Optomechanical sensitivity and tolerancing.

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ABSTRACT

Optomechanical sensitivity is the amount of image quality degradation in response to a certain deviation of any primary parameter of an optomechanical system. The sensitivity depends not only on the optical system but also on the mechanical layout. A technique for calculation of optomechanical sensitivity is presented. Only mechanical parameters are used as primary parameters. It is shown that airspaces or element tilts and decentrations cannot be used as primary parameters. The tolerances of an optomechanical system can be determined based on the computed sensitivity. Edge Function is used to evaluate the image quality for projection lenses. Software for realization of this technique has been developed. Examples of optimization of the optomechanical system of a projection lens using the developed technique are presented.

Keywords: sensitivity, primary parameter, optomechanical layout.

1. INTRODUCTION

The components of an optomechanical system cannot be made perfectly. Some deviations of the parameters of a lens assembly will lead to certain image quality degradation that can be quantified with sensitivity coefficients. Since sensitivity depends not only on the optical design but also on the optomechanical layout, the modification of the latter can be used to reduce the sensitivity of the lens assembly.

This project was an attempt to create a technique for optimization of a lens assembly by choosing the least sensitive optomechanical layout. It consists of three main procedures:

* Definition of the manufacturing errors as deviations of primary parameters;
* A technique to compute the image quality degradation and to generate a sensitivity table based on certain image quality criteria;
* Optimization of the system by choosing the least sensitive optomechanical layout based on the sensitivity table. Application of a statistical approach to budget the tolerances.

Special software was developed for practical implementation of this procedure. Some practical examples illustrate the application of the developed technique.

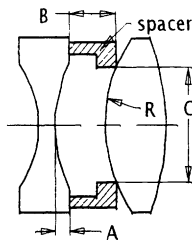## 2. DEFINITION OF PRIMARY PARAMETERS.

Fig. 1 Primary parameters that affect airspace for given mechanical layout

A primary parameter of an optomechanical system is defined as any parameter that does not depend on other parameters and can be specified and checked independently. Airspaces, tilt, and decentration are not primary parameters. Illustrations of this statement are shown in Figs 1, 2. For the mechanical layout shown in Fig. 1 the airspace depends on four primary parameters which are: 1) spacer thickness B, 2) contact diameter C, 3) sag A and 4) radii R. Fig. 2 illustrates that the lateral position of an optical element in presence of clearance also depends on the mechanical layout. The same clearance between barrel and element leads to different positions (tilt and decentration) of the element for various mechanical layouts schematically shown in Fig. 2: the element is positioned on the flat, on the left optical surface and on the right optical surface.
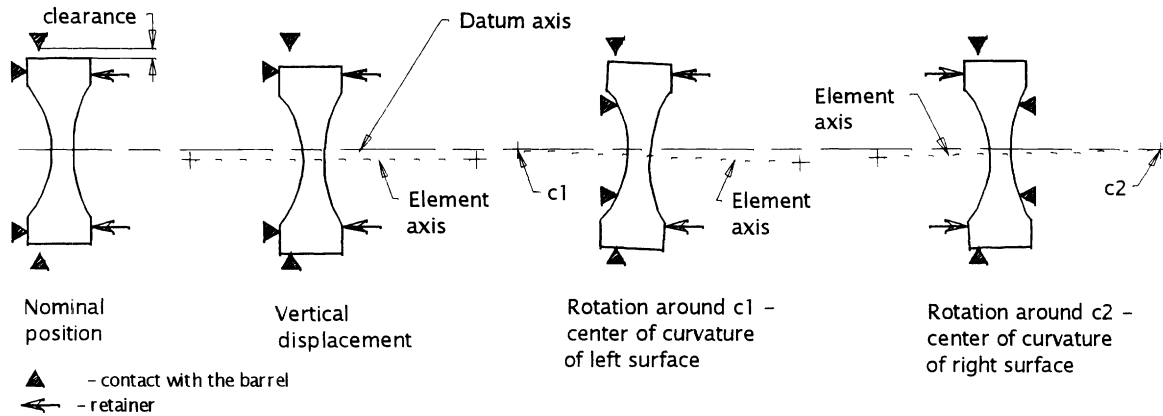
Fig. 2 Clearance effect on position of optical
element for various mechanical layouts

The developed technique includes twelve primary parameters, which are listed in Table 1 with abbreviations, definition and applicability. The optical surface profile is not included in this list (this topic is under development now). One of the parameters in Table 1 is an exception from the above definition - clearance is the combination of two primary parameters: barrel inside diameter and lens (or spacer) outside diameter. This exception is used to reduce the number of primary parameters without losing generality. The applicability of primary parameters for a lens assembly depends on the optomechanical layout that will be illustrated below. The definitions of datum axis and datum point can be found in [1].

Table 1. List of used primary parameters in lens assembly.

		Parameter	Definition	Applicable to:
1	CTH	Center thickness	Axial distance between two consecutive vertexes	Lenses, spacers
2	SAG	Surface sag	Axial distance from basis flat to vertex of optical surface	Lenses with flat contact surface(s)
3	REF	Index of refraction		Lenses
4	COD	Contact diameter	Diameter of the seat in contact with non-plano optical surfaces	Barrel, spacers
5	ASP	Axial seat position	Axial distance from mechanical assembly datum point to barrel seat	Barrel
6	CLE	Clearance	Clearance per side between barrel ID and lens (or spacer) OD	Barrel-spacer or barrel-lens interface
7	WED	Wedge	Edge thickness difference	Lenses, spacers
8	STI	Seat tilt	Tilt of flat seat relative to common mechanical axis (datum axis)	Barrel
9	CDE	Contact diameter eccentricity	Displacement of seat diameter axis from common mechanical axis (datum axis)	Barrel, spacers
10	S2S	Surface to surface displacement	Lateral displacement of optical surfaces of the lens	Aspherical lenses
11	BDE	Base diameter eccentricity	Displacement of base diameter axis from common mechanical axis (datum)	Barrel
12	SBC	Second barrel clearance	Clearance between main and secondary barrel	Primary/secondary barrel interface

3. IMAGE QUALITY CRITERIA

The image quality criteria (q-factor) used is based on the Edge Function (or Edge Gradient as described in [2]). The Edge Function is the intensity distribution across the image of the step border between wide white and black areas. The routine for the Edge Function calculation is well known [3] and consists of three steps:

1. Calculation of the Point Spread Function (PSF) based on the tracing of a certain number of rays through the system;
2. One-dimensional integration of the PSF to generate the Line Spread Function (LSF);
3. Appropriate integration of the LSF to generate the Edge Function (EF).

For a complete description of the image quality at a particular field position we have to use three different orientations of the white/black border: one in the sagittal direction and two in the tangential direction which are additive to each other (white to black and black to white). The Edge Function for these three directions can be obtained from the same Point Spread Function by changing the direction of integration.

Two numerical parameters – flare and slope – can be derived from the Edge Function as shown in Fig.3:

* flare is the linear dimension from the geometrical position of the image of the edge to the point where the intensity of the Edge Function is equal to given cut-off;

* slope is the average tilt of the Edge Function and can be calculated as:

$$slope = \int_{-\infty}^{+\infty} \left(|y| \times \frac{\partial EF(y)}{\partial y} \right) dy \qquad (1)$$

The developed software has the option to use contrast (MTF) at a certain frequency as the image quality criteria for all calculations described below. However, the Edge Function was chosen and used for this purpose because of the following practical reason: this theory was developed and applied mainly for projection lenses where the end user is able to see the Edge Function directly on the screen. Flare is the parameter that is easy to observe and measure. Slope can be used to quantify the image softness. Combination of flare and slope gives the image quality criteria (q-factor):

$$Q = flare + slope \qquad (2)$$

The cut-off intensity for the flare calculation determines the visibility of the flare and depends on a number of variables, such as the light condition in the room, the broad area contrast of the lens assembly, etc. It can vary from one observer to another. The value of the cut-off used in this project is equal to 2%.
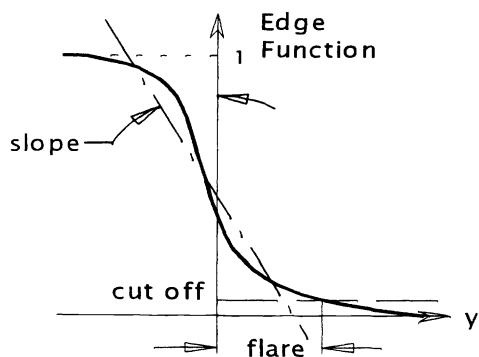

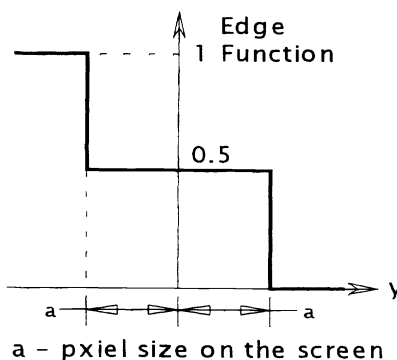Fig. 3. Edge Function,   Fig. 4. Edge Function for Qp.   Fig. 5. Definition of ΔQ
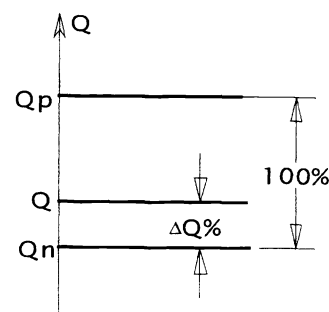flare and slope.

a – pxiel size on the screen

There are three image quality parameters (Qn, Qp, ΔQ) used for the calculation of the sensitivity table and the tolerance budgeting:

* The nominal image quality (Qn), which corresponds to the system with all nominal (designed) parameters. Qn can be calculated using equations 1,2.

* The limited image quality (Qp) is a measure of the worst acceptable image on the screen. This parameter is very important to establish the right scale for tolerancing, because it determines the total available budget for manufacturing errors. It is one of the central questions for tolerance analysis – how much a lens as manufactured can depart from its nominal design and still be acceptable [3]. One of the possible criteria to determine a value for Qp is to associate it with the pixel size on the screen (used in this project) as shown in Fig.4. This is an extreme case for an Edge Function, giving a double image of the edge. Application of equations 1 and 2 for this case gives the result:

$$Qp = 2 \times a$$

This value can (and probably should) be changed for a specific application without any modification of the developed technique. The appropriate value for the limited image quality depends on application, customer requirement, balance between image quality and production cost, etc. [7, 8].

* The image quality (Q) of the optical system with a given primary parameter deviation.

The relative image quality degradation (ΔQ) can be associated with the deviation of any given parameter. The relationship between Qn, Qp and Q is shown schematically in Fig.5. The value for ΔQ can be obtained from the equation:

$$\Delta Q = \frac{Q-Qn}{Qp-Qn} \times 100\% \tag{3}$$

The physical meaning of the parameter ΔQ is as follows: if the total budget of acceptable image quality degradation is 100% the value ΔQ corresponds to the percentage of this total budget used by a given deviation of a given primary parameter. Basically this parameter can be used as the measure of system sensitivity. For each field position there are three values of ΔQ – one in the sagittal and two in the tangential directions, and these values will vary across the screen. This will generate a large number of coefficients ΔQ. The following example illustrates the practical inconvenience of this approach:

number of field position used for practical analysis = 7 (on-axis, ±30%, ±50%, ±80% of full field);
number of values for ΔQ for deviation of one primary parameter: 7 x 3 = 21;
number of primary parameters in the system (typical): 50;
total size of sensitivity table: 50 x 21 = 1050.

It is obvious that this is too much information to be used for any practical purposes.
The routine to compute sensitivity with a reduced volume of final information is described in the following section.

4. ROUTINE FOR OPTOMECHANICAL SENSITIVITY CALCULATION.

The following abbreviations are used below:

k	order number of field position
s	sagittal direction
tp, tm	two additive tangential directions: white to black and black to white
QN	q-factor calculated for nominal system for a certain direction
$Qnom$	nominal q-factor associated with particular field position
ΔP	deviation of primary parameter
$\pm Q$	q-factor calculated for system with a particular deviation of a given primary parameter ΔP
Q	q-factor associated with particular field position of deviated system
$\Delta Q[k]$	relative q-factor degradation for given field position
ΔQ	relative q-factor degradation for entire screen associated with deviation ΔP of given primary parameter

Procedure of calculation implemented in the developed software:

1. Choose q-factor Qp corresponding to limited image quality.
2. For nominal system:
   * Find the best focus position of the screen.
   * Make q-factor calculation for four field positions and for three directions:
       $QNs[k]$, $QNtp[k]$, $QNtm[k]$, $k = 0, 1, 2, 3$.
   * For each field position the nominal q-factor is taken as the maximum value of the three directions:
       $Qnom[k]$ = maximum of $QNs[k]$, $QNtp[k]$, $QNtm[k]$
       This value corresponds to the worst orientation of the white/black border for each field position.
3. The system is rearranged in accordance with the primary parameter deviation ΔP and the given optomechanical layout. This step simulates the real position of the surfaces in the system. For example, if the primary parameter is the clearance between the element and the barrel, the position of the element is determined in accordance with how this element is fixed (see Fig. 2). Moreover, if this element serves as the basis for other components, these components will follow it. Another example: if the primary parameter is the center thickness, the positioning of the surfaces along the optical axis will depend on the optomechanical layout as illustrated on Fig. 6. If the center thickness 'A' is increased, three optical surfaces will move to the right and
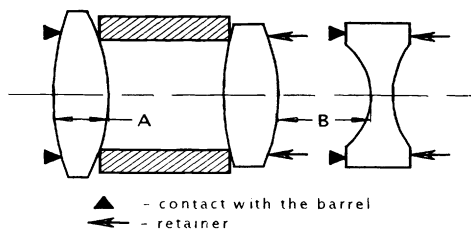

▲ – contact with the barrel
◀— – retainer

Fig. 6 Center thickness (A) and
corresponding airspace (B)

the airspace 'B' will correspondingly decrease. In other words, in this step the assembly is regarded as an optomechanical system, but not as an optical design. For the rearranged system the q-factor is calculated for three directions (one sagittal and two tangential) for each required field position. If the rearranged system is axially symmetrical (first 5 primary parameters as they are listed in Table 1), the system is refocused (screen is moved) to find the best focus on axis.

For the rearranged system, the q-factor is calculated for $+\Delta P$ and $-\Delta P$:

   * Make q-factor calculation for $\pm$ deviation of primary parameter:

$$\pm Qs[k], \ \pm Qtp[k], \ \pm Qtm[k], \ k = 0, 1, 2, 3$$

4. The overall image quality for a given field position is determined by the worst image quality for all three directions. This means that we need to choose the worst case out of three directions for each point on the screen. The same consideration is applicable to the worst out of both directions of primary parameter deviation (there is the assumption that tolerance fields are symmetrical).

$$Q[k] = \text{maximum of } \pm Qs[k], \ \pm Qtp[k], \ \pm Qtm[k].$$

5. Up to now we have used the q-factor as the parameter that describes the image quality. But the nominal image quality Qn for different field positions is different and the limited image quality Qp can vary across the screen. This means that the tolerance budget will vary from one field position to another. To estimate how much of the total budget will be used by a given deviation ΔP of given primary parameter, the results from the previous step are reformatted in ΔQ in accordance with the equation 4.

6. The final step is to pick the largest relative image quality deviation across the entire screen:

$$\Delta Q = \text{maximum of } \Delta Q[k] \}.$$

After application of this routine we generate one number (ΔQ) which describes the response of the optomechanical system to a deviation (ΔP) of one chosen primary parameter.

5. SENSITIVITY TABLE AND RUDIMENTARY OPTIMIZATION OF OPTOMECHANICAL SYSTEM.

The sensitivity table is simply the set of ΔQ, computed as described above, for given deviations ΔP of each primary parameter of the optomechanical system. Table 2A and 2B exemplify this for mechanical layouts of the lens assemblies shown in Fig.7A and 7B.
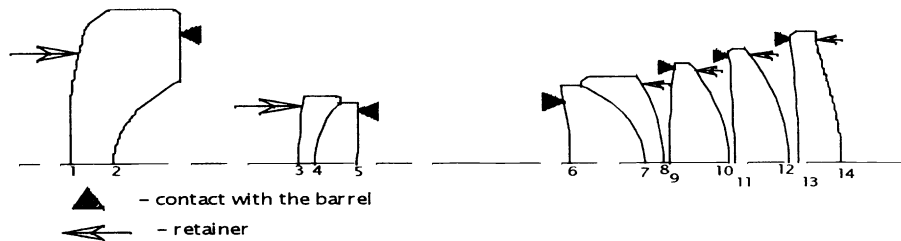

▲   – contact with the barrel

◄   – retainer

Fig. 7A Optomechanical layout 'A'

▲   – contact with the barrel

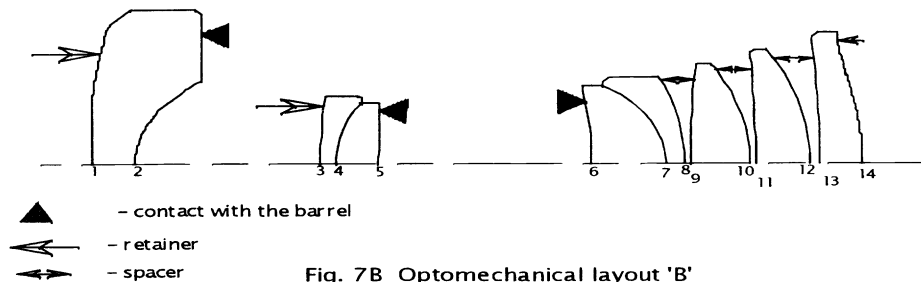◄   – retainer

◄►   – spacer   Fig. 7B Optomechanical layout 'B'

The abbreviations of the primary parameters correspond to Table 1 and all deviations ΔP equal 0.025 mm, except for the refractive index (REF) $\Delta P = 0.0005$. Values in the table are ΔQ - the percentage of total image quality budget used by a deviation ΔP of a particular primary parameter.

The value of ΔQ corresponds to the surface specified in the left column or to the space after this surface. An empty cell in the table means that this deviation is not applicable to the current surface or to the space after this surface. Center thickness (CTH) corresponds to the element after the current surface, for instance (see table 2B and Fig. 7B) CTH[9] is the deviation of the center thickness for the element with surfaces 9 and 10, but CTH[8] belongs to the spacer thickness after the second doublet. The surface sag is applicable only when the flat of the lens is used for positioning purposes. The contact diameter (COD and CDE) is significant when the barrel or spacer contacts with a non-planar optical surface of an element. In these cases the contact diameter deviation (COD) will change the axial positioning of the element(s) and contact diameter eccentricity (CDE) will cause a tilt. The axial seat position (ASP) and seat tilt (STI) is the position and tilt of the barrel flat, which serves as the seat for the optical element. Wedge is applicable for all elements and spacers. Base diameter eccentricity (BDE) is the lateral displacement of the internal barrel diameter relative to the datum axis of the barrel. Surface to surface displacement is applicable only to elements with aspherical surfaces.

Table 2A. Sensitivity table for mechanical layout "A".

Surface #	CTH	SAG	REF	COD	ASP	CLE	WED	STI	CDE	S2S	BDE
1	1		1			2	1			6	2
2		1			1			11			
3	0		1			1	30				1
4	1		1								
5				0	1			1	0		
6	3		3	2	3	21	11	4			21
7	2		2								
8											
9	1		5	2	2	24	27	4	1		24
10											
11	1	2	4		2	22	27	14			22
12											
13	0	1	2		0	27	13	7		28	27

Table 2B. Sensitivity table for mechanical layout "B".

Surface #	CTH	SAG	REF	COD	ASP	CLE	WED	STI	CDE	S2S	BDE
1	1		1			2	1			6	2
2		1			1			11			
3	0		1			1	30				1
4	1		1								
5				0	1			1	0		
6	1		3	0	1	13	14	4	2		13
7	4		2								
8	4			2		6	13		6		6
9	1		5	0		49	55		1		49
10	0			1		22	25		21		22
11	1	0	4			30	42				30
12	1			0		5	7		6		5
13	0	1	2			27	13			28	27

Comparison of data in Tables 2A and 2B shows that introducing spacers in the rear group of elements will:

* increase the sensitivity of the system to clearance between elements and barrel
* increase the sensitivity to wedge in the elements
* not change the sensitivity to center thickness deviation.

The element after surface #9 in Table 2B shows significant increasing of sensitivity to wedge and clearance. A wedge of this element will tilt surface #10 on layout 'A', but on layout 'B' the same wedge will cause tilt of the following two elements also ("domino" effect).

Comparison of these tables allows choosing the less sensitive mechanical layout that is the example of elementary step of an optomechanical system optimization, with the goal of reducing sensitivity. In parallel with implementation of this technique the mechanical/manufacturing/cost constraints must be taken into consideration.

6. STATISTICAL BUDGETING OF TOLERANCES.

Assumptions for statistical analysis:

* There is a linear relationship between a deviation of a primary parameter ΔP and the system response ΔQ. Four graphs on Fig. 8 present the results of calculating $\Delta Q(\Delta P)$ for four different primary parameters. Each graph has three curves corresponding to three different orientation of the Edge Function. These graphs do not prove that this relationship is always linear, but illustrate that such assumption can be used.
* Fabrication errors will follow normal Gaussian statistics, which means that parameters vary about their means in a Gaussian fashion as outlined in [5]. For many fabrication processes this is not true, and this assumption will not work.
* Tolerances ΔT are considered to be a multiple of standard deviations σ:

$$\Delta T = A \times \sigma ,$$

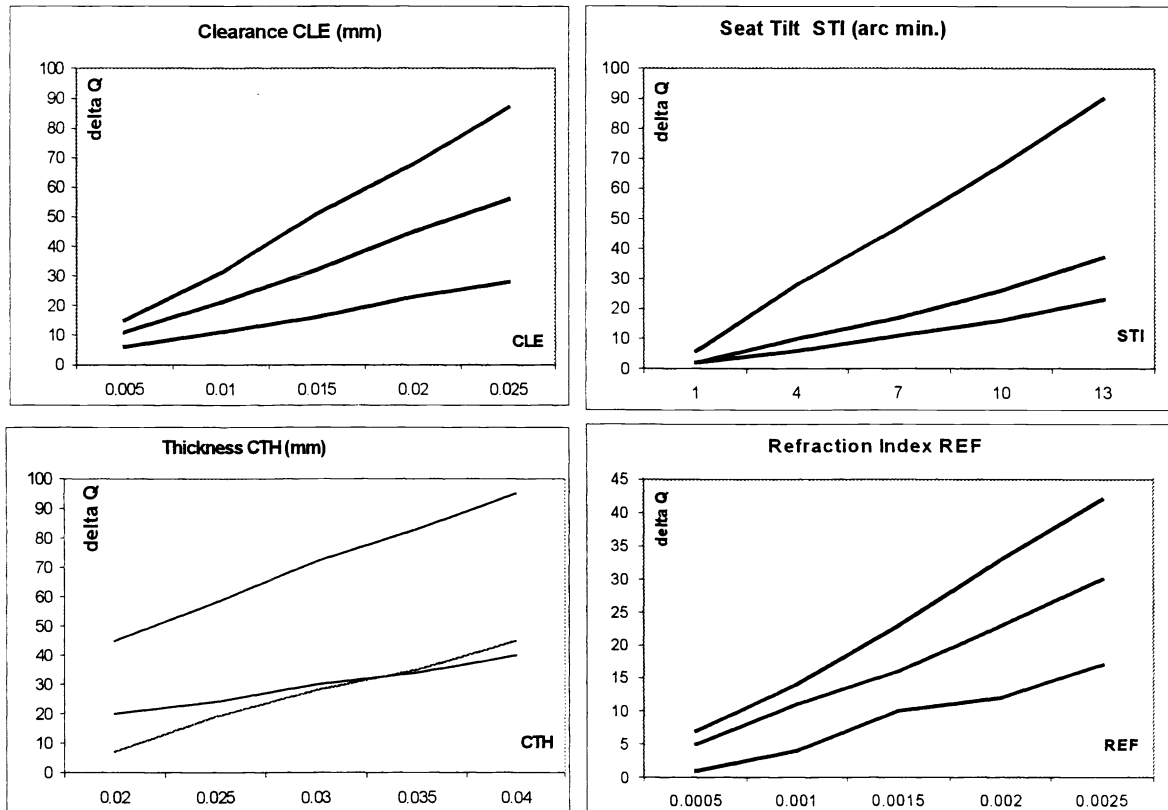where A is the constant that controls the probability that the value will fall within the allowed range.

Fig.8. Linearity of ΔQ versus ΔP

The procedure of tolerance calculation is well developed [6, 7] and consists of the following:

1. The sensitivity table contains the number of sensitivity coefficients

$$S[k] = \frac{\Delta Q[k]}{\Delta P[k]},$$

where k corresponds to certain primary parameters of the optomechanical system.

These coefficients provide relationships for independent, individual changes of the image quality in response to the changes of primary parameters.

2. The relationship between the standard deviation σ_{qk} of the q-factor resulting from the standard deviation σ_{pk} of the kth primary parameter is

$$\sigma_{qk} = \sigma_{pk} \times S[k].$$ (4)

The total standard deviation of q-factor resulting from deviations of all primary parameters is:

$$\sigma_\Sigma = \sqrt{\sum_k \sigma_{qk}^2},$$ (5)

where the sum is taken over all the primary parameters in the system.

As it described in [6] the tolerance budgeting is a two-step process:

1) Calculation of the tolerances for individual primary parameters as equal contributors to an acceptable level of image quality degradation;

2) Redistribution of tolerances between individual primary parameters with the purpose of obtaining the required image quality and relaxing the tolerances for the most critical parameters as much as possible.

Table 3 is an example of results from the first step calculations. These data belong to the optomechanical system shown in Fig. 7B with sensitivity coefficients from Table 2B. The data in Table 3 are calculated in accordance with the above equations and under the assumption that every single primary parameter contributes the same amount of q-factor degradation. Analysis of these data shows that some of them are impractical. For instance, a tolerance of ±0.323 mm for the center thickness of an optical element or spacer looks too loose, but 0.007 mm for the clearance between barrel and element 9/10 is definitely too tight. This table is basically a good illustration of the necessity to rebalance tolerances to obtain practical results within the same total budget.

Table 3. Tolerances (mm) for optomechanical layout "B" (equal contributions).

Surface #	CTH	SAG	REF	COD	ASP	CLE	WED	STI	CDE	S2S	BDE
1	0.323		0.0065			0.161	0.323			0.054	0.161
2		0.323			0.323			0.029			
3			0.0065			0.323	0.011				0.323
4	0.323		0.0065								
5					0.323			0.323			
6	0.323		0.0022		0.323	0.025	0.023	0.081	0.161		0.025
7	0.081		0.0032								
8	0.081			0.161		0.054	0.025		0.054		0.054
9	0.323		0.0013			0.007	0.006		0.323		0.007
10				0.323		0.015	0.013		0.015		0.015
11	0.323	0.323	0.0016			0.011	0.008				0.011
12	0.323					0.065	0.046		0.054		0.065
13		0.323	0.0032			0.012	0.025			0.012	0.012

An example of tolerance budgeting is shown in Table 4 showing one of the possible solutions of redistributing tolerances between different primary parameters. The total amount of q-factor degradation is the same as in Table 3, but detailed analysis shows that this approach avoids unreasonable tolerances. The data in Table 4 are only one of the many possible solutions.

Table 4. Tolerances (mm) for optomechanical layout "B" (balanced budgeting).

Surface #	CTH	SAG	REF	COD	ASP	CLE	WED	STI	CDE	S2S	BDE
1	0.025		0.0010			0.025	0.025			0.015	0.015
2		0.050			0.025			0.015			
3			0.0005			0.025	0.025				0.015
4	0.050		0.0005								
5					0.025			0.015			
6	0.050		0.0005	0.050	0.025	0.025	0.025	0.015	0.015		0.015
7	0.050		0.0005								
8	0.025			0.025		0.0025	0.025		0.015		0.015
9	0.050		0.0005			0.016	0.015		0.015		0.015
10	0.025			0.025		0.025	0.025		0.015		0.015
11	0.050	0.050	0.0005			0.025	0.019				0.015
12	0.025					0.025	0.025		0.015		0.015
13	0.025	0.050	0.0010			0.025	0.025			0.015	0.015

In general, budgeting is one of the most difficult parts of a tolerancing process, since the main part of the budget decision involves the judgment based on experience, specific technology requirements, vendors, etc. The developed software has a spreadsheet type interface to make simultaneous evaluations of the combinations of assigned tolerances in accordance with equations 4, 5.

7. CONCLUSION

The developed technique permits evaluation of the sensitivity of a system to deviation of primary optomechanical parameters. Original software generates the sensitivity table that corresponds to a certain optomechanical layout of the system and can be used to optimize (desensitize) the mechanical design. The software can utilize the Edge Function of MTF to compute the sensitivity. A statistical approach can be applied to the optomechanical system in order to budget the tolerances.

ACKNOWLEDGEMENTS

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