermal bond thickness for mounting optical elements. All of the previously developed closed-form solutions use the assumption that the bondline is thin with respect to the width of the bond in the axial direction. While this assumption is mathematically convenient, it is not empirically or theoretically supported. To compensate for the inaccuracies of these closed-form solutions, recent research using test data and finite element analysis has centered on generating empirically determined correction factors that are applied to the closed-form solutions for a zero-stress bond. In this paper an alternative closed-form solution that incorporates the bond aspect ratio is presented. The values generated from this formula are compared to the empirical results of a finite element analysis (FEA) study. An example case is used to compare the results provided by the different methods for calculating the ideal bond thickness.

Keywords: Athermal, bond, elastomer, adhesive, RTV, epoxy, stress

Table 1. Symbol Definitions

α	Coefficient of thermal expansion (CTE)	r	Subscript: Radial direction
ν	Poisson ratio of bond material	θ	Subscript: Tangential direction
ε	Normal strain (strain)	z	Subscript: Axial direction
σ	Normal stress (stress)	o	Subscript: Optical element
δ	Deflection from stress-free state	b	Subscript: Bond
h	Bond thickness (radial direction)	c	Subscript: Cell
L	Bond width (axial direction)	k_{11}, k_{12}, k_{13}	Correction factors[5]

1. INTRODUCTION

Determining the athermal bond thickness has been the topic of several papers since the subject was first addressed by Bayar[1]. When using a bond to secure an optical element around its circumference, the goal is to eliminate the radial stress on the element that can occur due to the differences in the coefficients of thermal expansion (CTEs) of the element, the bond material, and the supporting cell. The elimination of stress, or athermalization, is accomplished by sizing the bond thickness so that the expansion of the bond material matches the difference in the growth of the cell and the optical element. Most analytical solutions to this problem are derived from Hooke's Law matrix for three dimensional stress. Referring only to normal stresses, Hooke's Law relates the stresses on a body in three directions to the three strains on that body.

This paper first compares the existing derivations for the athermal bond thickness that are based on Hooke's Law. The different equations result from the use of different assumptions regarding how the bond is constrained. Second, a new formula for the athermal bond thickness is derived by using a novel way of constraining the bond. It is shown that the limiting upper and lower bounds for athermal bond thickness are provided, respectively, by this new equation and the existing van Bezooijen equation. The new upper bound is called the "modified van Bezooijen equation". Next, a derivation is presented for a new approximation for the ideal athermal bond thickness; this approximation provides a solution that spans the space between the two limiting equations. This new approximation takes into account the ratio

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of the bond width to the bond thickness, a factor that has previously been ignored except for empirically determined compensating correction factors. The new approximation is compared to the results of finite element analysis (FEA). Finally, there is a sensitivity analysis and a discussion of the stress induced in a non-athermalized system. For consistency, the diagrams and language in this paper reflect the affects of a system that is *heated* from its original bonded temperature. All effects and equations also apply as the system is cooled.

2. BACKGROUND INFORMATION

The published equations for athermal bond thickness that can be derived directly from Hooke's Law are listed in table 2. The equations are listed in the order of the complexity of the assumptions used to generate them; this is also the order of increasing accuracy. A complete list of the assumptions used for the equations discussed in this paper is found in table 4 in the conclusion section.

Table 2. Existing closed-formed solutions for athermal bond thickness

Equation	Major Assumption
Bayar	No axial or tangential strain on the epoxy
Modified Bayar[4]	Complete constraint of epoxy in axial and tangential directions
van Bezooijen*	Epoxy is fully constrained to the lens and the cell in the axial and tangential directions

*Frequently referred to as the Muench equation, this equation was originally derived by Roel van Bezooijen.

Another solution was created by DeLuzio. DeLuzio employed a different approach than using Hooke's Law. While DeLuzio's equation is mathematically different than van Bezooijen's, the calculated ideal bond thickness from the two equations is almost identical. DeLuzio's solution will only be discussed in this paper for comparison to the van Bezooijen equation and the other equations derived in this paper; it will not be derived or discussed in detail. It is interesting that the DeLuzio equation reduces to the van Bezooijen equation for a Poisson ratio of 0.5, but the two equations deviate slightly for other Poisson ratios. The DeLuzio equation is shown here for reference only[4]:

$$h = \left(\frac{1-\nu}{1+\nu}\right)\frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_o + \dfrac{(7-6\nu)\cdot(\alpha_c - \alpha_o)}{4(1-\nu)}}$$

3. BAYAR'S EQUATION AND RADIAL STRAIN

The simplest of the three equations commonly used for calculating bondline thickness is the Bayar equation. Bayar considers only the radial thermal expansion, ignoring the affects of constraining the bond axially and tangentially. Figure 1 defines the terms that will be used for discussing Bayar's Equation and all other equations. Cross-section views are shown that provide views from the top of the system and also a cut through the system. For simplicity, a flat element is shown and the cell is shown to be the same height as the bond and the element; neither of these is a requirement of the system. The radial ('r'), tangential ('θ'), and axial ('z') directions are also defined in figure 1.

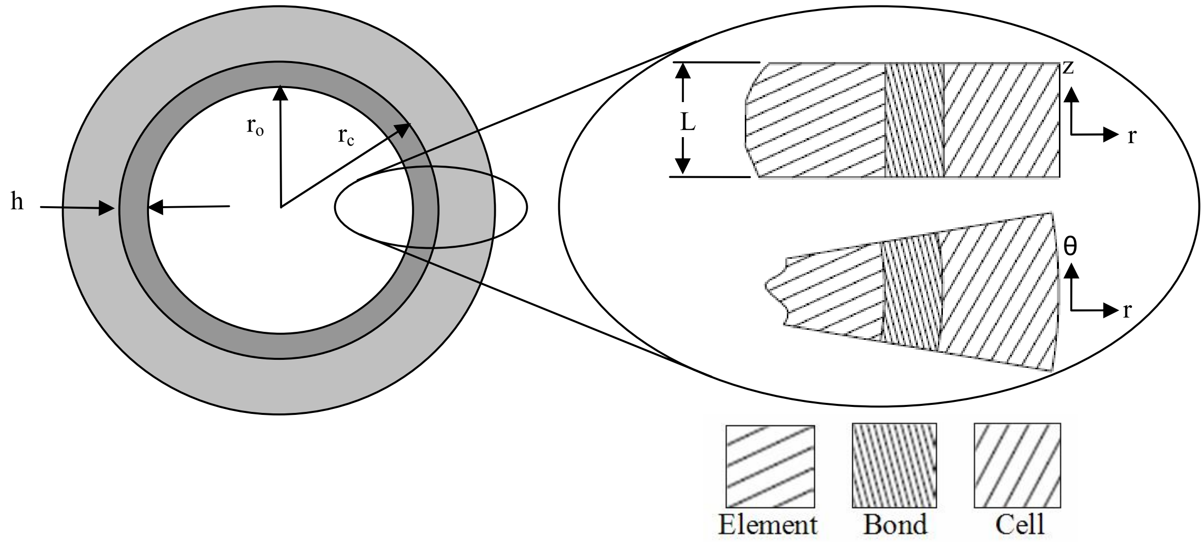

Figure 1. Typical athermal lens mount. Two cross sections are shown for a small "slice" of the system.

The Bayar equation is derived by solving for the bondline thickness when equating the change in the bondline thickness over temperature to the difference in the changes of the cell and the optical radii over temperature:

$$\Delta h = \Delta r_c - \Delta r_o$$
$$h\alpha_b \Delta T = (r_o + h)\alpha_c \Delta T - r_o \alpha_o \Delta T \quad (1)$$

Solving for h, it can be seen that the thickness of the bond is a function of the radius of the element and the three CTEs, as follows:

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c} \quad (2)$$

Equation (1) can also be used to calculate the radial strain, if it is assumed to be non-zero, where radial strain is a function of radial deflection. The radial deflection, δh, is defined as the change in thickness of the bond material from its unconstrained thickness with any given change in temperature. The radial strain, ε_r, is given by the ratio of the deflection to the bond thickness. The resulting equation (3) for radial strain will be used later in the paper.

$$\delta h = \Delta T\left(h(\alpha_b - \alpha_c) - r_o(\alpha_c - \alpha_o)\right)$$
$$\varepsilon_r = \frac{\delta h}{h} = \Delta T\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right) \quad (3)$$

It is worth noting that if it were possible to match the CTEs of all three of the materials then any bond thickness would be athermal. Unfortunately this is typically not possible because of the inherent properties of the available materials and because of cost limitations. For most situations, and for the purposes of this paper, we assume that the CTEs of the three materials are different. We know that the CTE of the cell must be greater than that of the optical element; if this were not the case, then the CTE of the bond material would need to be negative. Since negative CTE bond materials are generally not available, it follows that $\alpha_c > \alpha_o$. Finally, the CTE of the bond must be the greatest CTE of the three because its thickness will always be less than the inner diameter of the cell. Accordingly, the following relationship must apply: $\alpha_b > \alpha_c > \alpha_o$. We are fortunate that materials are readily available that meet these criteria. Metals are typically used as cell materials and glasses are commonly used for lenses and as mirror substrates. Most metals have higher CTEs than glasses and bonding materials tend to have CTEs that are at least an order of magnitude greater than both the lens and the cell.

4. USING HOOKE'S LAW AND INCLUDING BULK AFFECTS

4.1 Hooke's Law

In order to derive the several other formulas for athermal bond thickness, Hooke's Law for stress is used. Hooke's Law is frequently presented as a matrix that defines all six of the normal and shear stresses, but in this case we are only concerned with radial stress. For radial stress, Hooke's Law reduces to equation (4):

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_r + \nu(\varepsilon_z + \varepsilon_\theta)\right] \tag{4}$$

The following methodology can be used to derive all of the different solutions for athermal bond thickness: expressions for the three strains are substituted into Hooke's Law, equation (4), and the resulting equation is solved for the zero radial stress condition. In each derivation the portion of the stress equation that is outside of the brackets, $\frac{E}{(1+\nu)(1-2\nu)}$, drops out. It should be noted that when calculating stress, equation (4) requires a correction factor to the elastic modulus; this correction factor is discussed by Michaels and Doyle[5]. This correction factor drops out along with the modulus so the correction factor does not affect any of the results for athermal bond thickness that are discussed in this paper. A further discussion of this correction factor is provided in section 8.

For all of the different athermal bond thickness derivations, equation (3) is substituted for the radial strain. The derivations differ only in the expressions for axial and tangential strain that are used; these expressions are summarized in table 3. The following sections discuss the difference in the assumptions for axial and tangential strain that are used for deriving each solution.

Table 3. Expressions for Tangential and Axial Strain

	Axial Strain		**Tangential Strain**	
Athermal Equation	**Eqn. #**	**Expression**	**Eqn. #**	**Expression**
Bayar		0		0
Modified Bayar	5	$\Delta T\alpha_b$	5	$\Delta T\alpha_b$
Van Bezooijen	7	$\Delta T\left(\alpha_b - \frac{\alpha_o+\alpha_c}{2}\right)$	7	$\Delta T\left(\alpha_b - \frac{\alpha_o+\alpha_c}{2}\right)$
Modified van Bezooijen		0	7	$\Delta T\left(\alpha_b - \frac{\alpha_o+\alpha_c}{2}\right)$
Aspect Ratio Approximation	11	$\Delta T\left(1-\frac{h}{L}\right)\left(\alpha_b - \frac{\alpha_o+\alpha_c}{2}\right)$	7	$\Delta T\left(\alpha_b - \frac{\alpha_o+\alpha_c}{2}\right)$

4.2 Bayar equation

Bayar considers the strain in the axial and tangential directions to be zero. In other words, the bond layer is free to expand or contract in these directions; see figure 2. To solve for the Bayar equation for athermal bond thickness, equation (3) is substituted into equation (4), the axial and tangential strains and the radial stress are all set equal to zero, and the resulting equation is solved for the bond thickness, *h*. The Bayar equation (2) was already derived in a simpler way in section 3.

All of the equations discussed here, excluding the De Luzio equation, reduce to the Bayar equation for a Poisson ratio of zero. Because most bond materials have a Poisson ratio that is much greater than zero, typically between 0.4 and 0.5, the assumptions of the Bayar equation generate unrealistic values for ideal bond thickness when the optimal element is surrounded by a complete circumferential bond and a supporting cell. However, the Bayar equation does serve as a good approximation for an upper limit for the athermal bond thickness in a highly segmented system, as would occur if the optical element were surrounded by small cubical or cylindrical pads of bond material. The Bayar equation's use as an upper limit for the athermal bond thickness will be more apparent after the discussion of the modified van Bezooijen equation.

4.3 Modified Bayar equation – including bulk affects

The modified Bayar equation, as presented by Herbert[4], affords a major improvement over the original Bayar equation. The modified Bayar equation accounts for strains normal to the radial direction and it incorporates the Poisson ratio. The assumption used for this equation is that the strains in the tangential and axial directions are equal to the expansion of the bond layer in those directions. In other words, the bond is fully constrained to its original unheated size in both the axial and tangential directions; see figure 2. The axial and tangential strains are derived below; equation (3) is used again to describe the radial strain. The terms for the three strains are substituted into equation (4) and stress is set to zero to derive the modified Bayar equation (6).

$$\varepsilon_z = \varepsilon_\theta = \frac{\delta L}{L} = \frac{1}{L}\left(L\alpha_b \Delta T\right) \quad \rightarrow \quad \varepsilon_z = \varepsilon_\theta = \Delta T \alpha_b \tag{5}$$

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\frac{1+\nu}{1-\nu}\cdot\alpha_b - \alpha_c} \tag{6}$$

The axial and tangential constraint used for the modified Bayar equation results in greater radial bond growth than the unconstrained assumption that Bayar used. The increased growth causes the calculated ideal athermal bond to be thinner. This fact can be seen in equation (6): the denominator gets bigger when compared to the Bayar equation (2) for Poisson ratios between 0 and 0.5.

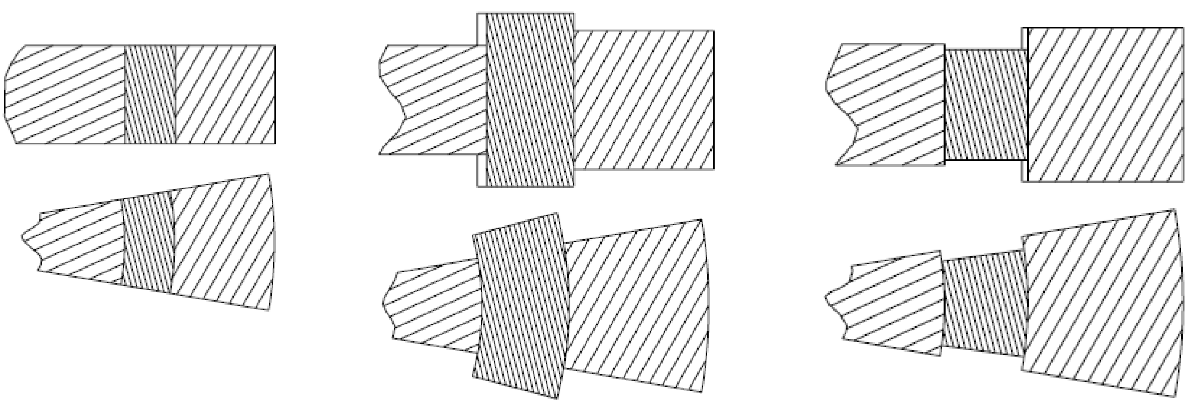

Figure 2. Comparison of an unheated system (left), a heated system under the Bayar assumption (center), and a heated system under the modified Bayar assumption (right). Bayar allows for the bond material to grow unconstrained tangentially and axially, while the modified Bayar fully constrains the bond in those directions. Figure is not to scale.

The modified Bayar equation makes the assumption that the optical element and the cell have CTEs that are small enough that they can be ignored for purpose of refining the axial and tangential constraints on the bond material. Because the CTE of the bond material is typically one to two orders of magnitude greater that that of the bond's substrates, this assumption is often fairly reasonable. The calculated bond thickness of the modified Bayar equation tends to be very close to that calculated by the van Bezooijen equation, which corrects for the assumption of the complete constraint. The modified Bayar equation serves as a good first order estimate of the van Bezooijen equation.

4.4 Van Bezooijen Equation – including the CTE of the lens and cell

Van Bezooijen[7] improves the approximation of the modified Bayar equation by accounting for the expansion of the lens and the lens cell in the tangential and axial directions. In order to estimate the constrained length of the bond in the heated condition, the average of the bond length at the surface of each of the bond's two substrates is used. This average is a good approximation considering the bond is thin compared to the radial distance from the lens axis. The strain in these directions is described by equation (7).

$$\varepsilon_z = \varepsilon_\theta = \frac{\delta L}{L} = \frac{1}{L}\left(L\alpha_b \Delta T - \frac{L\alpha_o \Delta T + L\alpha_c \Delta T}{2} \right) \quad \rightarrow \quad \varepsilon_z = \varepsilon_\theta = \Delta T \left(\alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \tag{7}$$

In equation (7) there is a small reduction in strain from the modified Bayar assumption, equation (5). Figure 3 graphically shows the nature of the affects of temperature change on the system.

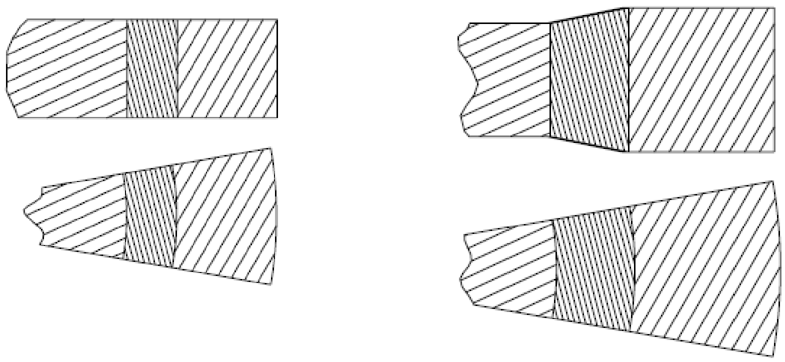

Figure 3. Comparison of an unheated system (left) and a heated system under the van Bezooijen assumption (right). Van Bezooijen constrains the bond to the surfaces of the element and the cell. Figure is not to scale.

The van Bezooijen equation (8) is a result of substituting equations (3) and (7) into equation (4) and solving using the method described for the other equations.

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{2\nu}{1-\nu}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2} \right)} \tag{8}$$

For the van Bezooijen assumption the element and the cell expand, allowing the bond material to expand slightly in the tangential and axial directions along with them. Therefore, the bond expands slightly less in the radial direction than it does under the modified Bayar assumption. The reduced radial growth results in a calculated ideal bond thickness that is greater than the ideal thickness calculated by the modified Bayar equation.

The van Bezooijen equation is the closest of the equations discussed thus far to correctly accounting for all of the constraints on the bond. The assumption of van Bezooijen is that the bond is fully constrained to the lens and the cell. In reality this assumption is not entirely accurate because the bond is allowed to bulge or shrink at its exposed surfaces. Because of its assumption of *complete* constraint, the van Bezooijen equation does not calculate the ideal bond thickness, but rather it serves as a limit. The assumptions of the van Bezooijen equation include a constraint on the bond which is slightly too severe; if the bond were actually constrained as much as the van Bezooijen equation suggests, the bond would grow more than it does in reality and thus the equation calculates too thin of a bond gap. The thickness calculated by the van Bezooijen equation is therefore a lower limit for the athermal bond thickness for a complete circumferential bond.

5. MODIFIED VAN BEZOOIJEN – THE UPPER LIMIT

The van Bezooijen equation was shown to be the lower limit to the ideal bond thickness; an upper limit is also needed. The upper limit is derived by making the opposite assumption of the van Bezooijen for axial strain; rather than the bond being completely constrained to the surfaces of the bond's substrates, the bond is completely unconstrained axially. This means that the axial strain is assumed to be zero, as it was assumed to be for the derivation of the Bayar equation. The tangential and radial strains are the same as for the original van Bezooijen derivation. When the stress equation is solved for h with zero radial stress, the result is the modified van Bezooijen equation (9).

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)} \tag{9}$$

The difference between the van Bezooijen equation (8) and this newly proposed modified equation (9) is a factor of 2 on the third term in the denominator. The van Bezooijen and modified van Bezooijen equations define the lower and upper bounds for the correct athermal bond thickness. The factor that determines where the actual ideal bond thickness lies between these two extremes is the bond aspect ratio. All of the standard athermal bond thickness equations (Bayar, modified Bayar, and van Bezooijen) neglect the impact of the aspect ratio on the performance of the bond. The aspect ratio is defined as the ratio of the bond width to the bond thickness: $R_{aspect} = \frac{L}{h}$. The published equations for athermal bond thickness all assume a large aspect ratio for the bond and thus the edge affects of the exposed surfaces are negligible. However, if the aspect ratio is small, the axial strain can no longer be assumed to be completely constrained in this way. For extremely low aspect ratios the strain in axial direction can be set to zero because as the width of the bond, L, approaches zero, bulge of the bond at temperature is the dominant affect. For low aspect ratios the axial constraint of being adhered to the lens and cell can be neglected; the modified van Bezooijen represents this case.

6. ASPECT RATIO APPROXIMATION

Now that the two limiting cases have been defined, a relationship is needed that spans the region between them. The aspect ratio is required in order to derive a new equation that spans this space. To incorporate the aspect ratio, we let the modified van Bezooijen equation represent an aspect ratio of one. In other words, the assumption is made that, if the width of the bond is equal to the thickness of the bond, then the bond behaves as though it is not constrained in the axial direction. The "ratio of axially constrained bond" is then defined as follows:

$$R_{constrained} = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - R_{aspect}^{-1} \tag{10}$$

This relationship is shown graphically in figure 4, where the unconstrained portions of the bond are highlighted. This is clearly an idealization; in reality, the edges of the bond are not completely unconstrained and the center of the bond is not fully constrained.

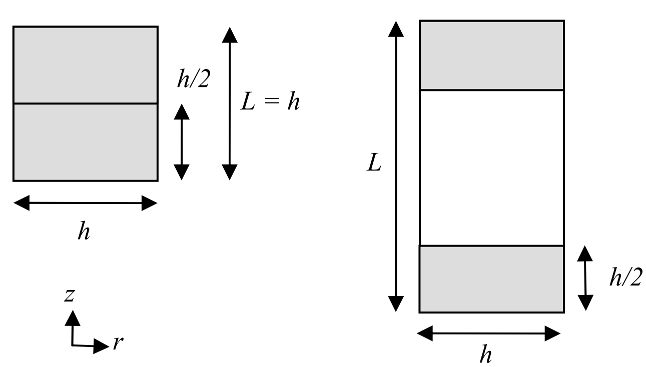

Figure 4. "Unconstrained portions" (gray) of a bond with an aspect ratio of 1 compared to a more typical bond cross section.

The full derivation of the aspect ratio approximation follows. First, the expressions for strain are substituted into equation (4). Equations (3) and (7) are used for the radial and tangential strains respectively, as they were for the derivation of the van Bezooijen equation. Axial strain is calculated by multiplying equation (10) by equation (7), the result is equation (11). The formula for stress follows:

$$\Delta T\left(1-\frac{h}{L}\right)\left(\alpha_b-\frac{\alpha_o+\alpha_c}{2}\right) \tag{11}$$

$$\sigma_r=\frac{E\Delta T}{(1+\nu)(1-2\nu)}\left[(1-\nu)\left(\alpha_b-\alpha_c-\frac{r_o}{h}(\alpha_c-\alpha_o)\right)+\nu\left(2-\frac{h}{L}\right)\left(\alpha_b-\frac{\alpha_o+\alpha_c}{2}\right)\right]$$

As stated before, the equation for stress is lacking a correction factor, which drops out in the next step. Note that the axial and tangential strains have been combined in the second term inside the brackets. The stress equation is set equal to zero and the result is solved for *h*. Equation (12) is shown to demonstrate the similarities between this formula and the previously derived formulas for bond thickness; it does not explicitly solve for *h*, however.

$$h=\frac{r_o(\alpha_c-\alpha_o)}{\alpha_b-\alpha_c+\frac{\nu}{1-\nu}\left(2-\frac{h}{L}\right)\left(\alpha_b-\frac{\alpha_o+\alpha_c}{2}\right)} \tag{12}$$

Note the coefficient of the last term is $\left(2-\frac{h}{L}\right)$, which can assume values between 1 and 2 for values of *L* between *h* and infinity. Recall that the coefficient of this last term in the denominator of the van Bezooijen equation (7) is 2 and coefficient of the same term the modified van Bezooijen equation (8) is 1. Computations of *h* will be bounded by those two equations. The quadratic formula is required to formulate the complete closed-form solution. The original stress equation is multiplied by *h* and rearranged into the following quadratic equation.

$$0=h^2\left(\frac{-\nu}{L}\left(\alpha_b-\frac{\alpha_o+\alpha_c}{2}\right)\right)+h\left((1-\nu)(\alpha_b-\alpha_c)+2\nu\left(\alpha_b-\frac{\alpha_o+\alpha_c}{2}\right)\right)-r_o(1-\nu)(\alpha_c-\alpha_o)$$

$$\left.\begin{aligned}a&=\frac{-\nu}{L}\left(\alpha_b-\frac{\alpha_o+\alpha_c}{2}\right)\\ b&=(1-\nu)(\alpha_b-\alpha_c)+2\nu\left(\alpha_b-\frac{\alpha_o+\alpha_c}{2}\right)\\ c&=r_o(1-\nu)(\alpha_c-\alpha_o)\end{aligned}\right\}\quad h=\frac{-b+\sqrt{b^2-4ac}}{2a} \tag{13}$$

The aspect ratio approximation solution (13) for the athermal bond thickness is shown in expanded form. The alternate solution to the quadratic formula (the solution having the minus sign before the square root) is associated with unrealistic values of the bond width and is a result of the aspect ratio approximation only being valid for aspect ratios between 1 and infinity. Figure 5 compares the assumptions of the van Bezooijen, modified van Bezooijen, and aspect ratio approximation equations. The aspect ratio approximation expands less in the radial direction because it is allowed to bulge at the exposed top and bottom surfaces. The aspect ratio approximation's allowance for bulging results in a calculated ideal bond thickness that is greater than the thickness calculated by the van Bezooijen equation.

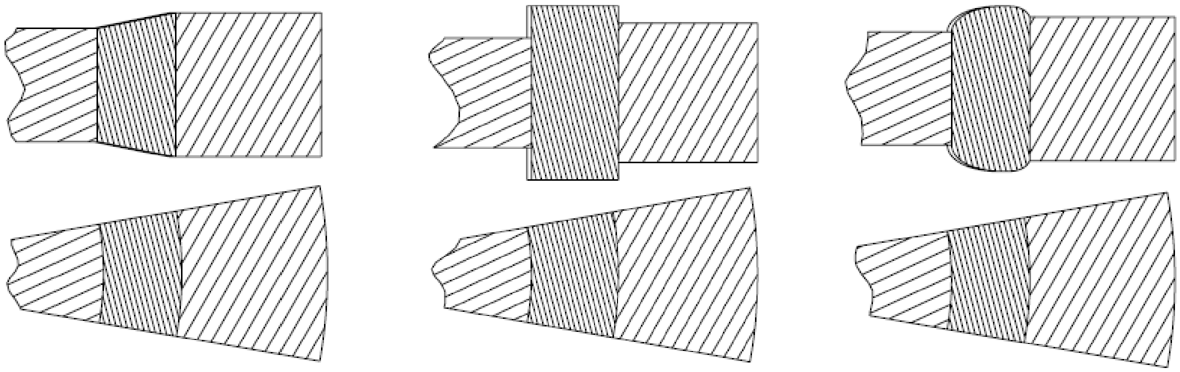

Figure 5: Comparison of a heated system under the van Bezooijen assumption (left), a heated system under the modified van Bezooijen assumption (center), and a heated system under the aspect ratio approximation (right). The three derivations all use the same tangential and radial constraint; they vary only in axial constraint. Figure is not to scale.

A graphical comparison of the van Bezooijen, modified van Bezooijen, and aspect ratio approximation methods is shown for an example system in figure 6. As the aspect ratio approaches one (towards the left of the graph), the approximate solution approaches the result of the modified van Bezooijen equation.

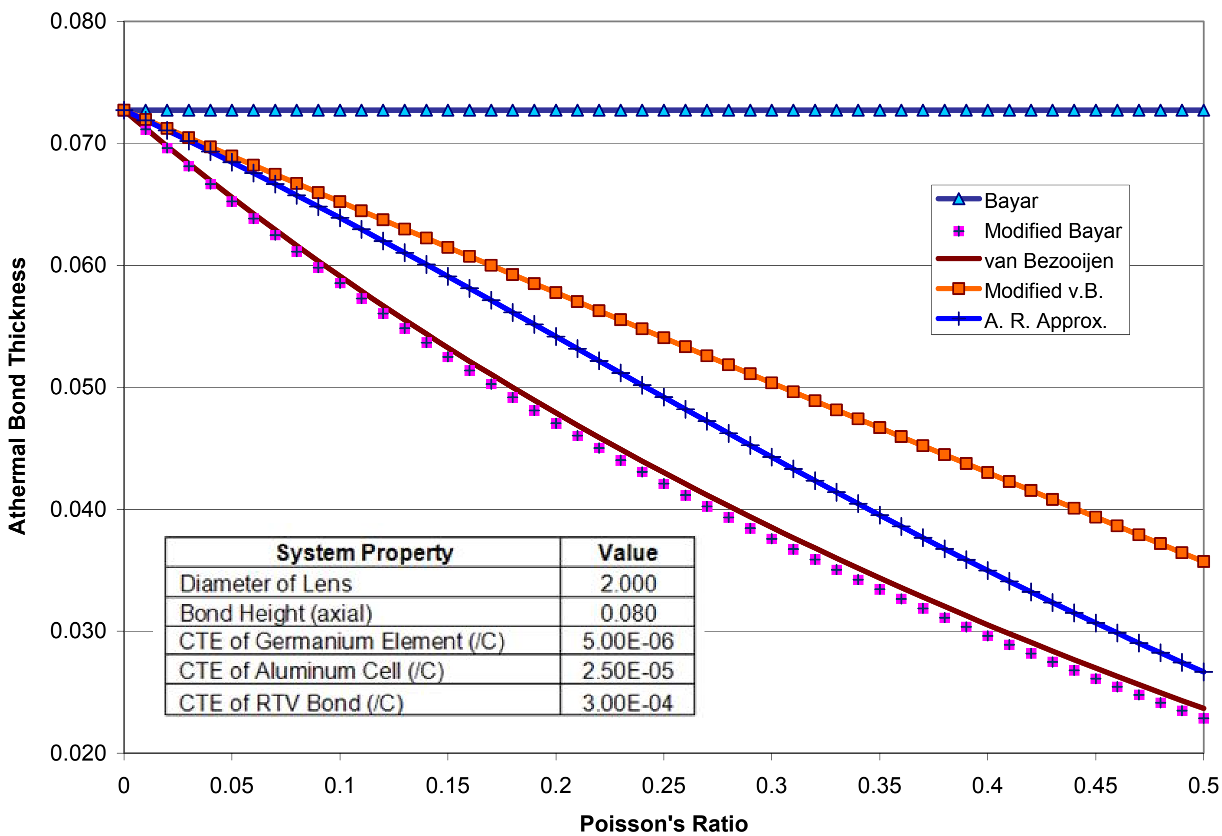

Figure 6. Comparison of results for a sample system from the Bayar, modified Bayar, van Bezooijen, modified van Bezooijen, and aspect ratio approximation solutions for athermal bond thickness.

7. COMPARISON TO RESULTS FROM FINITE ELEMENT ANALYSIS

A modified Hooke's Law matrix was developed by Michaels and Doyle[5], who developed empirical correction factors to be applied to Hooke's Law based on finite element analysis (FEA). The result of their work for radial stress is:

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}\left[k_{11}(1-\nu)\varepsilon_r + k_{11}\nu(k_{12}\varepsilon_z + k_{13}\varepsilon_\theta)\right] \tag{14}$$

The correction factors k_{11}, k_{12}, and $k_{13,}$ are available in tabular form as functions of the bond aspect ratio and Poisson's ratio. In order to correlate the FEA results to the aspect ratio approximation, equations (3) and (7) are substituted into the radial stress equation, resulting in a equation (14) for stress in terms of the correction factors. The strains are the same as those used in the van Bezooijen derivation. This equation is then solved for the bond thickness.

$$\sigma_r = \frac{k_{11}E\Delta T}{(1+\nu)(1-2\nu)}\left[(1-\nu)\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right) + \nu\left(k_{12}+k_{13}\right)\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)\right]$$

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu}\left(k_{12}+k_{13}\right)\left(\alpha_b - \frac{\alpha_o+\alpha_c}{2}\right)} \tag{15}$$

The correction factor k_{11} drops out of the equation for bond thickness. The result is a direct connection between equations (12) and (15), with the term $(2 - h/L)$ in equation (12) replaced by $(k_{11} + k_{11})$ in equation (15). The two expressions are compared in Figure 7. The FEA data were collected for several values of the Poisson ratio, all values in the relatively narrow range of 0.45 to 0.5.

The coefficient calculated by the aspect ratio approximation is shown to be fairly close to the tabulated coefficients gathered from FEA data. The benefit of the aspect ratio approximation is that it provides a closed-form solution. When using the aspect ratio approximation there is no need to refer to a table of correction factors that are used to iterate to a solution.

Referring to figure 7, it can be seen that for aspect ratios greater than 4 there is little difference between the correction factor calculated by the aspect ratio approximation the correction factors that Michaels and Doyle present. For aspect ratios below 4 the calculated athermal bond thickness will be slightly smaller for the aspect ratio approximation than for the tabulated correction factors. For comparison, the "correction factor" for the van Bezooijen equation is always equal to 2.

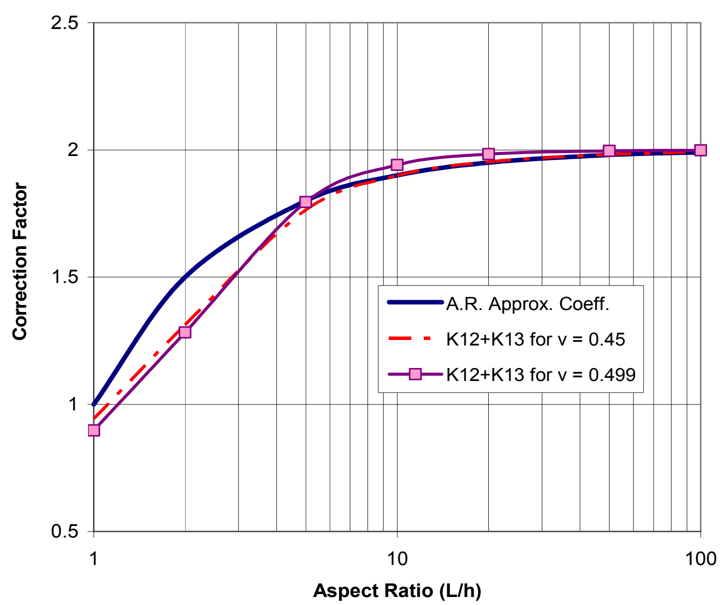

Figure 7. Comparison of the correction factor as calculated by the aspect ratio approximation to the correction factor from FEA data for several Poisson ratios

8. DISCUSSION OF STRESS AND SENSITIVITY

8.1. Stress

As stated before, the first part in all of the stress equations, $\frac{E}{(1+\nu)(1-2\nu)}$, drops out when calculating athermal bond thickness. When calculating stress, this part of the equation approaches infinity as the Poisson ratio approaches 0.5. In reality, the bulging of the bond that occurs allows for a great reduction in this stress. In the stress calculation presented by Michaels and Doyle[5], the correction factor that has the largest impact is the factor k_{11}. The k_{11} correction factor is shown graphically for three different Poisson ratios in figure 8. It can be seen that the correction factor plays the roll of greatly reducing tendency towards infinity as long as the aspect ratio is low. The lesson to be learned is that small aspect ratios have the affect of greatly reducing thermally induced stress on the element. Low aspect ratios in optical element mounting have affectively been "saving" systems that were designed using the Bayar equation. The other reason that systems have tended to succeed in the past is that the rubbery bond materials with the highest Poisson ratios tend to also have the lowest elastic moduli.

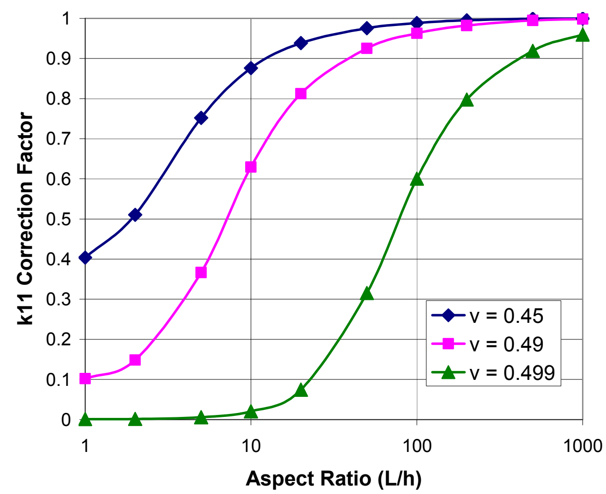

Figure 8. Stress correction factor[5] vs. aspect ratio for three high Poisson ratios.

8.2 Sensitivity

Values for the CTE of the element and cell materials are typically available and reliable. Obtaining accurate information on the CTE and Poisson ratios of the bond material is more difficult and the values are unlikely to be highly accurate. A sensitivity analysis was performed on the system that is defined in figure 6. The baseline Poisson ratio that is used for the analysis is 0.45. It is shown that for this system there is high sensitivity to both the bond CTE and the bond Poisson ratio. A 10% inaccuracy in the bond material's CTE corresponds to a 15% error in the calculated bond thickness. This suggests that perhaps not too much more effort should be spent in refining the calculations for the athermal bond thickness because the sensitivity of any equation to its inputs will overshadow any improvement.

Table 4: Sensitivity analysis for example system that is shown in figure 6. Baseline values for Poisson ratio and bond thickness are .45 and .031 respectively.

Input Parameter	**Change (% Change) in Bond Thickness After a 10% Change in the Parameter**
CTE Element	.001 (3%)
CTE Cell	.004 (12%)
CTE Bond	.005 (15%)
Poisson Ratio of Bond	.004 (12%)

9. CONCLUSION

A new closed-form solution for the ideal athermal bond thickness was developed. The solution was generated by incorporating the bond aspect ratio in order to span the space between the two limiting cases for athermal bond thickness. The lower limit was shown to be the equation originally developed by van Bezooijen; a newly derived upper limit was formulated by allowing for the bond to be unconstrained in the axial direction. The aspect ratio approximation solution was compared to empirical data derived from FEA. The closed-form solution is shown to be quite close to the tabulated results from the FEA data.

The closed-form solution derived in this paper is the closest approximation to the ideal athermal bond thickness presented to date. It should be used for calculations when the zero radial stress condition is critical. The new equation still makes several assumptions; the assumptions of all the equations discussed in this paper are summarized in table 5. One of the most important assumptions, common to all of equations presented in this paper, is that the material properties of the system's materials are constant with temperature. Since this assumption generally is valid only when temperature changes are relatively small, the athermal bond thickness becomes less accurate when applied across wide temperature ranges. It was also shown that the equations are generally sensitive to inaccuracies in the CTE and Poisson ratio of the bond, which are sometimes difficult to determine. The detailed description of the assumptions used for each derivation should aid the reader in deriving approximations for more complicated geometries.

Table 5. Table of Athermal Bond Equations: Assumptions and Usage Guideline

Name of Equation	#	Assumptions	Recommended Use
All	NA	1. Constant CTEs 2. Constant Poisson ratio 3. Cell and lens are infinitely stiff (not an assumption of the FEA analysis) 4. Zero stress bond at cure temperature (zero bond shrinkage) 5. No thermal gradient	N/A
Bayar	2	1. Unconstrained in axial and tangential directions	Theoretical upper limit for an unconstrained bond such as a low aspect ratio bond with small pads of bond material
Modified Bayar	6	1. Perfectly constrained in axial and tangential directions – no thermal expansion of cell or lens in those directions 2. No axial bulging of bond – high aspect ratio	No major benefit; the van Bezooijen equation should be used in place of the modified Bayar equation
Van Bezooijen	8	1. Perfectly constrained to the expanding and contracting lens and cell 2. No axial bulging of bond – high aspect ratio	Closer to the real solution than the modified Bayar and serves as the lower limit to the correct solution for a complete circumferential bond
Modified van Bezooijen	9	1. Perfectly constrained in tangential direction to the expanding and contracting lens and cell 2. Unconstrained in the axial direction – low aspect ratio	The upper limit to the correct solution for a complete circumferential bond
Aspect ratio approximation	13	1. No specific assumptions other than those stated for all of the equations, but the correction factor is an approximation	The closest closed-form solution for athermal bondline thickness
FEA data corrected	15	1. FEA methodology is correct	Empirically determined calculation for athermal bond thickness – has no limiting assumptions.

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