

Calculation of image position, size and orientation using first order properties

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OPTI.521 Tutorial

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1. Introduction

Image position, size and orientation are very important for initial first order design. This report discusses the calculation using first order properties. Analysis methods include Gaussian optics and paraxial optics. Small angle approximations are used.

2. Use Gaussian properties calculating image position and size

Many optical systems are first modeled as a thin lens. A thin lens is a surface with zero thickness that has refracting power. It is almost always used in air ($n = n' = 1.0$) and is characterized by its focal length f .

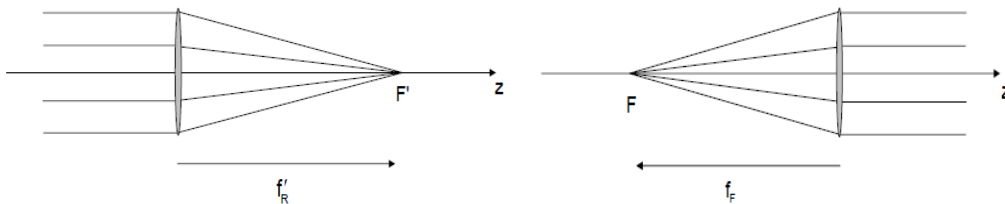


Fig.1 An object at infinity is imaged to the Rear Focal Point of the lens F' . Fig.2 An object at infinity is imaged to the Rear Focal Point of the lens F' .

Object and its image are conjugate. The relationships between the object position, the image position, the magnification, and the focal length can be determined from the properties of the focal points.

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \quad m \equiv \frac{h'}{h}$$

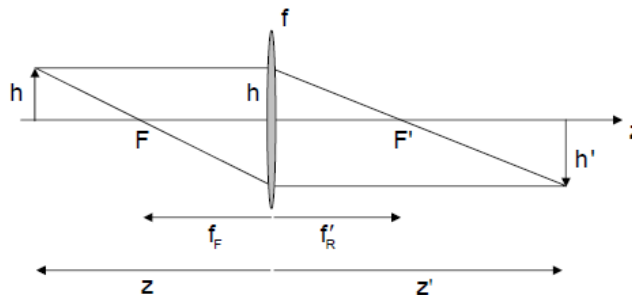


Fig.3 Object and image relationship

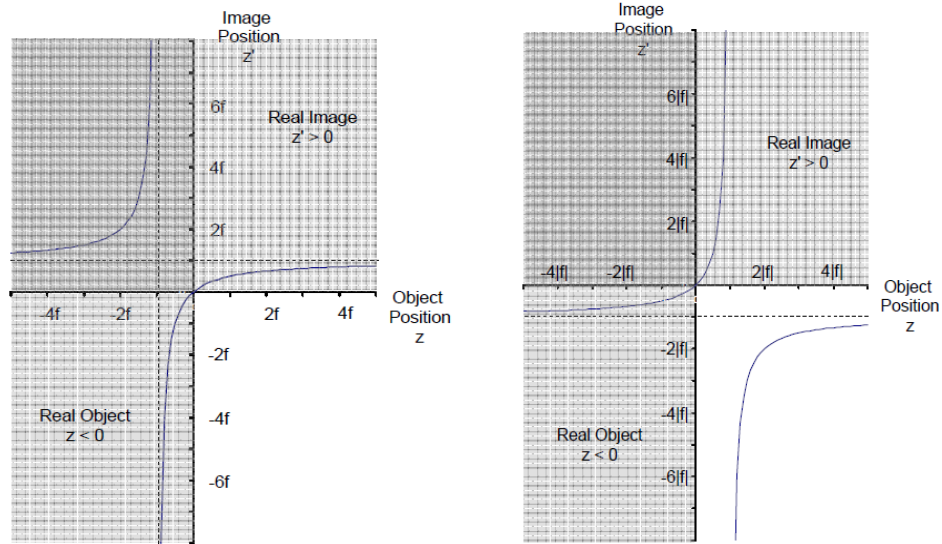


Fig. 4 Imaging with (a) positive and (b) negative lens

Object-Image Approximations

When the magnitude of the object distance z is more than a few times the magnitude of the system focal length, the image distance z' is approximately equal to the rear focal length. A positive thin lens in air is assumed ($n = n' = 1$):

$$|z| \gg |f|: \quad z' \approx f \quad L = z' - z \approx f - z \quad m = \frac{z'}{z} \approx \frac{f}{z}$$

Object Distance	Image Distance	Object-Image Distance	Approx. Object-Image Distance	Magnification	Approx. Magnification
z	z'	$L = z' - z$	$L \approx f - z$	m	$m \approx f/z$
$-f$	Inf	Inf	Inf	Inf	-1
$-2f$	$2f$	$4f$	$3f$	-1	$-1/2$
$-3f$	$1.5f$	$4.5f$	$4f$	$-1/2$	$-1/3$
$-4f$	$1.33f$	$5.33f$	$5f$	$-1/3$	$-1/4$
$-5f$	$1.25f$	$6.25f$	$6f$	$-1/4$	$-1/5$
$-10f$	$1.11f$	$11.11f$	$11f$	$-1/9$	$-1/10$
$-20f$	$1.05f$	$21.05f$	$21f$	$-1/19$	$-1/20$
$-100f$	$1.01f$	$101.01f$	$101f$	$-1/99$	$-1/100$

Table. 1 Object-Image Approximations

$$|f|/|z|$$

The fractional error in these approximations is about $|f|/|z|$, so they are very useful when the object distance more than 10-20 times the focal length. Most imaging problems can be solved with little or no computation.

Scheimpflug principle

First order optical systems image points to points, lines to lines and planes to planes. This condition holds even if the line or plane is not perpendicular to optical axis. The Scheimpflug condition states that a tilted plane images to another tilted plane, and for a thin lens, the line of intersection lies in the plane of the lens.

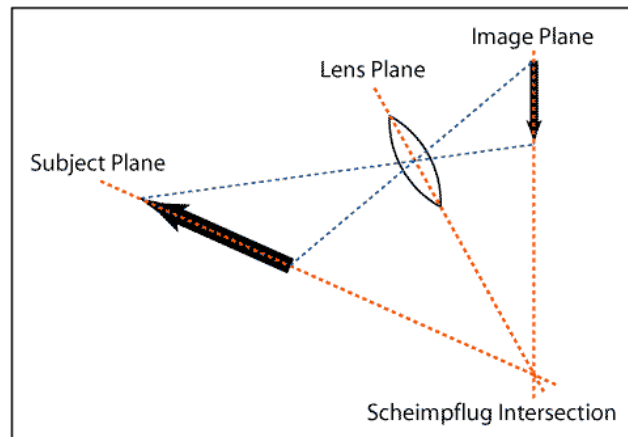


Fig. 5 Scheimpflug condition

Lens motion

1. Lateral motion

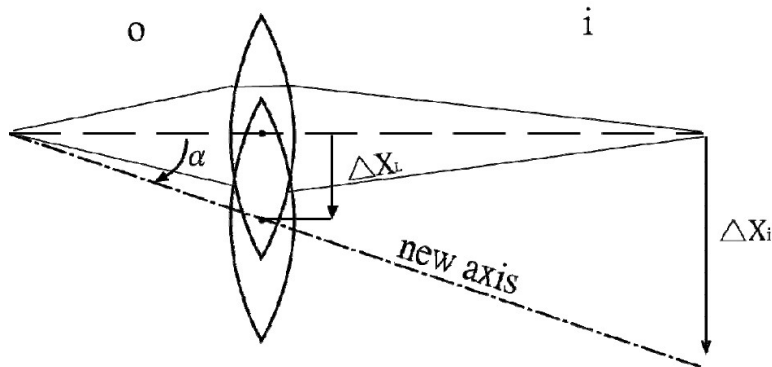


Fig. 6 Laterally shifting an lens by an amount, ΔX_L , will cause the image shift by motion caused by ΔX_i

$$\Delta X_i = \Delta X_L (1 - m)$$

2. Longitudinal motion

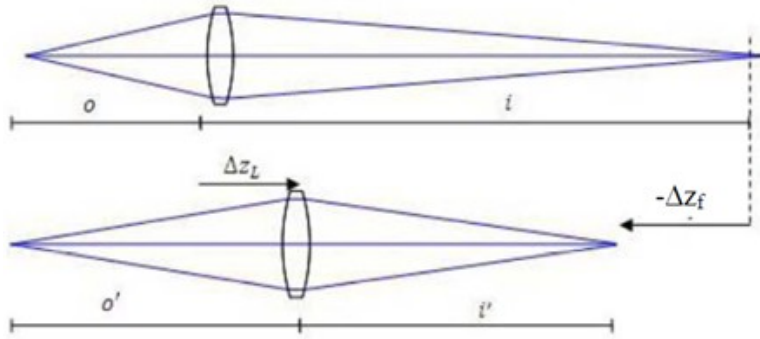


Fig. 7 Axially shifting an lens by an amount, Δz_L , will cause the focus to shift by Δz_f .

$$\Delta z_f = \Delta z_L (1 - m^2)$$

Tilt of optical element

For thin lens, there is no significant effect if you tilt an element about its center. But large tilt will cause aberrations.

For mirror, the image will tilt 2θ if you tilt the mirror by θ . You can trace the chief ray to prove it. So, we

should be more careful when dealing with mirror systems.

3. Image orientation in Prism systems

Plane mirrors

In addition to bending or folding the light path, reflection from a plane mirror introduces a parity change in the image. An image seen by an even number of reflections maintains its parity. An odd number of reflections change the parity. The rules of plane mirrors are used sequentially at each mirror in a system of plane mirrors.

Prism systems

Prism systems can be considered systems of plane mirrors. If the angles of incidence allow, the reflection is due to TIR. Prisms fold the optical path and correct the image parity.

Prisms are classified by the overall ray deviation angle and the number of reflections.

(1) 90° Deviation Prisms

Right angle prism (1 R) – the actual deviation depends on the input angle and prism orientation. Image is inverted or reverted depending on prism orientation.



Fig. 8 Right angle prism and its tunnel diagram

Amici or Roof prism (2 R) – a right angle prism with a roof mirror. The image is rotated 180° . No parity change.



Fig. 9 Roof prism and its tunnel diagram

Pentaprism (2 R) – two surfaces at 45° produce a 90° deviation independent of the input angle. It is the standard optical metrology tool for defining a right angle. The two reflecting surfaces must be coated. No parity change.

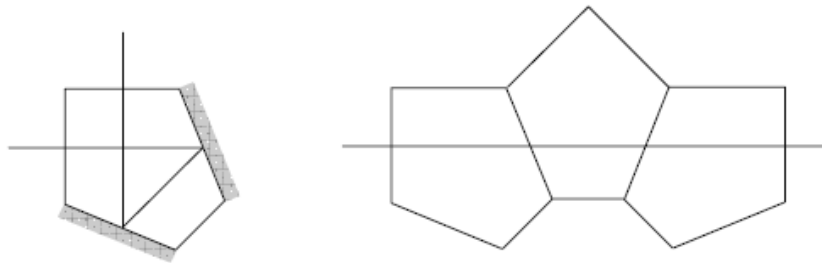


Fig. 10 Penta prism and its tunnel diagram

Reflex prism (3 R) – a pentaprism with an added roof mirror. Used in single lens reflex (SLR) camera viewfinders to provide an erect image of the proper parity. The roof surfaces must also be coated.

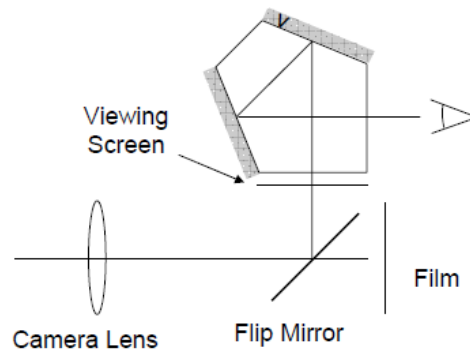


Fig. 11 A reflex prism used in SLR

(2) 180° Deviation Prisms

Porro prism (2 R) – a right angle prism using the hypotenuse as the entrance face. It controls the deviation in only one dimension.



Fig. 12 Porro prism and its tunnel diagram

Corner cube (3 R) – three surfaces at 90° . The output ray of this retroreflector is truly anti-parallel to the input ray. The deviation is controlled in both directions, and light entering the corner cube returns to its source.



Fig. 13 Corner cube and its tunnel diagram

(3) 45° Deviation Prisms

45° Prism (2 R) – half a pentaprism.



Fig. 14 45° prism and its tunnel diagram

Schmidt prism (4 R) – a 3 R version without a roof also exists. No coated surfaces



Fig. 15 Schmidt prism and its tunnel diagram

(4) Image Rotation Prisms

As the prism is rotated by θ about the optical axis, the image rotates by twice that amount (2θ). In all of these prisms, the input and output rays are co-linear (the light appears to go straight through) and there are an odd number of reflections (and therefore a parity change).

Symmetry and the parity change explain the image rotation. Each prism has an inversion direction and flip axis associated with it.

As the prism rotates relative to the object, this flip axis rotates, and the object is inverted about this axis. By symmetry, the object must rotate twice as fast (the prism at 0° and 180° must give the same output).

Dove prism (1 R) – because of the tilted entrance and exit faces of the prism, it must be used in collimated light. Lateral chromatic aberration is introduced.



Fig. 16 Dove prism and its tunnel diagram

Pechan prism (5 R) – a small air gap provides a TIR surface inside the prism. This compact prism supports a wide FOV. The two exterior surfaces must be coated. The Pechan prism is a combination of a 45° prism and a non-roof Schmidt prism (3 R).

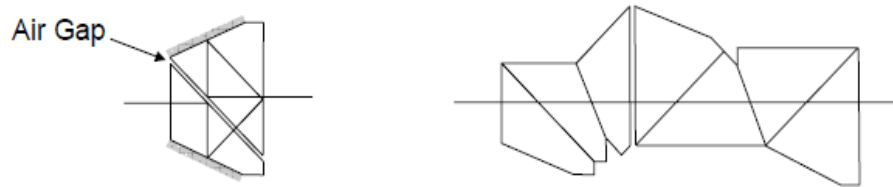


Fig. 17 Pechan prism and its tunnel diagram

(5) Image Erection Prisms

Image erection prisms are inserted in an optical system to provide a fixed 180° image rotation. They are commonly used in telescopes and binoculars to provide an upright image orientation. No parity change.

Porro system (4 R) – two Porro prisms. The first prism inverts the image and the second reverts the image. This prism accounts for the displacement between the objective lenses and the eyepieces in binoculars.

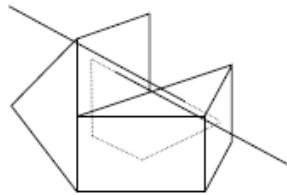


Fig. 18 Layout of Porro system

Porro-Abbe system (4 R) – a variation of the Porro system where the sequence of reflections is changed.

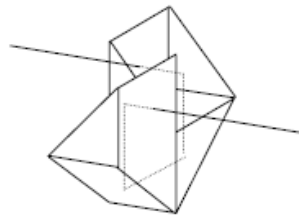


Fig. 19 Layout of Porro-Abbe system

Pechan-roof prism (6 R) – a roof is added to a Pechan prism. This prism is used in compact binoculars and provides a straight-through line of sight. It is a combination of a 45° prism and a Schmidt prism. Note that the roof surface does not need to be coated.

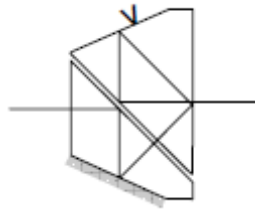
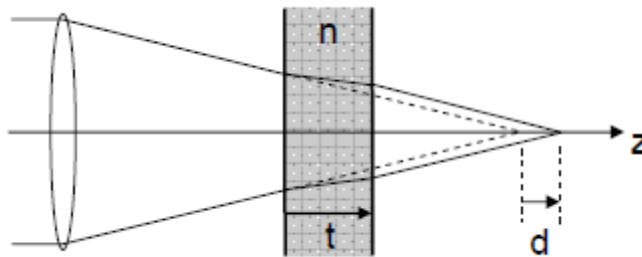


Fig. 20 Layout of Pechan-roof prism

Prisms with entrance and exit faces normal to the optical axis can be used in converging or diverging light. They will, however, introduce the same aberrations as an equivalent thickness plane parallel plate. Spherical aberration and longitudinal chromatic aberration are introduced into an on-axis beam. TIR often fails when prisms are used with fast $f/\#$ beams. In polarized light applications, TIR at the prism surfaces will change the polarization state of the light. In both these situations, silvered or coated prisms must be used. This situation occurs frequently with corner cubes.

Plane Parallel Plate (PPP)

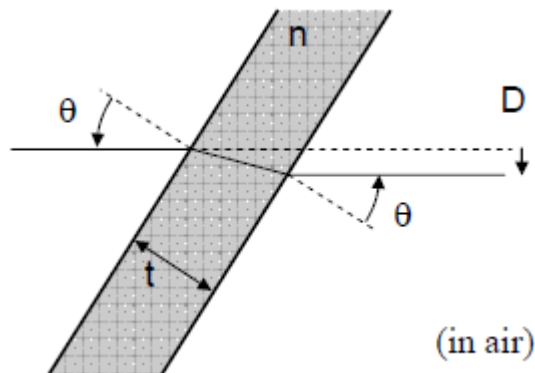
An image formed through a plane parallel plate is longitudinally displaced, but its magnification or size is unchanged.



$$d \approx \left(\frac{n-1}{n} \right) t \quad d \approx \frac{t}{3} \quad \text{for } n=1.5$$

Fig. 21 Focus shift caused by PPP

A ray passing through a plane parallel plate is displaced but not deviated; the input and output rays are parallel.



$$D \approx t \theta \left(\frac{n-1}{n} \right)$$

Fig. 22 Light displacement caused by PPP tilt

Reduced Thickness

The reduced thickness τ gives the air-equivalent thickness of the glass plate. A reduced diagram shows the amount of air path needed to fit the plate in the system, and no refraction is shown at the faces of the plate. Reduced diagrams can be placed directly onto system layout drawings to determine the required prism aperture sizes for a given FOV.

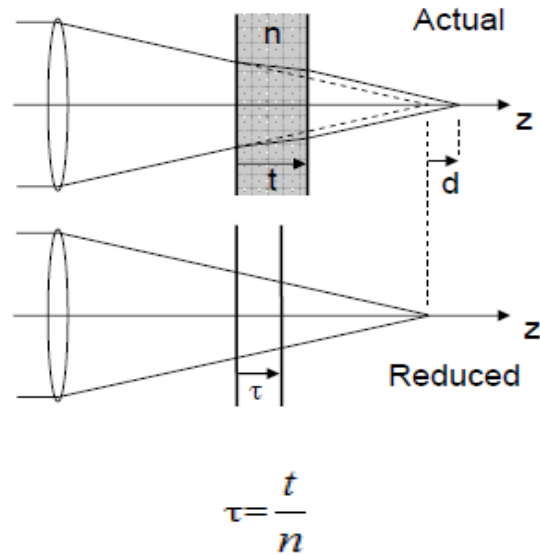


Fig. 23 Schematic of reduced thickness

The reduced thickness is useful to determine whether a certain size plate or prism will fit into the available airspace in an optical system (between elements or between the final element and the image plane). Since the plate makes some extra room for itself by pushing back the image plane, the required space is less than the actual plate thickness.

4. Paraxial raytrace

The Gaussian properties of an optical system can be determined using a paraxial raytrace with particular rays.

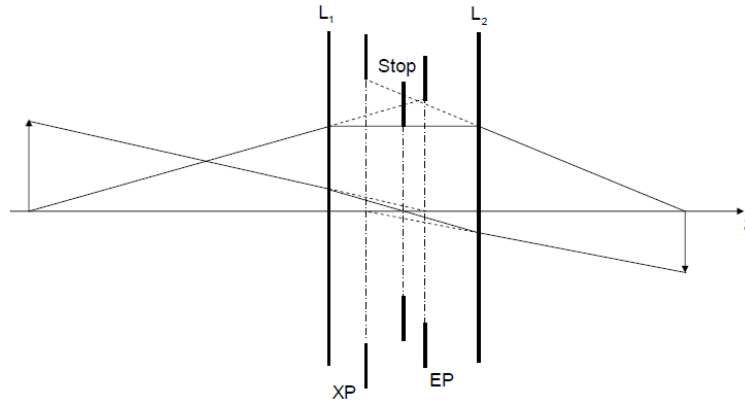


Fig. 24 Example of paraxial raytrace

As it is shown above, chief ray determines image size and marginal ray determines image location. Also, we can determine the pupil size and location by tracing chief ray and marginal ray.

5. Conclusion

This tutorial discussed the methods to calculation image location, size and orientation using first order properties. Gaussian properties and paraxial raytrace are used. Especially, image orientation and parities are mainly discussed in multiple prism systems. Also, some miscellaneous topics like Scheimpflug principle and effects of plane parallel plate are also discussed.

Reference:

- [1] John E. Greivenkamp "Field Guide to Geometrical Optics" SPIE
- [2] Prof. Jim Burge, class notes and lectures of "Introductory opto-mechanical engineering", Fall 2010
- [3] Prof. John Greivenkamp, class notes and lectures of "Optical Design and Instrumentation I", Fall 2009
- [4] Wikipedia, webpage http://en.wikipedia.org/wiki/Scheimpflug_principle