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Estimation of mounting-induced axial contact stresses in multi-element lens assemblies

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ABSTRACT

Prior publications dealing with techniques for estimating axial contact stresses in lenses mechanically clamped axially near their rims have assumed the lenses to be single elements. In this paper, we expand the theory to include multiple-element lenses such as cemented doublets and optomechanical designs with spacers or equivalent cell features between separated lenses. Examples are given to illustrate the use of these new analytical techniques.

Keywords: optomechanical design, lens-mount interface, lens cell; axial contact stress, multi-element lens mounting

A. INTRODUCTION

In previous publications,[1-5] techniques for estimating axial contact stresses in single element lenses were discussed. That theory is here extended to include multiple lens designs such as cemented doublets and optomechanical designs with spacers or equivalent cell shoulders between separated lenses. The applicable equations and the procedures for use thereof are summarized.

B. DISCUSSION

1. The Cemented Doublet

Figure 1 shows a typical cemented doublet of total edge thickness, $t_E = t_{E1} + t_{E2}$, clamped between a cell shoulder and a threaded retainer. For simplicity, the contact heights, y, are assumed to be the same at both interfaces. The stressed region in the glass is the annulus of radial width $t_{E1} + t_{E2}$ as indicated by the dashed diamond. Equation 1 or 2, as appropriate, is used to calculate the area, A_G, of this region as if the lens were a homogeneous single element.

$$\text{If } (2y + t_E) < D_G\text{, then: } A_G = 2\pi y t_E \qquad 1$$

$$\text{If } (2y + t_E) \geq D_G\text{, then: } A_G = (\pi/4)(D_G - t_E + 2y)(D_G + t_E - 2y) \qquad 2$$

Equation 3 is then used to determine A_M, the annular area of the stressed region in the cell wall.

$$A_M = 2\pi t_C\,((D_M/2) + (t_C/2)) \qquad 3$$

These areas, pertinent component dimensions and the applicable material properties can then be substituted into the following equation to determine the doublet's temperature sensitivity factor K_3:

$$K_3 = \frac{-(\alpha_M - \alpha_{G1})t_{E1} - (\alpha_M - \alpha_{G2})t_{E2}}{\dfrac{2t_{E1}}{E_{G1}A_G} + \dfrac{2t_{E2}}{E_{G2}A_G} + \dfrac{(t_{E1} + t_{E2})}{E_M A_M}} \qquad 4$$

Given the total axial preload, P_A, at assembly temperature, the new P_T and the corresponding linear preload, p, at the height, y, around the rim of the lens at any new temperature can be calculated using Eqs. 5 through 8. Note that this preload is the same at both the first and third surfaces of the doublet.

$$\Delta P = K_3\,\Delta T \qquad 5$$

$$\Delta T = -\ (T_A - T) \tag{6}$$

$$P_T = P_A + \Delta P \tag{7}$$

$$p = P_T / 2\pi y \tag{8}$$

Then, the applicable value of K_2 at either of these surfaces can be estimated by Eq. 9:

$$K_2 = K_G + K_M = [(1 - \upsilon_G^2)/E_G] + [(1 - \upsilon_G^2)/E_M] \tag{9}$$

where υ_G, E_G, υ_M and E_M are Poisson's ratio and Young's modulus values for the contacting glass and metal respectively.

Knowing the type of interface and surface radius at each surface, the value for K_1 can be calculated from the applicable form of Eq. 10. Note that K_1 is always assigned a positive sign.

For a "sharp corner" interface,[1, 4]

$$K_1 = (D_1 \pm 0.004)/0.004 D_1 \tag{10(A)}$$

where D_1 = 2(surface radius) and the radius is in inches. The "+" sign is used for convex surfaces and the "−" sign is used for concave surfaces.

For a tangential interface,[4]

$$K_1 = 1/D_1 = 0.5/R. \tag{10(B)}$$

For a toroidal surface,[2]

$$K_1 = 0.55/R \text{ for a convex surface} \tag{10(C)}$$

$$K_1 = 0.5/R \text{ for a concave surface.} \tag{10(D)}$$

The contact stress, S_A, at any interface can be estimated through use of Eq. 11 adapted by Yoder[4] from Roark[6]:

$$S_A = 0.798(K_1 p/K_2)^{1/2}. \tag{11}$$

In general, the stresses at the two surfaces will differ because the glasses have different elastic and thermal properties.

If the temperature rises sufficiently to dissipate assembly preload, an axial gap between the doublet and the mount develops for additional temperature increases, ΔT, in accordance with the following equation:

$$\Delta x = [(\alpha_M - \alpha_{G1})t_{E1} + (\alpha_M - \alpha_{G2})t_{E2}]\Delta T \tag{12}$$

2. Two Air-Spaced Singlets

A simple mounting for two air-spaced, unequal diameter elements with differing edge thicknesses is illustrated in Fig. 2. The spacer material may be different from that of the cell. The glasses also may be different. The contact heights at both surfaces of a given lens are assumed equal and the cell wall thickness is assumed to be constant in this example. If the contact heights at the individual lenses are the same, a cylindrical spacer with parallel OD and ID may be used. In the figure, the spacer has a cylindrical OD and a tapered ID since the contact heights are not equal. The assembly preload, P_A, is the same at all lens surfaces.

The following equations give the applicable values of K_3 and the axial gap, Δx, for temperature increases above that for which the preload reaches zero:

$$K_3 = \frac{-(\alpha_M - \alpha_{G1})t_{G1} - (\alpha_M - \alpha_S)t_S - (\alpha_M - \alpha_{G2})t_{G2}}{\dfrac{2t_{G1}}{E_{G1}A_{G1}} + \dfrac{t_S}{E_S A_S} + \dfrac{2t_{G2}}{E_{G2}A_{G2}} + \dfrac{(t_{G1} + t_S + t_{G2})}{E_M A_M}} \qquad 13$$

$$\Delta x = [(\alpha_M - \alpha_{G1})t_{G1} + (\alpha_M - \alpha_S)t_S + (\alpha_M - \alpha_{G2})t_{G2}]\Delta T \qquad 14$$

where all terms are as defined above.

Cross sectional views of two simple types of lens spacers are shown in Fig. 3. Both are solid cylinders fitting closely into the ID of the lens cell. The version shown in View (B) has a tapered ID to accomodate different heights of contact with the lenses. It also shows tangential interfaces. Equations 15 through 20 allow the annular areas, A_S, to be calculated for each of these spacers.

For spacer version (A):

$$w_S = (D_M/2) - y \qquad 15$$

For tapered spacer version (B), the wall thickness is taken as its average annular thickness calculated as follows:

$$\Delta y_i = (D_{G\,i}/2) - y_i \qquad 16$$

$$y_i' = y_i - \Delta y_i \qquad 17$$

$$w_S = (D_M/2) - ((y_1' + y_2')/2) \qquad 18$$

In both cases:

$$r_S = (D_M/2) - (w_S/2) \qquad 19$$

$$A_S = 2\pi r_S w_S \qquad 20$$

By following the same sequence of calculations as described for the cemented doublet, the contact stresses at the four air-glass interfaces can be estimated.

3. Two Air-Spaced Doublets

Fig. 4 provides a schematic example for stress estimation in a more complex multiple element design. Here, different cemented doublets, "A" and "B", are separated by a spacer of uniform annular thickness, t_C, in a cell with constant ID adjacent to the lens rims. A single retainer applies axial preload. The interfaces are shown as "sharp corners". The cross-sectional areas of the lenses are A_A and A_B while those for the cell wall and spacer are A_M and A_S. These areas are calculated with the aid of Eqs. 1 or 2, 3, 15, 19 and 20.

The applicable equation for K_3 of this design is:

$$K_3 = \frac{-(\alpha_M - \alpha_1)t_1 - (\alpha_M - \alpha_2)t_2 - (\alpha_M - \alpha_3)t_3 - (\alpha_M - \alpha_4)t_4 - (\alpha_M - \alpha_5)t_5}{\dfrac{2t_1}{E_1 A_A} + \dfrac{2t_2}{E_2 A_A} + \dfrac{t_3}{E_3 A_3} + \dfrac{2t_4}{E_4 A_B} + \dfrac{2t_5}{E_5 A_B} + \dfrac{t_M}{E_M A_M}} \qquad 21$$

The axial gap, Δx, for temperature increases above that for which the preload reaches zero can be calculated for the Fig. 4 design by the following equation:

$$\Delta x = [(\alpha_M - \alpha_1)t_1 + (\alpha_M - \alpha_2)t_2 + (\alpha_M - \alpha_3)t_3 + (\alpha_M - \alpha_4)t_4 + (\alpha_M - \alpha_5)t_5]\Delta T \quad 22$$

By following the same sequence of calculations as described for the cemented doublet, the contact stresses at the four air-glass interfaces can be estimated.

4. General Formulation for Multiple Elements

All of the above equations for K_3 have, in their numerators, the sum of negative terms comprising the axial thicknesses of each lens element and of any spacers at the applicable heights of contact multiplied by the pertinent differences in thermal expansion coefficients for those parts relative to that of the cell. In the denominators, are found the sums of reciprocals of the spring constants for each part of the subassembly. These spring constants have the following forms: for lens elements, $C_i = 2t_i/E_i A_i$, for spacers, $C_i = t_i/E_i A_i$ and for the cell wall, $C_i = (\Sigma_1^n t_i)/E_M A_M$. Note that lenses and spacers represent parts in compression so have positive signs while segments of the cell wall are in tension so have negative signs. Using this type notation, the equation for K_3 of any design can be rewritten as:

$$K_3 = \frac{-\Sigma_1^n(\alpha_M - \alpha_i)t_i}{\Sigma_1^n C_i} \qquad 23$$

Similarly, the equations for Δx also can be rewritten in more general form as:

$$\Delta x = [\Sigma_1^n((\alpha_M - \alpha_i)t_i]\,\Delta T \qquad 24$$

For any lens-mount design, the calculations leading to estimation of the axial contact stress at each interface involve first the sequential application of Eqs. 1 or 2, 3, 23 and 5 through 8 to determine the linear preload, p, for all air-glass interfaces at any temperature given the total assembly preload, P_A. Then, the applicable value of K_2 at each interface is calculated by Eq. 9 using the material properties prevailing at that interface. Finally, knowing the type of interface and surface radius at each surface, the value for K_1 can be calculated from the applicable form of Eq. 10 and the contact stress at each surface can be estimated through use of Eq. 11. Equation 24 allows the potential axial freedom of the lens at elevated temperature to be evaluated.

C. EXAMPLES

Example #1 - Cemented Doublet (see Fig. 1)

Pertinent design parameters are given in Table 1. The lens to cell interfaces at R_1 and R_3 are tangential. Assuming a total preload at assembly (68° F) of 44 lb, estimate the axial stress at each interface (1) at assembly and (2) at -80° F.

Computations are summarized in Table 2.

Example #2 - What is the axial preload on the cemented doublet of Example #1 at 160° F ? How much can the lens move axially?

By Eq. 6, $\Delta T = 92°F$
By Eq. 5, $\Delta P = -500.2$ lb. This is greater than P_A so $P = 0$ (i.e.,contact is lost) when the temperature rises by $P_A/K_3 = 8.1°F$. The effective ΔT thereafter is 76.1° F.

By Eq. 12, $\Delta x = 2.2\times10^{-4}$ in. The lens can thus move axially by this amount under shock or vibration.

Example #3 - Two Air-Spaced Singlets (see Fig. 2)

Pertinent design parameters are given in Table 3. All lens to cell interfaces are tangential. Assuming a total preload at assembly (68° F) of 40 lb, estimate the axial stress in each lens at -80° F.

Calculations are summarized in Table 4.

D. ACKNOWLEDGEMENTS

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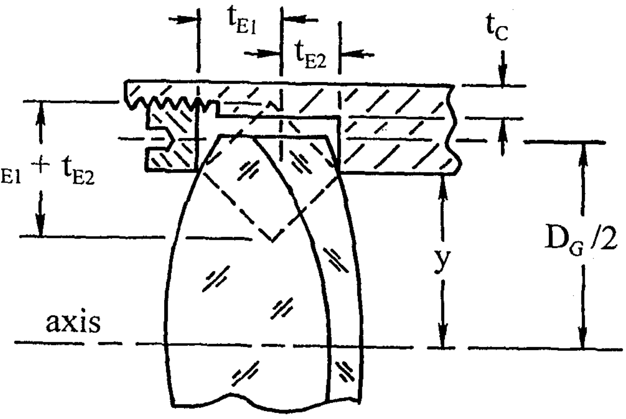


Fig. 1 - Schematic of a typical cemented doublet lens clamped between a cell shoulder and a retainer.

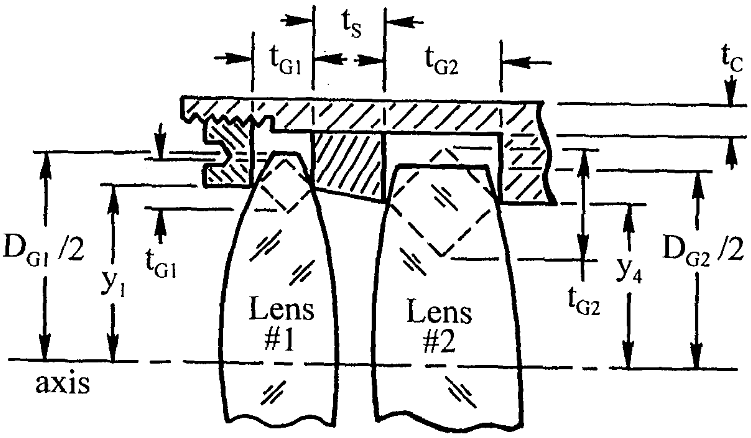
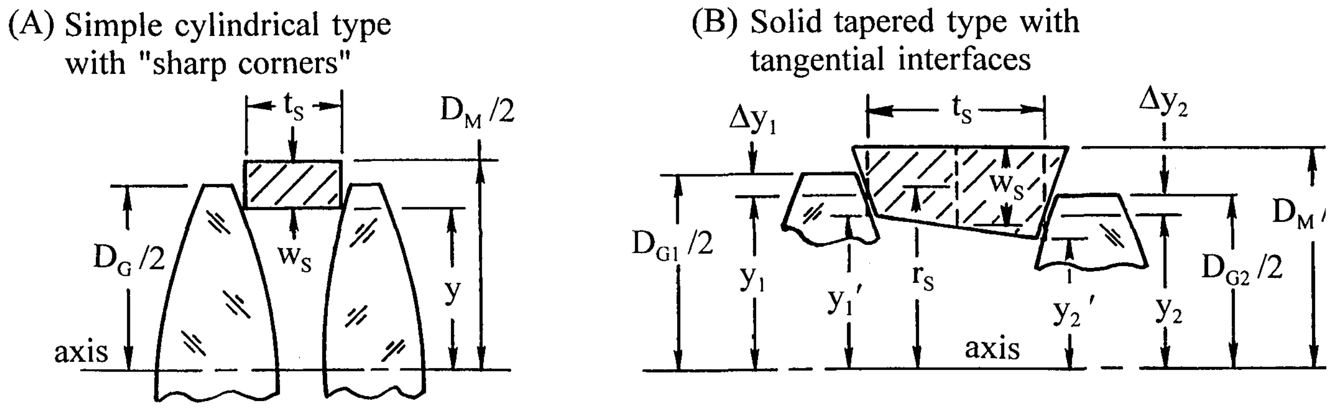


Fig. 2 - Schematic of two singlet lenses air-spaced by a spacer and clamped between a cell shoulder and a retainer.


(A) Simple cylindrical type with "sharp corners"

(B) Solid tapered type with tangential interfaces

Fig. 3 - Schematics of typical lens spacers. (A) Simple cylindrical type with "sharp corner" interfaces, (B) Solid tapered type with tangential interfaces.

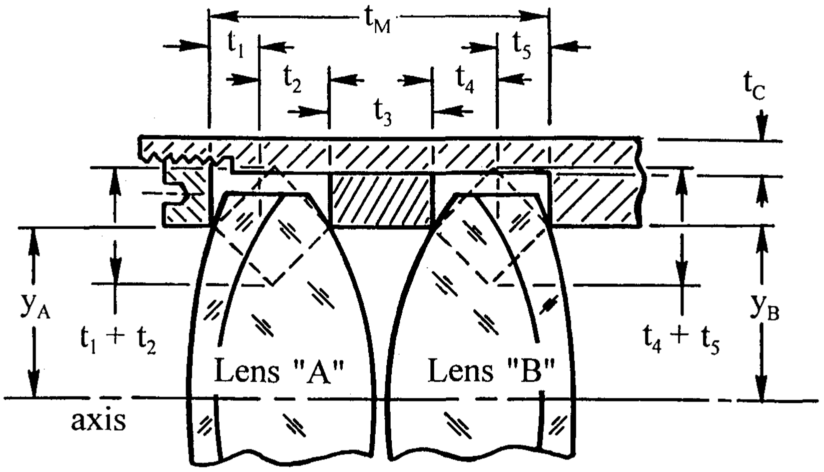


Fig. 4 - Schematic of two cemented doublets air-spaced by a spacer and clamped between a cell shoulder and a retainer.

Table 1 - Design Parameters for Example #1, Cemented Doublet

Parameter	Units	Cell	Lens #1	Lens #2
Material		Al6061	SF2	K5
E	lb/in.^{2}	9.9×10^6	7.98×10^6	1.03×10^7
υ		0.332	0.231	0.227
α	ppm/° F	1.3×10^{-5}	4.7×10^{-6}	4.6×10^{-6}
K_M & K_G	in.^{2}/lb	8.988×10^{-8}	1.186×10^{-7}	9.208×10^{-8}
R_1 & R_3	in.		2.174	-1.730
t_C & t_i in.		0.094	0.187	0.124
t_E	in.	0.311		
$y_i = y_{i+1}$	in.		0.517	0.517
D_M & D_G	in.	1.104	1.100	1.100
P_A	lb		44	44

Table 2 - Calculations for Example #1, Cemented Doublet

Parameter	Units	Cell	Lens #1	Lens #2	Use Eq.
$2y_i + t_E$	in.		1.345 ($> D_G$)		
A_G	in.^{2}		0.540		2
A_M	in.^{2}	0.354			3
K_3	lb/° F		-5.44		4
K_1	/in.		0.230	0.289	10(B)
K_2	in.^{2}/lb		2.085×10^{-7}	1.820×10^{-7}	9
(1) At 68° F					
p	lb/in.		13.545	13.545	8
S_A	lb/in.^{2}		3085	3701	11
(2) At -80° F					
ΔT	° F	-148			6
ΔP	lb	805			5
P_T	lb	849			7
p	lb/in.		261.4	261.4	8
S_A	lb/in.^{2}		13,550	16,258	11

Table 3 - Design Parameters for Example #3, Two Air-Spaced Singlets

Parameter	Units	Cell	Lens #1	Spacer	Lens #2
Material		Ti6Al4V	SF2	Ti6Al4V	BK7
E	lb/in.^{2}	$1.65\text{x}10^7$	$7.98\text{x}10^6$	$1.65\text{x}10^7$	$1.17\text{x}10^7$
υ		0.340	0.231	0.340	0.208
α	ppm/° F	$4.9\text{x}10^{-6}$	$4.7\text{x}10^{-6}$	$4.9\text{x}10^{-6}$	$3.9\text{x}10^{-6}$
K_M & K_G	in.^{2}/lb	$5.36\text{x}10^{-8}$	$1.186\text{x}10^{-7}$	$5.36\text{x}10^{-8}$	$8.14\text{x}10^{-8}$
R_1 & R_3	in.		8.125		4.755
t_C & t_i in.		0.100	0.200	0.750	0.550
y_1 & y_3	in.		2.000		1.880
D_M & D_G	in.	4.310	4.300		4.085
P_A	lb		40	40	40

Table 4 - Calculations for Example #3, Two Air-Spaced Singlets

Parameter	Units	Cell	Lens #1	Spacer	Lens #2	Use Eq.
$2y_i + t_E$	in.		4.200 $(< D_G)$		4.310 $(> D_G)$	
A_G	in.^{2}		2.513		5.013	2
Δy_i	in.		0.150		0.162	16
y_i'	in.		1.850		1.718	17
w_S	in.			0.371		18
r_S	in.			1.970		19
A_M & A_S	in.^{2}	1.385		4.591		3 & 20
K_3	lb/° F	-5.165				4
K_1	/in.		0.061		0.105	10(B)
K_2	in.^{2}/lb		$1.722\text{x}10^{-7}$		$1.354\text{x}10^{-7}$	9
At -80° F						
ΔT	° F	-148				6
ΔP	lb	764				5
P_T	lb	804				7
p	lb/in.		64.0		68.1	8
S_A	lb/in.^{2}		3800		5800	11