Overview

This paper will review Chapter 7 from Max Riedl’s book Optical Design Fundamentals for Infrared Systems, which covers the topic of thermal effects on infrared imaging systems. The effects of temperature change on the performance of infrared imaging systems is particularly notable since many of the materials used in these wavelength bands have high $dn/dT$. In addition, the manufacture of infrared components does not have the same quality control as more common components used in the visible wavelength band and places even more importance on effective athermalization during the design process. In general, the topics discussed here can be applied to any wavelength band though specific attention is placed on considerations at long wavelengths.

Effects of Temperature Change

For a lens with coefficient of expansion $\alpha_L [10^{-6} \text{mm/mm/°C}]$ and a housing with coefficient of expansion $\alpha_H$ the initial design parameters will change by some amount as the temperature departs from the design value. Table 1 shows the departure of design parameters given a temperature shift $\Delta T$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value at T</th>
<th>Value at T+(\Delta T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>$r(1 + \alpha_L \Delta T)$</td>
<td>$r(1 + \alpha_L \Delta T)$</td>
</tr>
<tr>
<td>$d$</td>
<td>$d(1 + \alpha_L \Delta T)$</td>
<td>$d(1 + \alpha_L \Delta T)$</td>
</tr>
<tr>
<td>$n$</td>
<td>$n + (dn/dT)\Delta T$</td>
<td>$n + (dn/dT)\Delta T$</td>
</tr>
<tr>
<td>$s'$</td>
<td>$s'(1 + \alpha_H \Delta T)$</td>
<td>$s'(1 + \alpha_H \Delta T)$</td>
</tr>
</tbody>
</table>

Table 1: Value of design parameters for a temperature change of $\Delta T$.

Additionally, a parameter called the thermal glass constant (TGC) is defined which takes into account the change in refractive index as well as the coefficient of thermal expansion.

$$TGC = \frac{dn/dT}{n-1} - \alpha_L$$

For a single thin element the change in focal length is given by $\Delta f = -f \Delta T(TGC + \alpha_H)$. In the case of most IR materials, the TGC is positive and indicates a negative contribution to the change
in focal length for increasing temperature. Therefore, in an IR singlet the TGC shifts focus towards the lens with increasing temperature while the housing length expands. The net effect is that both contribute to significant thermal defocus, as shown in Figures 1 and 2.

The equation for change in focal length can also be generalized for a system of thin elements to give:

$$\Delta f = -[f^2 \sum_{i=1}^{j} \frac{TGC_i}{f_i} + \alpha_H f] \Delta T$$ (1)

In the case of a simple germanium singlet in an aluminum housing the effect of thermal defocus for a temperature change of tens of degrees is on the order of tenths of a millimeter. As a reference, the tolerance to defocus for diffraction limited operation in the long wave infrared is given by \(\delta z = 2\lambda(f/\#)^2 \approx 0.045mm\) for an \(f/1.5\) system. Despite the 10\(\mu m\) wavelength used here, \(f/1.5\) is common in the IR since the optics must perform well for detector limited operation.

Figure 1: Nominal system design.

Figure 2: System after temperature increase of \(\Delta T\).

The difficulty for IR systems is that the CTE for housing materials is typically much less than that for the TGC of IR transmitting optical materials. Table 2, a reproduction from Riedl, clarifies this difficulty by comparing the values for typical visible, IR and housing materials. BK7, a typical crown glass for use in the visible, has a negative TGC as defined in this convention and clearly acts
to compensate a small amount for the expansion of the housing material. Germanium, a common IR material, suffers from an unusually high TGC. Many IR systems use Germanium lens elements due to its good IR transmission, durability and extremely high index of refraction ($n_{lwir} = 4$), which means that many IR systems also have poor athermalization qualities.

<table>
<thead>
<tr>
<th>Lens Material</th>
<th>TGC $\times 10^{-6}$</th>
<th>Housing Material</th>
<th>$\alpha_H \times 10^{-6} mm/mm/°C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK7</td>
<td>-1</td>
<td>Aluminum</td>
<td>24</td>
</tr>
<tr>
<td>ZnS</td>
<td>25</td>
<td>Steel 1015</td>
<td>12</td>
</tr>
<tr>
<td>ZnSe</td>
<td>36</td>
<td>Invar 36</td>
<td>1.3</td>
</tr>
<tr>
<td>Si</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ge</td>
<td>126</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Table of lens materials TGC and housing materials CTE. Reproduced from Riedl.

**Athermal Doublet**

It is possible to use Equation 1 to determine the condition for athermalizing a doublet lens simply by applying the summation for elements A and B.

$$\Delta f = -f[f(TGC_A + TGC_B) + \alpha_H] \Delta T$$

This equation can be solved for zero focal shift to give the conditions for an athermal doublet (Equations 2 and 3).

$$f_A = \frac{T_B - T_A}{T_B - \alpha_H} f$$

(2)

$$f_B = \frac{T_A - T_B}{T_A - \alpha_H} f$$

(3)

It is important to note the relation between the focal length and athermalization. When Equations 2 and 3 are used to determine a starting point for computer optimization it is critical that the lens element focal lengths be maintained or the condition for athermalization will be lost.

**Athermal Achromat**

Three conditions must be met in order to athermalize an an achromat. These conditions are provided below.
\[ \phi_i = \sum_{i=1}^{j} \phi_i \quad (\text{Power}) \]  

(4)

\[ \sum_{i=1}^{j} (T_i \phi_i) = -\alpha_H \phi \quad (\text{Athermal}) \]  

(5)

\[ \sum_{i=1}^{j} \left( \frac{\phi_i}{\nu_i} \right) = 0 \quad (\text{Achromatic}) \]  

(6)

Solving these conditions results in a set of clumsy equations for the powers of each element in the lens but which can be used to define a starting point for computer optimization. Again, it is critical to maintain the focal length of each element in the system in order to keep the athermal and achromatic properties.

Conway et al.\(^1\) have discussed the case where an athermal achromat can be produced using a single material in a hybrid lens configuration. The solution to the three requirements can be satisfied when \( \nu_1(TGC_1 + \alpha_H) = \nu_2(TGC_2 + \alpha_H) \), which can also be written in the form \( \alpha_H = \frac{-(\nu_1 TGC_1 - \nu_2 TGC_2)}{(\nu_1 - \nu_2)} \).

When materials are plotted on a graph of \( \nu TGC \) vs \( \nu \), the slope of the line connecting two points gives the housing material CTE required for athermalization. Unfortunately, in the LWIR the slopes of the lines are generally far removed from common housing materials CTE’s, though new developments in IR materials have sought to replace germanium in part for this reason. An example of a \( \nu TGC \) vs \( \nu \) diagram for materials in the long wave IR is shown in Figure 3.

![Atthermalization Diagram for Common LWIR Materials](image_url)

**Figure 3:** \( \nu TGC \) vs \( \nu \) diagram for the LWIR.

In a similar approach, Hudyma\(^2\) has developed several rules of thumb for determining the three element materials from a plot of \( \nu TGC \) vs \( \nu \). These rules of thumb are listed below.

\(^1\)Arsenic sulphide athermal singlets for 3 - 5 \( \mu \)m imaging, SPIE Vol. 3061.

\(^2\)Athermal MWIR Objectives, SPIE Vol. 2540
• Materials 1, 2 and 3 do not lie in a straight line (or even close). The element powers will approach infinity.

• The area of the triangle bounded by materials 1, 2 and 3 should be as large as possible since this will have the effect of minimizing the element powers.

• The difference in $\nu$ values should be large to reduce the element powers.

• The slope of the line connecting two of the materials should be minimized with respect to the slope of the housing CTE.

The results of this study show that the initial selection of materials using this method give good starting points for athermal designs. Also of interest in this method is the inclusion of a diffractive "material" with $TGC_d = 2\alpha_L$. Simulations using a diffractive surface show optimal solutions maximizing color correction with the diffractive surface while minimizing its contribution to system power.

**Conclusion**

It is very clear that thermal analysis for infrared systems is a complex and important design process with many trade-offs being made. Closed form solutions are available for only the simplest of systems, though in the case of infrared applications good performance can be achieved in such a layout due to the unique characteristics of high index materials. Methods described by Conway and Hudyma show the utility of understanding the fundamental equations governing athermalization as Riedl has summarized in his book.