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Metrology mount development and verification for a large spaceborne mirror

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Abstract

This paper describes the development of a metrology mount system for large spaceborne optical elements operating at diffraction limit. Verification tests of the mount, together with confirmation of the finite element model of the mount and mirror system, demonstrate that it is capable of simulating a zero-g environment within an uncertainty band of $\lambda/300$ rms for a mirror whose characteristic gravity deformation is 12λ. Using a highly detailed finite element model of the sandwich mirror, the effects of stiffness and mass nonuniformity due to faceplate thickness variations and calibration uncertainties in each of the support points on gravity-release errors were assessed and/or compensated for. The mathematics of optimizing these forces to yield a minimum rms figure error and the method and results of the faceplate thickness mapping to determine the spatial weight and stiffness variations are also described. Finally, verification of the finite element model is discussed where predicted, and full aperture interferometrically measured figure changes due to discrete changes in the applied force field are compared.

Introduction

If we compare the specific precision, or diameter divided by the surface quality, of today's state-of-the-art mirrors, the Space Telescope primary mirror is by far the most accurate optic ever fabricated. This mirror, which has a 94.5-inch clear aperture, has been figured to achieve a surface quality commensurate with diffraction-limited performance. Crucial to the success of the mirror is the ability to preserve this figure quality in the weightless environment in which it will ultimately be placed. When one realizes that the characteristic self-weight deflection of this mirror is approximately 12λ, the importance of minimizing gravity release errors becomes increasingly apparent.

Many factors contribute to figure degradation subsequent to the initial manufacture in a spaceborne mirror. These include, in addition to gravity release, thermal distortions due to material inhomogeneity as well as temperature spatial variations, unwanted mechanical constraint forces due to the mirror/structure attachment system, residual deformations due to launch loads, and possibly temporal instability. Statistically subtracting these factors from the near-perfect laboratory figure, along with metrology uncertainty levels, one is left with only about $\lambda/350$ rms available as a gravity release error allocation.

The task facing the designer of the metrology mount, is to accurately determine the forces necessary to support the mirror in a quasi-strain free state. The accurate determination of these forces is crucial to ensuring confidence in controlling the gravity release error. To ensure that the forces have, indeed, been correctly determined, checkpoint tests have been developed to verify them, as well as to verify that they have been implemented or adjusted to their proper values in the actual hardware. This verification process is vitally important, and its need will become clearly apparent in the subsequent discussion of deflection sensitivities.

Metrology mount concept

Mirror blank description

Before we discuss the metrology mount system and the support force determination mathematics, it is instructive to understand the physical and structural characteristics of the mirror blank itself.

What is of paramount concern to metrology support design is the spatial variation of areal density and its resultant influence on the determination of the correct support forces. Unaccounted for areal density variations, or an uncertainty in the mirror mass distribution, will produce errors in the predicted self-weight deflections which are ultimately employed in the support force determination. For example, an 0.1 lb local areal density error could produce an rms figure error of 0.009λ. Local, in terms of the mirror under discussion, is an 8 x 8 or 64 in^2 area, equal to 0.9% of that of the total mirror.

As we will show later, this area corresponds (approximately) to the size of the region supported by each of the metrology mount supports and is mathematically convenient to refer to. For the Corning ULE glass mirror material, whose density is 0.079 lb/in^2, this 0.1 lb error is equivalent to a thickness uncertainty of only 0.02 inch. These deflection sensitivity values, of course, are dependent on where the force anomaly is with respect to the mirror registration constraint points.

If the mirror was strained by a constant amount during the figure measurement process in the absence of gravity on-orbit, the strain would be relieved, and what had appeared to be a "perfect" mirror in the laboratory would lose figure quality to the same extent but of opposite sign to the 1g strain. This points out another aspect of metrology mount design, namely, repeatability. If, due to hysteresis or stiction, the force application mechanisms cause the applied forces to vary, by say 0.1 lb between measurement cycles, then erroneous material removal profiles would be generated, again leading to figure error and uncertainty.

Returning to the discussion of the mirror blank itself, the following paragraph describes its nominal geometry. The overall blank diameter is 98 inches and the clear, or working, aperture is 94.5 inches, the difference allowing for tool roll-off at the edge. The central cutout is 28 inches in diameter and the overall mirror thickness is 12 inches. Its radius of curvature is 435 inches, resulting in a sag depth of 2.76 inches. The front and backplates are nominally 1 inch thick and are separated by an egg-crate core consisting of 10 inch deep, ¼ inch thick ribs on 4 inch centers. Inner and outer edge bands, also ¼ inch thick and equal in depth to the ribs, form circumferential borders. In addition to these edge bands, which increase the areal density at two local radii, three regions of increased thickness ribs are also employed in the blank to facilitate attachment of the flight, or operational, mounting hardware. The blank is constructed entirely from Corning 7941 ultra-low expansion (ULE) glass, and weighs 1850 lb, or approximately 25% of an equivalent solid.

The mass distribution variations due to the edge bands and rib reenforcements are deliberate design features, are highly deterministic, and therefore can easily be accounted for in the mirror finite element model. Of possibly greater interest are the weight variations that result from faceplate thickness variations. We discovered during the mount development activity that the front and rear faceplates, nominally 1 inch thick, were far from uniform with peak-to-peak differences on the order of 0.35 inches or 1.8 lb over a 64 in^2 area.

A further weight distribution uncertainty analysis was also conducted to determine the effect of a randomly distributed force or weight errors on the figure. In this case, the mirror was divided in 132 regions of equal area, and each area was randomly assigned a (+) or (-) one pound weight error. Using the finite element model, the rms surface error for each of five Monte Carlo random distributions was computed. The average of these five calculations was 0.0303λ with a high and low, respectively, of 0.044 and 0.019.

Thus the error sensitivity to a <u>random</u> weight distribution is $\lambda/33$ rms per lb. If, as will be discussed later, $\lambda/500$ is budgeted for this error, then the weight uncertainty needs to be limited to less than 0.066 lb. For ULE glass the thickness uncertainty cannot exceed 0.0159 inch. Front and backplate thickness variations as a consequence of the high temperature operation associated with fusing the mirror together and sagging it to the approximate curvature are 0.15 and 0.35 inch, respectively. Thus the need to accurately map the thickness distribution in the mirror and account for it in the finite element model is a real and necessary step in achieving precise deformation predictions.

In practice, these thickness measurements were made with an ultrasonic intervalometer whose accuracy is ± 0.002 inch. This accuracy is sufficient to limit errors due to thickness measurement uncertainties to no more than $\lambda/4000$. Actually, the thickness distribution in the back faceplate of the mirror was not random but three-lobed in nature. The thinnest regions were in the vicinity of the reinforced mounting rib areas as expected. The front face thickness variations were less than half the magnitude of the rear, but were indeed more randomly distributed with no traceability to the mount regions. This was also expected inasmuch as the sagging operation at Corning is performed with the front face down on a firebrick mold which restricts localized distortions.

<u>Determination of the support forces</u>

The essence of the metrology mount system is to apply an upward force field to the back of the mirror such that shears, and hence bending moments, are nominally zero in a 1g environment. Except for dilatation caused by a direct compressive stress parallel to the gravity vector (which results only in a miniscule power change), this simulates the effects of a gravity-free environment. In concept, this is similar to the methods by which ground-based astronomical telescope mirrors are supported.

The key to determining the required forces for a mirror whose areal density is non-uniform is a highly representative finite element model. What this implies is that areal density variations be accounted for, which was accomplished by specifying, on a 4 x 4 inch cell basis, the actual measured faceplate thickness data. This automatically defined a nodal density for each of the faceplates. It also enabled each core element to be accounted for so that the thickness of the deliberately heavier (thicker) ribs could be directly incorporated into the model. Ultimately, the model was verified by test; the results will be described at the end of this paper.

The first step in determining the required support forces is to use the finite element model to determine the magnitude and shape of the deflections to be corrected. This is precisely the self-weight deflection when the mirror is supported on its three (statically determinate) registration points. This deflection shape is designated as E_e in the subsequent analysis.

The mean square error which results from the self-weight error, E_e, and the compensating corrective deflections, E_c, produced by the force field is

$$E^2_{rms} = \frac{1}{n} \sum_{i=1}^{i=n} (E_{e_i} + E_{c_i})^2$$

where each unit area is identical in size. In reality, element sizes at the outer rim were of a different size than the basic 4 x 4 inch size used throughout the surface so that area weighting factors were employed in the ultimate analytical effort. Omitting this level of sophistication for clarity, the last equation can be expressed in matrix form:

$$\left[E^2_{rms} = \frac{1}{n} \quad \{E_e\}^T\{E_e\} + 2\{E_c\}^T\{E_e\} + \{E_c\}^T\{E_c\}\right]$$

where

$$\{E_c\} = [\alpha]\ \{P\}$$

and

$$\{E_c\}^T = \{P\}^T\ [\alpha]$$

$[\alpha]$ is the influence coefficient matrix where α_{mn} is the deflection of point m due to a unit force at n, only. Therefore, each column of $[\alpha]$ is the deflection surface due to a single force at n. $\{P\}$ is the applied force vector which is to be determined. It is emphasized that $[\alpha]$ is determined from the same model that was used to compute $\{E_e\}$ except that, in this case, the load was not the mirror weight but a unit force applied at each back surface node sequentially.

Rewriting:

$$E^2_{rms} = \frac{1}{n} \left[\{E_e\}^T\{E_e\} + 2\{P\}^T[\alpha]^T\{E_e\} + \{P\}^T[\alpha]^T[\alpha]\{P\}\right.$$

and differentiating with respect to $\{P\}$:

$$\left[d\ E^2_{rms} = \frac{2}{n}\{d\}^T \quad [\alpha]^T\{E_e\} + \quad [\alpha]^T[\alpha]\{P\}\right]$$

The minimum value of $E_{rms}{}^2$ is obtained by setting

$$[\alpha]^T\{E_e\} + [\alpha]^T\ [\alpha]\{P\} = 0$$

At this state of this analysis, the equation does not implicitly recognize that $\{P\}$ will ultimately be a sparse matrix, that is, not every node or element in the $[\alpha]$ matrix will be a force application point.

Condensing the notation such that

$$[\alpha]^T\ [\alpha] = [A]$$

and

$$-[\alpha]^T\{E_e\} = \{C\},$$

the prior equation becomes:

$$[A]\{P\} = \{C\}$$

Now {P} can be partitioned into those forces that we wish to be finite and those that we wish to be zero. Mathematically, this may be an arbitrary selection, or series of trial vectors, with the caution that if the applied forces are spaced too far apart, the inter-force uncompensated shear and bending deformations may become too large and cause unacceptable midfrequency errors. Performing this partitioning,

$$\left[A_1 \mid A_2\right] \begin{Bmatrix} P_1 \\ \hline P_0 \end{Bmatrix} = \{C\}$$

where the subscript "0" designates zero force. So then:

$$[A_1]\{P_1\} + [A_2]\{P_0\} = \{C\}$$

and since each element of $\{P_0\}$ equals zero,

$$[A_1]\{P_1\} = \{C\}$$

This partitioning scheme will lead to more equations than unknowns since $[A_1]$ is a vertically-oriented matrix. To resolve this dilemma in a deterministic way, {C} may also be partitioned into a region corresponding to the force application point and points where the force is zero.

Thus,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_0 \end{Bmatrix} = \begin{Bmatrix} C_1 \\ C_0 \end{Bmatrix}$$

which, after expanding, yields:

$$[A_{11}]\{P_1\} + [A_{12}]\{P_0\} = \{C_1\}$$

$$[A_{21}]\{P_1\} + [A_{22}]\{P_0\} = \{C_0\}$$

With $\{P_0\}$ equal to zero, the first equation becomes the linear simultaneous set:

$$[A_{11}]\{P_1\} = \{C_1\}$$

This may be directly solved for $\{P_1\}$. With $\{P_1\}$ determined, the compensating deflection vector $\{E_C\}$ is solved using the full $[\alpha]$ matrix.

Finally, the rms error is determined by putting the resulting $\{E_C\}$ back into the original discretized E^2 rms expression and taking its square root.

$$E_{rms} = \sqrt{\frac{1}{n} \sum_{i=1}^{i=n} \left(E_e{}^i + [\alpha]\{P_1\}\right)^2}$$

The actual software that was written to implement this analysis contains some additional features to make it more compatible with the metrology mount hardware. For example, restrictions on sign and/or amplitude of $\{P_1\}$ elements may be specified to avoid the necessity for a force application point to "pull" since the mirror only rests on the support devices or to overload the piezoelectric load cells employed at the registration "hardpoints".

Gravity release error

The theoretically minimum gravity release error achievable with this metrology mount system is equal to the midfrequency "ripple" caused by intersupport shear and bending deformations. These were found to be on the order of $\lambda/500$ p-p or $\lambda/1500$ rms. The gravity release estimate includes this, inherent in the force field correction residual error, and the other error sources previously discussed plus an allowance for modeling uncertainties. These factors are summarized in Table 1.

Table 1. Gravity Release Error Summary

Error Source	Sensitivity	Observed Effect	Error
Optimum force field correction residual error	-	0.00168λ	0.00168 λ rms
Force calibration	0.03λ/lb	0.0022 lb	0.000066λ
Vertical displacement	0.021λ/in	0.025 in	0.000525 λ
Tilt displacement	0.025λ/in p-p	0.04 in	0.001 λ
		RSS =	0.00202 λ
Modeling uncertainty		1.056	-
		Final value	0.002137 (λ/467)

Estimates were also made for secondary errors associated with thermal and temporal creep, registration point kinematic "imperfections", hysteresis in the pushrod pivots, overall system tilt with respect to gravity (side forces), and so on. These amounted to λ/1250 rms which, if added to the λ/467 value, results in a net overall gravity release prediction of λ/438 which exceeds the goal of λ/350 stated at the outset of this paper!

Mathematical model verification

The primary mirror was represented with a relatively sophisticated finite element model. It contains approximately 2400 nodes in each of three planes: the midthickness of the front and rear faceplates and the midheight of the core. These nodes are located at the core rib intersections resulting in 492 nodes, or deflection read-out coordinates, within the clear aperture. The majority of the elements used were NASTRAN QUAD-4 plates, but triangular TRIA-3's were also employed at the edges to better approximate the annular overhang of the faceplates beyond the edgebands. Boundary conditions included vertical and tangential constraints at each of the flight mounts, or hardpoints, on the rear surface. Rigid body motions of the optical surface due to direct compression effects of the core, although small, were removed from the data inasmuch as it has no influence on figure.

Since this model was used to generate both the error vector $\{E_C\}$ and the influence coefficient matrix $[\alpha]$ from which the support forces $\{P_1\}$ are calculated, it is important to verify its accuracy. How we accomplished this was extremely straightforward in concept. Using the analytic model, two points 180 degrees apart near the rim were increased in load, while two other rim points, 90 degrees away, were decreased the same amount. This loading condition principally drives astigmatism. Because the net force on the mirror remains unchanged and because of symmetry, no overturning moments are produced; it is also possible to physically produce these load changes in the metrology mount without disrupting its alignment with respect to the interferometer. Incidentally, the axis of astigmatism was designed to be inclined with respect to the rib direction since that results in a somewhat more complex structural behavior than a symmetrical condition. The predicted deflection data, in a compatible format, was subsequently reduced using Perkin-Elmer's Aberfit program to determine the Zernike coefficients, the orientation of the astigmatism terms (the fifth and sixth Zernike terms), and the overall rms figure error. This data reduction procedure is exactly identical to that used for evaluating interferograms with the exception that calculated, rather than measured, deflection data was, of course, used.

Next, the actual deflection test was performed, consisting of the following steps:

1. Interferometrically determine the baseline figure on the metrology mount.

2. Impose the counterbalance force changes.

3. Measure the deformed shape.

4. Obtain the difference between (1) and (3) (data reduction).

5. Return the counterbalance forces to the original setting.

6. Remeasure the baseline.

7. Obtain the difference between (1) and (6) (data reduction).

Step (4) yields the induced deformation shape and (7) discloses hysteresis, if any. It should be noted that this test was performed when the baseline figure was nearly at its specified level of quality so that the induced errors, approximately $\lambda/9$, were well out of the noise level. Finally, the results of (4), in terms of the Zernike coefficients, were compared to those analytically predicted. To the difference, we added step (7), which, as matters turned out, was too small to discern (naturally, that's what we expected!).

The results from this test were in splendid agreement with the pretest predictions as shown in Table 2.

Table 2. Analytical Model Verification Test Results - Surface Deformation (rms)

	Measured in Test	Prediction by Model
RMS figure	0.114λ	0.108λ
Magnitude of astigmatism	0.111λ	0.107λ
Orientation of astigmatism	32.1°	32.6°

The rms figure matched the predictions within 5.6 percent; the astigmatism term itself matched within 3.7 percent; and, for all intents and purposes, the orientation of the astigmatism axes were "right-on". Certain higher order aberrations, such as fifth order astigmatism (12th and 13th Zernike coefficients) and third order spherical, did not exhibit such excellent correlation. However, their absolute magnitudes were only 1/10 of the applied astigmatism and, consequently, less than the nominal figure error itself. We feel that these sources of poorer correlation have their origins in measurement noise, i.e., (4) minus (1). On the basis of the longer spatial frequency correlation, as well as that of the overall rms agreement, we deemed that this experiment successfully validated the mathematical model.

Conclusions

The development of the metrology mount described in this paper spanned several years of effort and involved considerable detail beyond the scope of this presentation. However, the important elements of the program have not been omitted. It should be noted that perhaps the singularly most critical element was attention to detail, both analytically and in hardware. To the maximum extent possible, all error or uncertainty contributors were identified and accounted for.