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Large stable mirrors: a comparison of glass, beryllium and silicon carbide

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ABSTRACT

The choice of a substrate material for large mirrors is a complex engineering task that must account for structural and thermal properties, as well as mirror blank fabricability, polishability and surface scatter. Nuclear hardness is also a consideration in some applications. Cost is almost always a concern.

The standard material for mirror substrates has always been glass. Beryllium technology, however, is well developed, and offers distinct advantages over glass in many applications. Reaction-bonded silicon carbide is a relatively new material that has matured to the point where it can now be considered as an alternative to either beryllium or glass in some large optics applications. The availability of these three different substrate materials offers the system designer a great deal of flexibility in optimizing the material for each particular application. In this paper we present a methodology for comparing the structural properties of mirror substrate materials and lightweighting designs. This methodology is used to compare glass, beryllium and silicon carbide.

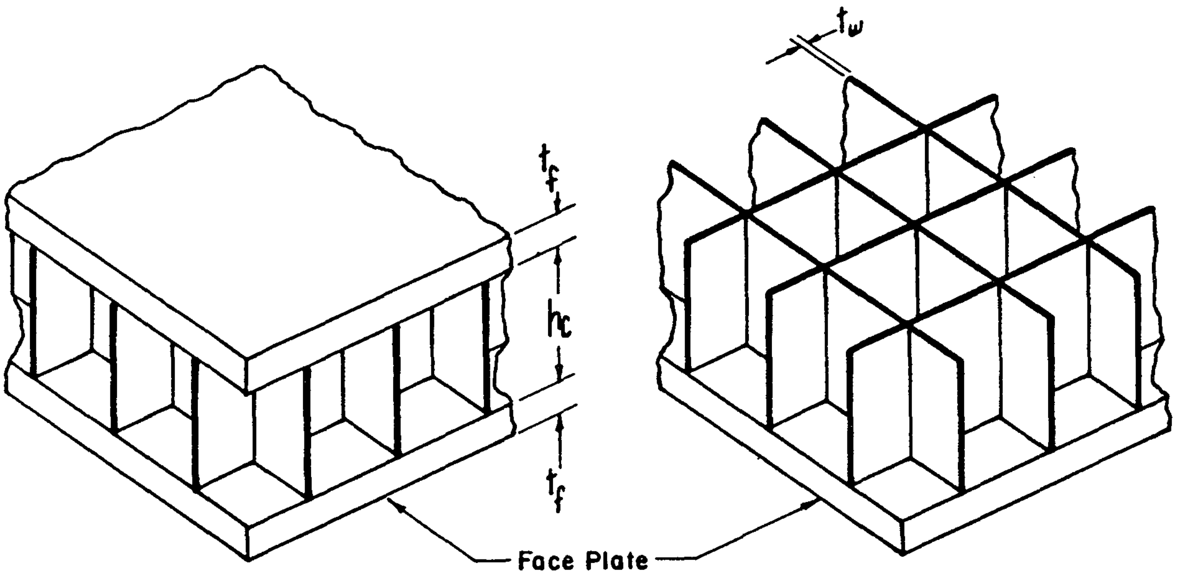
1. LIGHTWEIGHT MIRROR DESIGN PARAMETERS

Basic lightweight mirror design parameters are illustrated in Figure 1. Sandwich mirrors comprise two faceplates with a structured core in between. Although the faceplates are shown as having equal thicknesses t_f, they can, in general, be unequal. Open-back mirrors have a single faceplate. For both sandwich and open-back mirrors, the mirror core is characterized by its height, h_c, the web thickness t_w of the core elements, and the shape of the cells. Triangular cells provide the most isotropic structural properties. Regardless of shape, however, cell size is usually characterized by the diameter b of the largest inscribed circle.

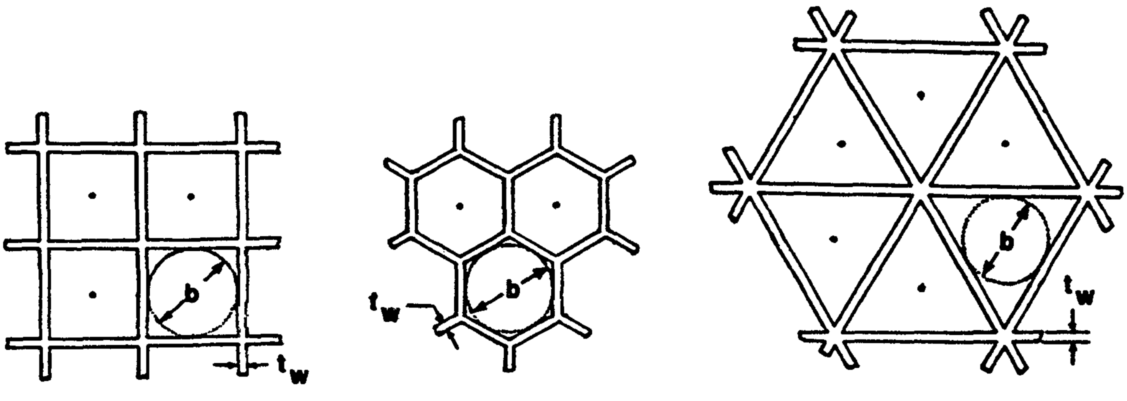
2. FIGURES OF MERIT

In comparing mirror structures made of different materials, we need to establish figures-of-merit to form the basis of comparison. Figures of merit include the following:

- Mirror weight W
- Area Weight Density W/A is the weight per unit area, where A = mirror area



(a) SANDWICH **(b) OPEN BACK**



Square Hexagonal Triangular

Figure 1. Lightweight mirror design parameters

- Flexural Rigidity, a measure of structural stiffness, is given by

$$D = IE/(1 - \nu^2) \qquad (1)$$

where: I = moment of inertia
E = Young's modulus
ν = Poisson's ratio

- Natural Frequency f_0 is an important property in any real system application, and is specified such that a mirror resonance is not excited due to launch conditions or other disturbances on board a spacecraft. Usually, f_0 needs to be higher than a specified minimum value. The frequency of the first structural mode is given by the approximation

$$f_o = (\lambda^2/2\pi)\sqrt{D/AW} \qquad (2)$$

where λ^2 is a constant that is a function of the mirror's shape and mounting conditions.

- Characteristic Length L is a property that is useful in characterizing active, deformable mirrors, and is a measure of the spatial extent of influence of a figure control actuator[1]. The accuracy with which the shape of a mirror can be controlled increases as the ratio of actuator spacing to characteristic length decreases. L is given by:

$$L = (R^2D/t_eE)^{1/4} \qquad (3)$$

where R = mirror radius of curvature
t_e = thickness of solid mirror having the same area weight density ("equivalent thickness")

We note that weight and natural frequency are the only figures-of-merit that are dependent upon the size of the mirror. Area weight density, flexural rigidity and characteristic length depend upon the structural design of the mirror, but not its size.

3. TRADE METHODOLOGY

We have developed a methodology for comparing the structural properties of lightweight mirrors fabricated from different substrate materials. This methodology is outlined below:

1. For each substrate material, we choose sets of faceplate and web thicknesses (t_f and t_w) that represent an "aggressive" design and a more "conservative" design. The aggressive thicknesses approach the practical limits of manufacturability for the substrate material in question, and represent a minimum weight design. The conservative thicknesses represent a design that may be less costly or lower risk.

2. Both sandwich and open-back designs are analyzed.

3. Core designs are chosen with the value of b adjusted to control quilting (or print-through) to a predetermined limit. In our analysis we have chosen b such that the cell deflection will equal $\lambda/80$ at 632.8 nm, given anticipated polishing tool pressures, and a faceplate of thickness t_f. Since tool pressure and faceplate thickness differ from material to material, all core sizes are different. For this study, we have chosen polishing pressures of 2.0 psi for glass, 1.0 psi for beryllium and 2.0 psi for silicon carbide. Although the trades presented are valid for all core shapes, triangular cores are preferred in open-back designs to maintain isotropic structural properties.

4. With mirror design parameters t_f, t_w and b held fixed for each substrate material, the core height h_c is varied parametrically to produce a continuum of point designs having different area weight densities. Figures of merit are plotted as a function of area weight density for each set of mirror design parameters.

5. For comparison of mirror weight and natural frequency, we have chosen a 1.5m diameter, f/1.5 configuration, with a 25% linear obscuration, as an example.

4. DESIGN PARAMETERS

The substrate materials we have chosen to compare are ULE and fused quartz (FQ) glasses, reaction-bonded silicon carbide (SiC), and beryllium. Since the mechanical properties of these materials are very weak functions of temperature, we have used room temperature values in our analyses. The pertinent mechanical properties are given in Table 1. While beryllium possesses the best mechanical properties, silicon carbide shares a fairly large specific stiffness E/ρ, as well as fracture toughness K_{Ic}, making it both stiffer and less fragile than glass.

MATERIAL	YOUNG'S MODULUS E (GPa)	DENSITY (g/cc)	SPECIFIC STIFFNESS E/ρ	FRACTURE TOUGHNESS K_{IC} (MPa $\sqrt{M}$)	POISSON'S RATIO ν
ULE	67	2.21	30.3	<1	0.17
FUSED QUARTZ (FQ)	72	2.19	32.9	<1	0.17
SILICON CARBIDE (SiC)	324	2.93	110.6	≥5	0.20
BERYLLIUM (Be)	287	1.85	155.1	>10	0.07
PREFERRED	HIGH	LOW	HIGH	HIGH	LOW

TABLE 1 - Mirror substrate mechanical properties

Mirror design parameters for these materials are presented in Table 2. Values of b have been chosen to control quilting, as described previously. We have chosen one design each in ULE and fused quartz, to represent the range of available faceplate and web thicknesses available in glass substrates. It should be noted that the six designs presented in Table 2 are chosen in order to provide an example of the trade methodology, and that definitions of "aggressive" and "conservative" will change in time as manufacturing processes mature.

5. PARAMETRIC TRADES

Parametric trades were performed by independently varying core height h_c for each of the sets of fixed mirror design parameters listed in Table 2. Flexural rigidity, natural frequency and characteristic length were then plotted as a function of area weight density, which varies linearly with core height. Figures 2 and 3 show this linear relationship for sandwich and open-back designs, respectively. Each curve terminates at an area weight density for which the core height h_c is zero, and only one or two faceplates remain.

MIRROR * DESIGN	FACEPLATE THICKNESS (INCHES)	WEB THICKNESS (INCHES)	b (INCHES)
ULE	0.275	0.080	2.5
FQ	0.200	0.040	2.0
SiC #1	0.200	0.080	2.86
SiC #2	0.125	0.060	2.0
Be #1	0.200	0.065	3.3
Be #2	0.120	0.040	2.25

* ULE & #1 DESIGNS ARE CONSERVATIVE, FQ & #2 DESIGNS ARE AGGRESSIVE

TABLE 2 - Mirror design parameters

The flexural rigidities of sandwich and open-back designs are compared in Figures 4 and 5, respectively. Beryllium's flexural rigidity exceeds that of both silicon carbide and glass, for the most part by at least an order of magnitude. The SiC #2 and FQ designs are fairly close. Even though silicon carbide has a higher specific stiffness, the thinner web thickness of the FQ design provides more structural efficiency. We note that for each class of materials, the more aggressive design has a higher flexural rigidity than the conservative design, for equal area weight densities. In Figure 6, we directly compare sandwich and open-back forms of the aggressive designs. We see that open-back construction dominates at low area weight densities, but that sandwich designs become more rigid at large area weight densities.[2] We will see this "crossover" behavior in other structural properties, as well.

The natural frequency of a mirror is proportional to the square root of the ratio of its flexural rigidity to area weight product, as seen in equation (2). Given a set of mirror designs having the same area weight product, the design having the largest flexural rigidity will have the highest natural frequency. Conversely, given a set of mirror designs having the same natural frequency, the design having the largest flexural rigidity will weigh the least. This latter point, which is a key consideration in the design of spaceborne optical systems, is illustrated with the aid of Figures 7 and 8. Here we have plotted the natural frequencies of sandwich and open-back designs, respectively, as a function of area weight density. The values of A and λ^2 in equation (2) correspond to a 1.5m diameter, f/1.5 mirror with a 25% linear central obscuration, rigidly mounted at 3 points located at 75% of the mirror radius. NASTRAN was used to calculate the value of λ^2, including bending and shear, for the SiC #2 mirror design. The values of

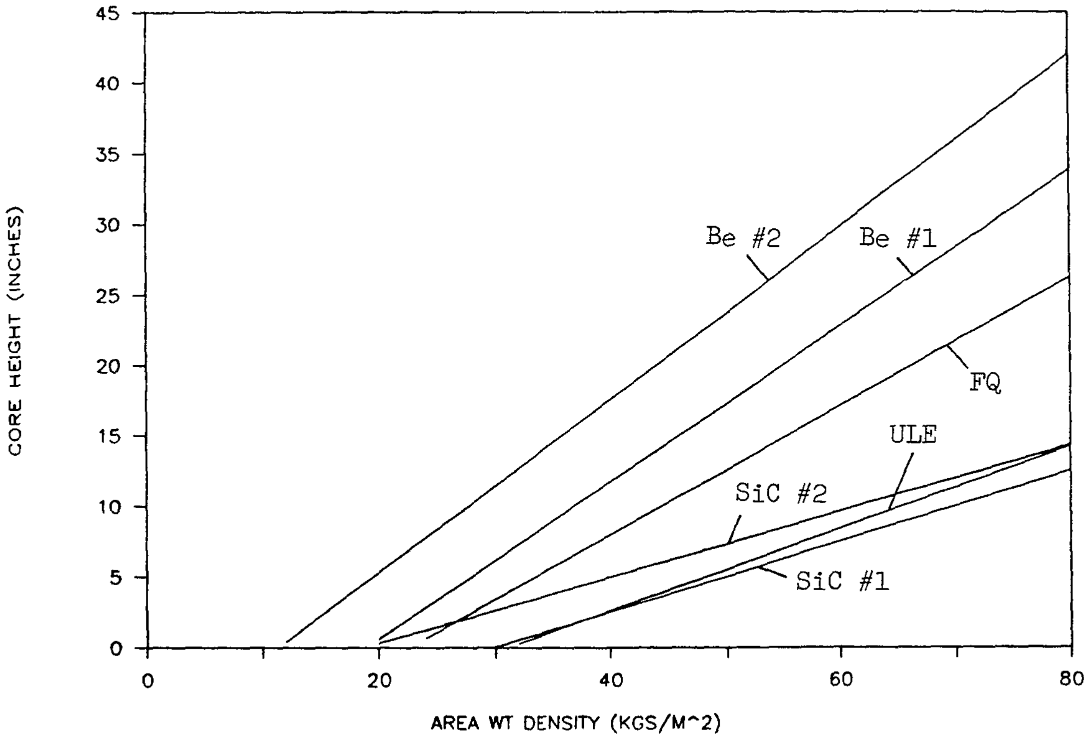


Figure 2. Core height of sandwich mirrors vs. area weight density

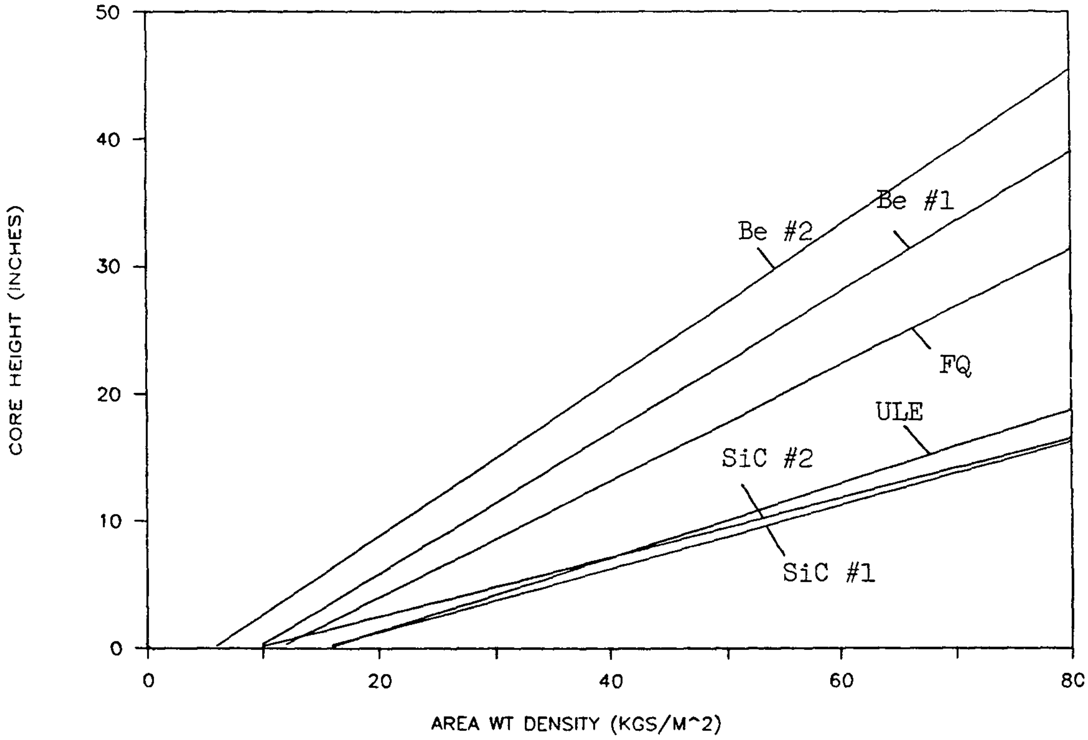


Figure 3. Core height of open-back mirrors vs. area weight density

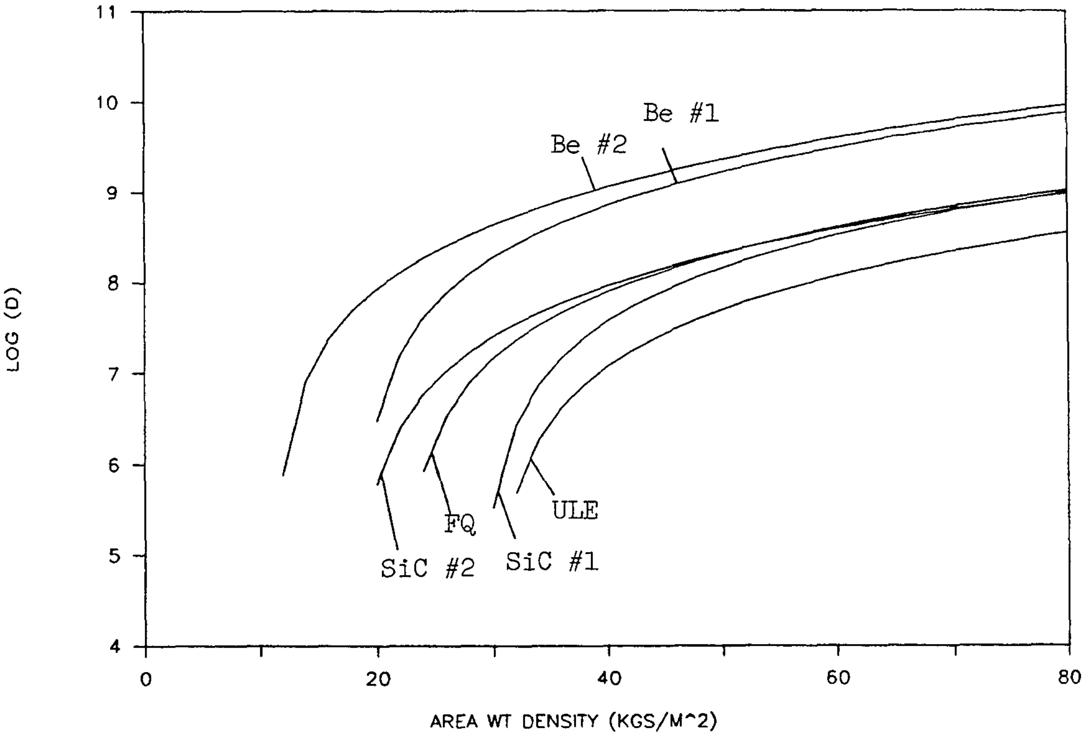


Figure 4. Flexural rigidity of sandwich mirrors vs. area weight density

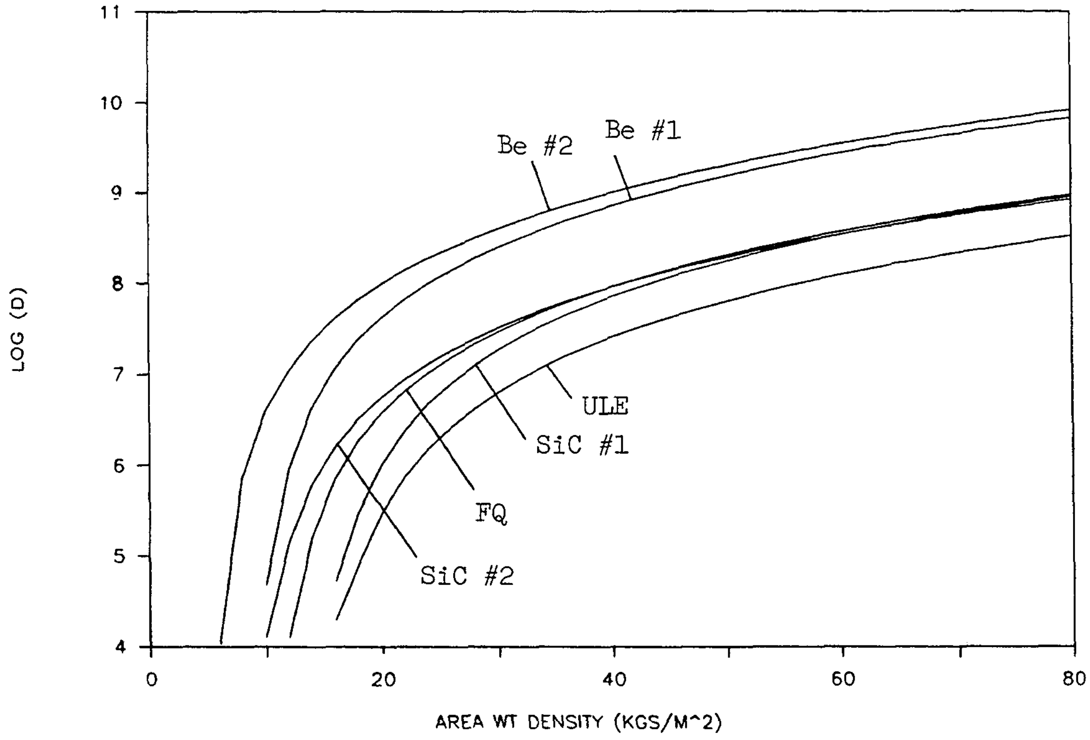


Figure 5. Flexural rigidity of open-back mirrors vs. area weight density

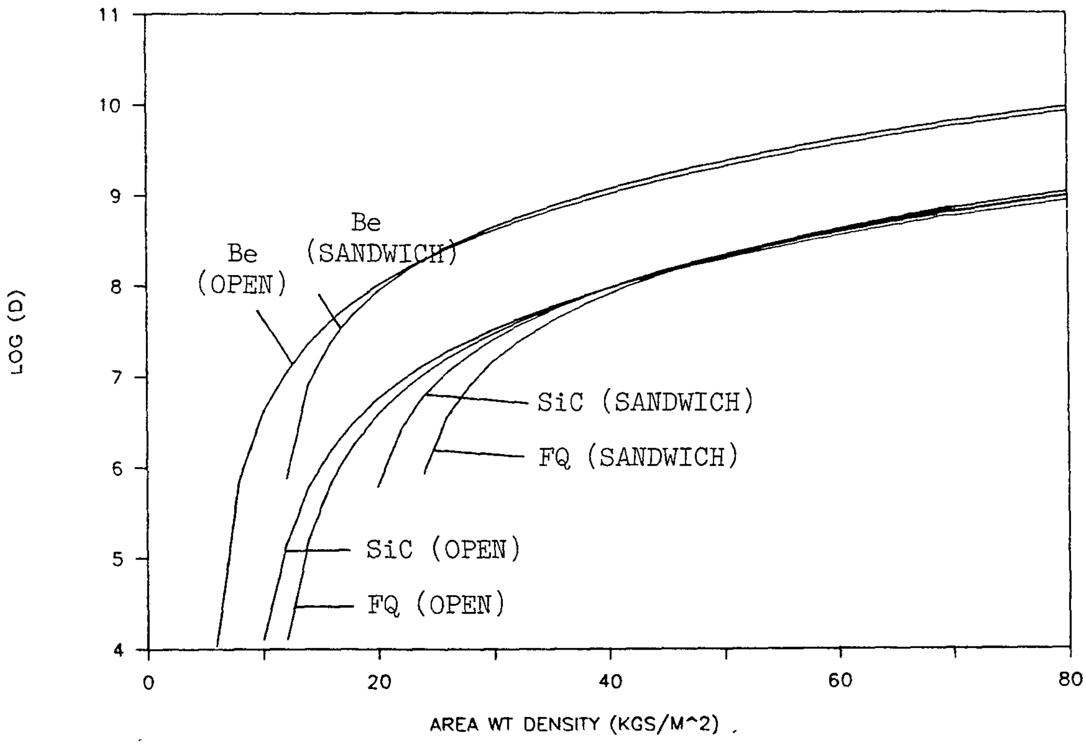


Figure 6. Comparison of flexural rigidities of sandwich & open-back designs

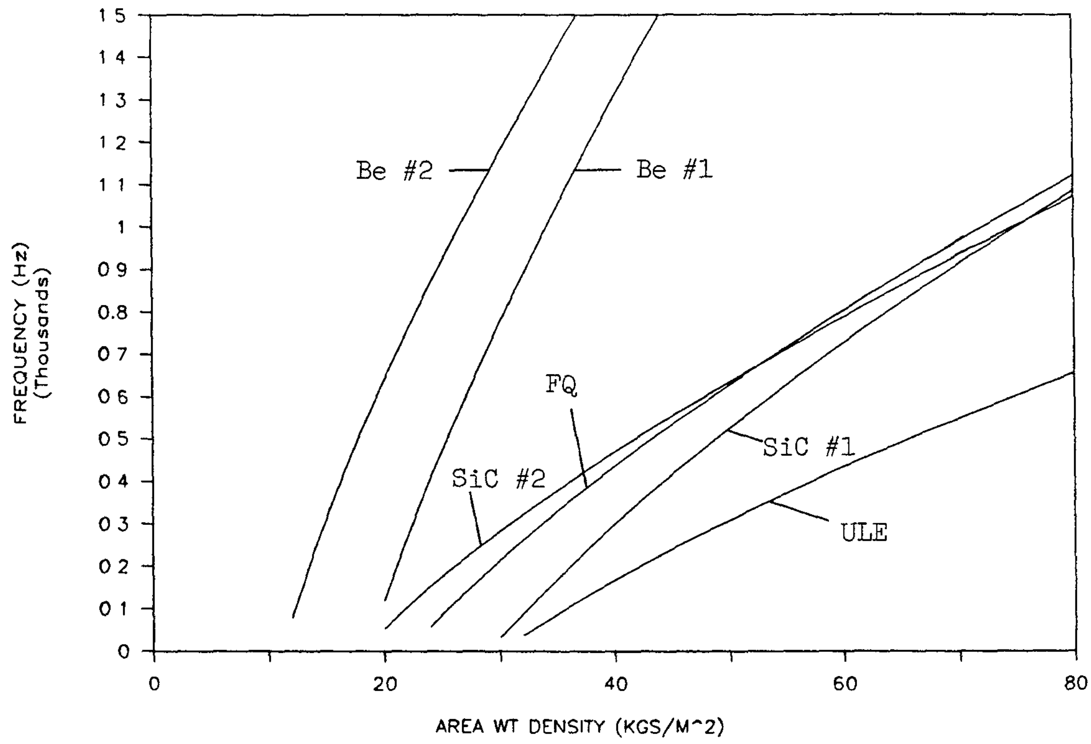


Figure 7. Natural frequency of sandwich mirrors vs. area weight density

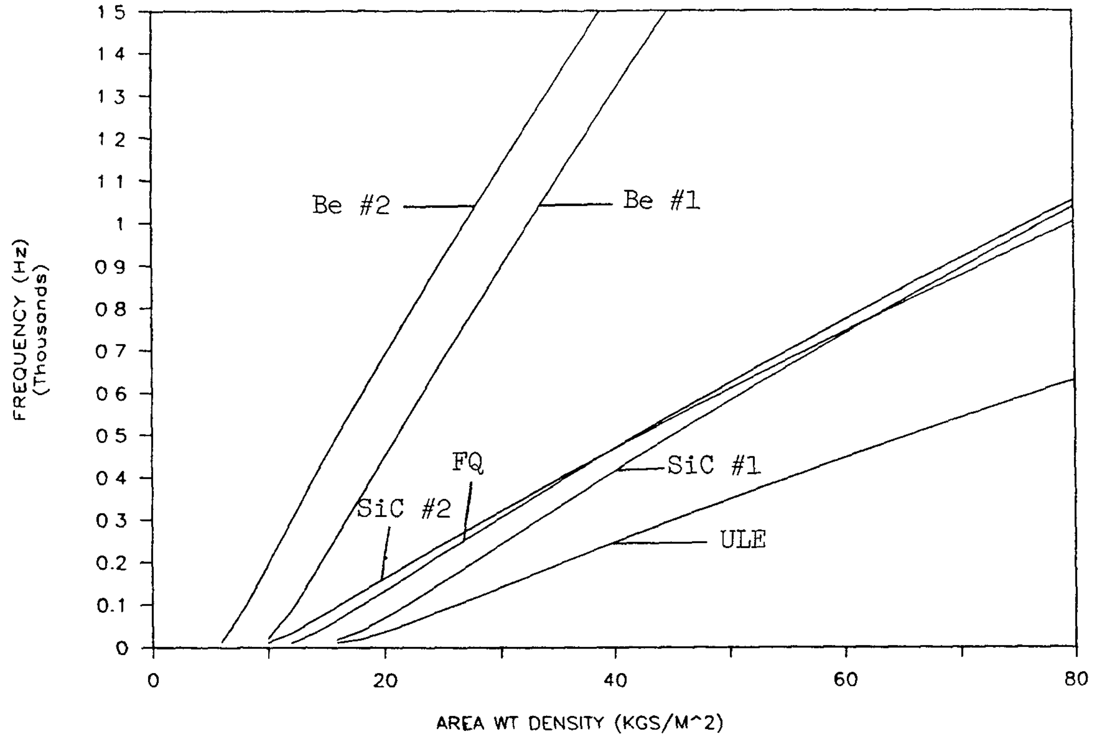


Figure 8. Natural frequency of open-back mirrors vs. area weight density

f_0 for the other mirror designs were scaled from equation (2) according to their values of $\sqrt{D/AW}$. We see from the figures that the beryllium mirror designs have the largest values of natural frequency, by quite a large margin. The FQ and SiC #2 designs are very close in natural frequency for both sandwich and open-back forms. The "crossover" behavior seen in flexural rigidity is repeated with natural frequency, as illustrated in Figure 9. Open-back designs are more efficient at low area weight densities, and sandwich designs are more efficient at high area weight densities.

We can compare the weights of different mirror designs of equal natural frequency by drawing horizontal lines of fixed frequency on Figures 7 and 8, and noting the value of area weight density corresponding to each mirror point design. Since natural frequency requirements can vary considerably depending upon application, we have made this comparison for natural frequencies of 100, 300 and 500 Hz. The results are presented in Tables 3 and 4, for sandwich and open-back designs, respectively. We note, for both sandwich and open-back designs, that the beryllium designs are, by far, the lightest. The conservative ULE design is the heaviest, also by a large margin. The aggressive silicon carbide design (SiC #2) is lighter than both glass designs, but close to the aggressive FQ design. Examination of the core heights for each of the point designs indicates larger core heights for open-back designs, as expected, and the largest core heights for the glasses, which require more depth to achieve the same stiffness as silicon carbide or beryllium designs. As mentioned previously, the thin web thickness and large core height of the FQ design is structurally efficient, resulting in performance almost equal to the aggressive silicon carbide (SiC #2). However, even though the aggressive

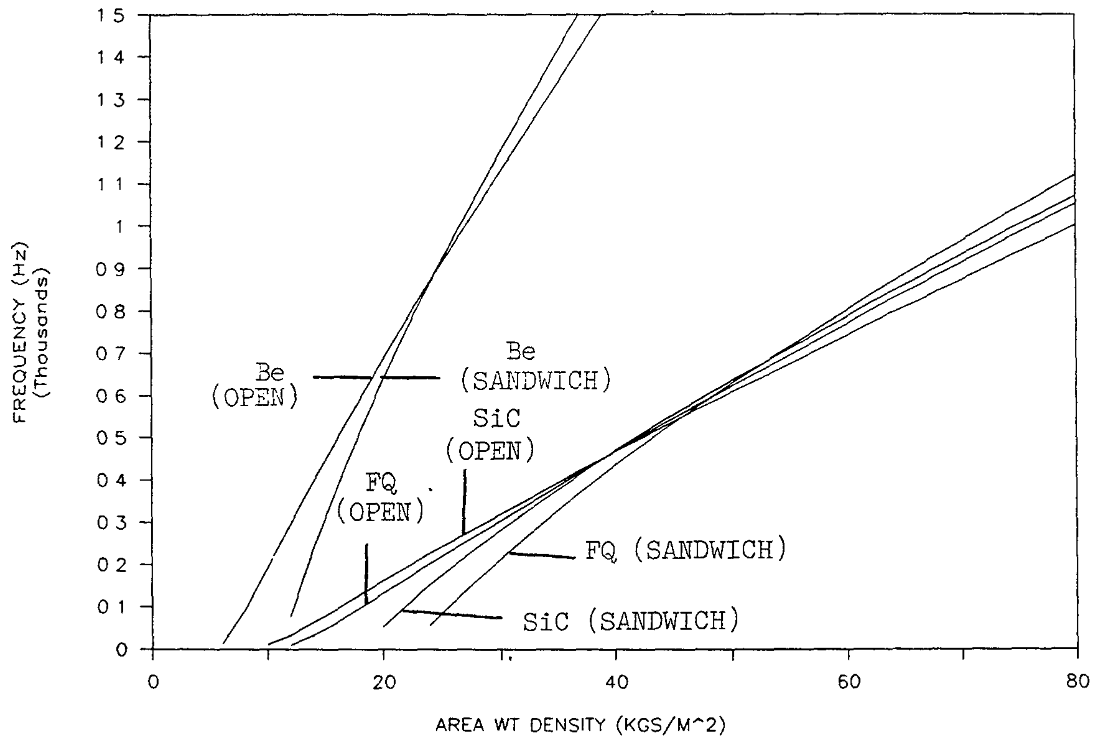


Figure 9. Comparison of natural frequency of sandwich & open-back designs

FQ web thickness of 0.040" may be manufacturable, its use in a large core height structure may be impractical. These designs, then, although structurally efficient, may not be realizable. We also note that the variety of material properties and structural designs considered results in a very wide range of mirror weights, any one of which may be appropriate for a particular system application.

6. ACTIVE MIRRORS

In some applications, it may be of advantage to design the mirror to be active (deformable). Such an approach may possibly save weight, or offer the flexibility to actively compensate for distortions or misalignments within the optical system. A key trade that must be considered is the stiffness of the mirror versus actuator spacing. The characteristic length of a mirror is a measure of the region of influence of an actuator, and varies as the fourth root of the mirror's flexural rigidity, as seen in equation (3).

In general, the characteristic length approximates the spacing at which figure control actuators should be placed. Figures 10 and 11 plot the characteristic length of sandwich and open-back mirrors, respectively, as a function of area weight density. Beryllium mirrors have the largest characteristic length, and therefore require the fewest actuators. The FQ design is seen to have a larger characteristic length than SiC #2, except at the lowest area weight densities. This is a result of its lower value of Young's modulus E.

		ULE	FQ	SiC#1	SiC#2	Be#1	Be#2
F_0 = 100 Hz	h_c (cm)	3.3	3.6	1.5	1.8	*	1.5
	W/A (Kg/m^2)	36	26	32	22	*	12
	W (KG)	**60.9**	**43.6**	**55.3**	**37.2**	*	**21.0**
F_0 = 300 Hz	h_c (cm)	13.5	13.0	6.3	7.1	5.1	5.6
	W/A (Kg/m^2)	49	34	40	31	23	15
	W (KG)	**84.1**	**57.4**	**68.3**	**52.7**	**38.5**	**25.5**
F_0 = 500 Hz	h_c (cm)	26.9	23.9	11.9	13.7	9.4	10.2
	W/A (Kg/m^2)	66	43	49	42	26	18
	W (KG)	**112.0**	**73.7**	**83.3**	**71.3**	**43.5**	**30.5**

* mirror with zero core height (i.e., faceplates only) exceeds 100 Hz

TABLE 3 - Comparison of sandwich mirror weights vs. substrate material and natural frequency

		ULE	FQ	SiC#1	SiC#2	Be#1	Be#2
F_0 = 100 Hz	h_c (cm)	7.9	7.9	4.3	4.1	4.1	4.1
	W/A (Kg/m^2)	26	18	22	16	12	8
	W (KG)	**44.6**	**30.9**	**37.4**	**27.6**	**21.2**	**14.0**
F_0 = 300 Hz	h_c (cm)	21.8	21.3	11.7	11.4	10.4	9.9
	W/A (Kg/m^2)	45	30	33	29	17	12
	W (KG)	**77.0**	**50.7**	**56.9**	**48.9**	**28.8**	**20.7**
F_0 = 500 Hz	h_c (cm)	37.1	35.6	19.1	19.6	16.5	16.3
	W/A (Kg/m^2)	66	42	45	42	21	16
	W (KG)	**112.2**	**71.7**	**77.0**	**72.2**	**36.2**	**27.6**

TABLE 4 - Comparison of open-back mirror weights vs. substrate material and natural frequency

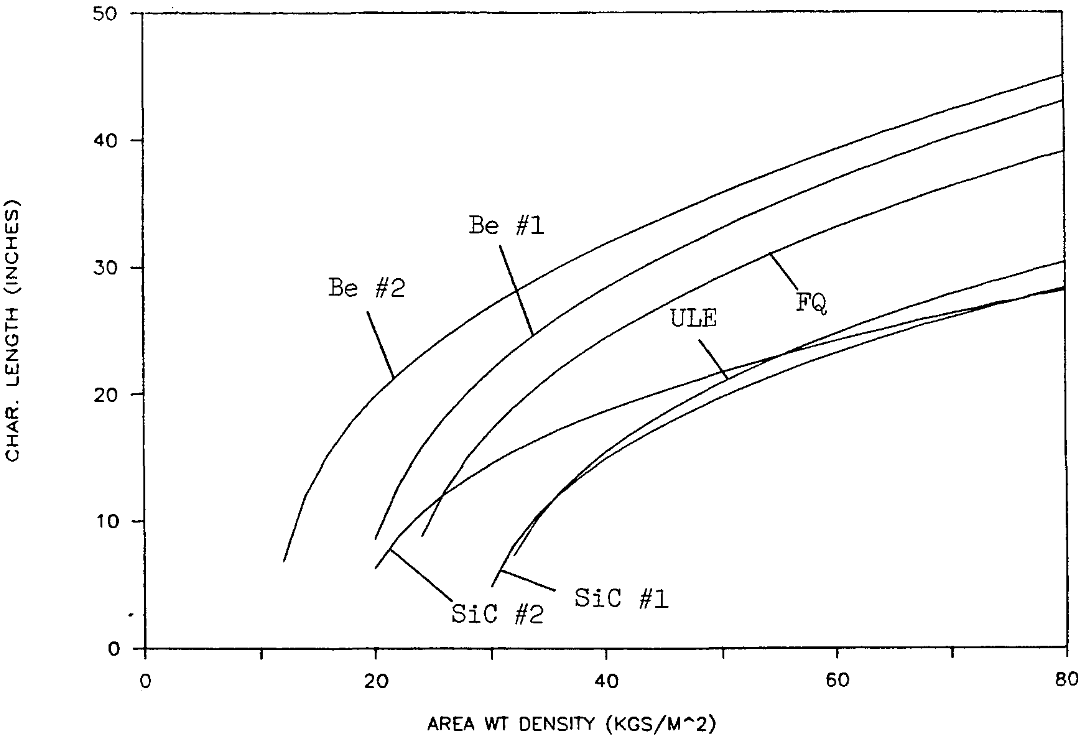


Figure 10. Characteristic length of sandwich mirrors vs. area weight density

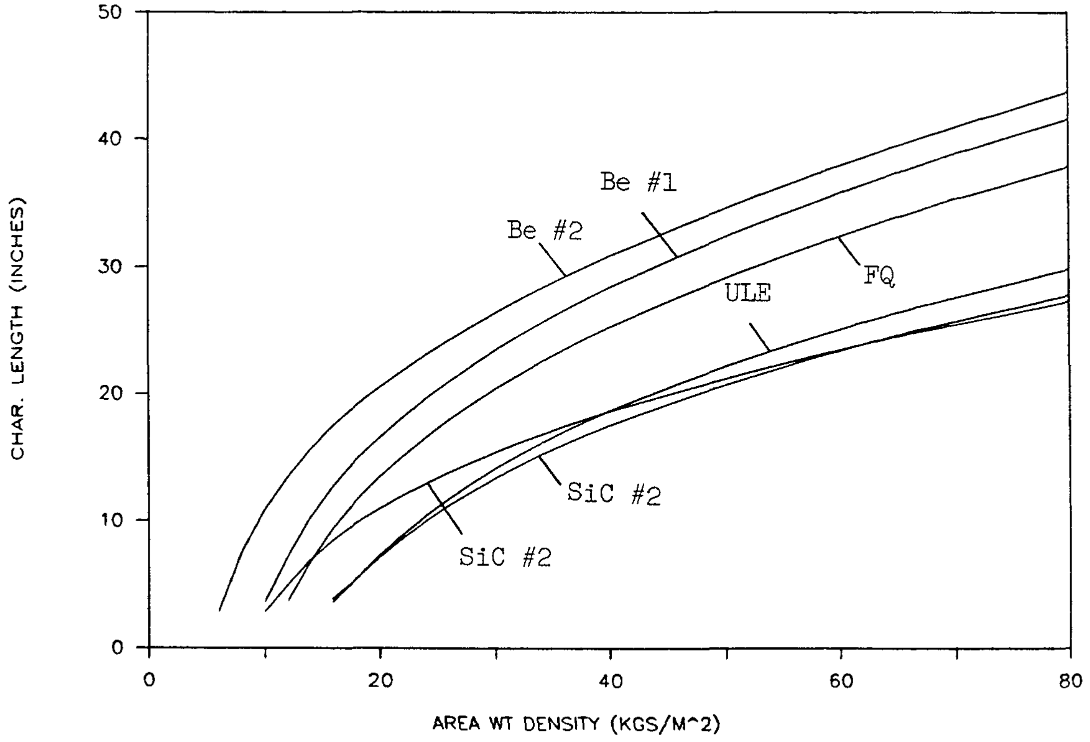


Figure 11. Characteristic length of open-back mirrors vs. area weight density

7. CONCLUSIONS

A methodology for comparing the structural properties of mirror substrate materials and lightweighting designs has been presented. This methodology applies to passive and active mirror applications. We have seen that silicon carbide offers lightweight mirror structural characteristics that fall between those of glass and beryllium. While beryllium has the best mechanical properties, and therefore can yield the lightest mirror designs, silicon carbide offers some of beryllium's advantages over glass, including higher specific stiffness and fracture toughness. We have also seen that open-back designs are more efficient than sandwich designs at low area weight densities.

It must be realized that structural properties alone do not always dominate the choice of substrate material. The differences in the thermal properties of the materials considered in this study can also be of significance in making this choice.

8. REFERENCES:

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