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# Minimized cost through optimized tolerance distribution in optical assemblies

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## Abstract

The cost to fabricate most mechanical systems and components is in inverse proportion to the tolerances applicable. Each system performance requirement has various sensitivities to errors in certain parameters (dimensions). The total cost to produce a system is the sum of the cost of each tolerance plus certain constant factors that cannot be reduced. The performance errors of the system are the sum of the contributions of each tolerance. This paper describes a methodology that allows the designer to assign and distribute the optical/mechanical tolerances in a system to minimize the cost of production while maintaining required system performance. The practical application of this method can lead to significantly reduced cost and increased ease of manufacture without risking or sacrificing performance.

## Introduction

The purpose of this paper is to introduce and illustrate a methodology by which one may reduce the production cost of most optical systems. The cost to produce an optical system increases rapidly as the mechanical and optical tolerances are made more stringent. The ultimate performance of a system is a function of each of the tolerances held for each component. Since the tolerances have an interdependent effect on the performance parameters, it is generally possible to trade or adjust tolerances while staying within the performance requirements. The cost of each tolerance and its effect on performance can be quantified. This system of relations can be optimized in general to achieve the minimum cost-set of tolerances that will satisfy the system requirements.

This paper describes a model of the relationships that is accurate to an engineering approximation and that is amenable to manual or computer optimization. The methodology demonstrated is suitable for practical application by the design engineer. It also forms a basis for further refinement of detail when required. The general principles are applicable to a far greater field than just optical/mechanical systems. The references from other fields illustrate the generic value of the methodologies.

The objective of this methodology is to assign tolerances in a system to 1) minimize the production cost and 2) satisfy the performance requirements.

## Formulation

### System performance versus component tolerances

A system generally will have one or more performance requirements to meet for it to perform its function. In an optical/mechanical system, these might include resolution (including focus), boresight, etc. A laser beam expander/telescope is used here as a simple illustration of the method (Figure 1). Table I lists the design data. In this illustration the principal requirement is to form a HeNe laser beam (0.65mm diameter) into a beam that does not exceed 25mm diameter over a 200 meter range. The nominal system design produces an image of approximately 13mm at 200 meters. It is expanded 20 times at the aperture, which gives a 13mm beam at that point. A 13mm increase in the nominal image size at 200 meters due to tolerances has been allowed. The performance requirement, therefore, is simply that the performance errors added by dimensional errors should not exceed 13mm. There are other performance factors, such as beam boresight, that, for the sake of simplicity, will not be considered at this point.

Each performance requirement is designated by an $E_j$. $E_1$ in the illustration is the image size increase allowed at 200mm and $E_1$ = 13mm. $E_2$ will be the allowed boresight error from a mounting surface.

Each dimensional or parameter tolerance may have an effect on the performance. The sensitivity of the performance $E_j$ to some tolerance error $T_i$ will be $C_{ij}$. Some requirements (E) may not be effected at all by some errors (T), and, therefore, the associated sensitivity coefficients (C) will be zero (Ø). The general system is described by the following equations:

$$
\begin{aligned}
E_1 &= C_{11}T_1 + C_{21}T_2 + C_{31}T_3 + C_{41}T_4 \cdot\cdot\cdot\cdot\cdot + C_{n1}T_n \\
E_2 &= C_{12}T_1 + C_{22}T_2 + C_{32}T_3 \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot + C_{n2}T_n \\
&\vdots \\
E_m &= C_{1m}T_1 + C_{2m}T_2 \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot + C_{nm}T_n
\end{aligned}
\qquad (1)
$$

Many problems will only have a few $E_j$ and may even reduce to one in a practical sense.

In summary, any performance error might be described as being the sum of the contributions of each tolerance error. The contribution of each tolerance error is the product of that error times the sensitivity of the performance error to that tolerance error. The performance error must be less than or equal to the requirement, that is to say:

$$E_j \geq \sum_{i=1}^{n} C_{ij} \; T_i \qquad (2)$$

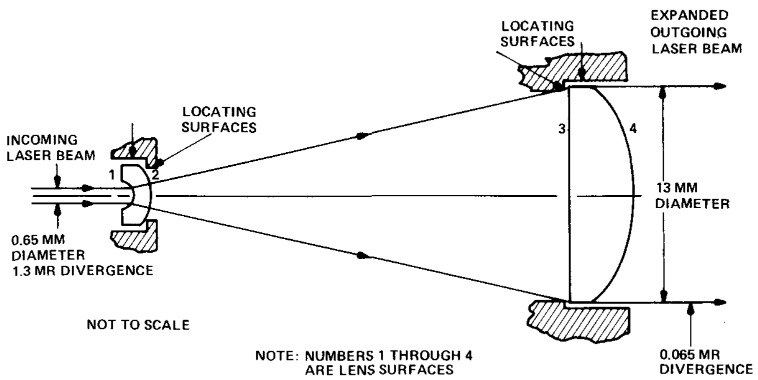


Figure 1. Example 20X Laser Beam Expander to Illustrate Tolerance Distribution Method.

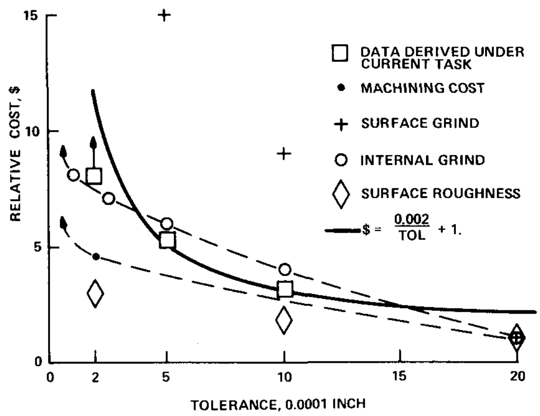


Figure 2. Cost Sensitivity to Changes in Dimensional Tolerances.

Table 1. Design data for beam expander

| Surface | Curvature 1/mm | Distance to Next Vertex (mm) | Refractive Index | Lens Decentration | Tilt |
|---|---|---|---|---|---|
| 1 | -1.2 $\pm$ $T_1$ | 1.0 $\pm$ $T_5$ | 1.515 $\pm$ $T_8$ | $\pm$ $T_{10}$ | |
| 2 | -0.6 $\mp$ $T_2$ | 55.24 $\mp$ $T_6$ | 1.0 | | |
| 3 | 0 $\mp$ $T_2$ | 3.0 $\mp$ $T_7$ | 1.515 $\pm$ $T_9$ | $\pm$ $T_{11}$ | $\pm$ $T_{12}$ |
| 4 | - .03 $\mp$ $T_4$ | 200000 | 1.0 | | |

$T_{13}$ is the tilt of the entire system with respect to the laser beam axis
$T_{14}$ is the decentration of the lens locating bore of lens 1-2 and, therefore, the same as $T_{10}$
$T_{15}$ is the decentration of the lens locating bore of lens 3-4 and, therefore, the same as $T_{11}$

## Total system cost versus component tolerances

It is common experience that the cost to produce any mechanical or optical dimension increases with the reduction of the allowed deviation from nominal. It can be assumed safely that an allowed deviation of zero will result in an infinite cost to produce. It can be said also that even if any and every deviation from nominal is permitted, to produce the system there will be some base cost that cannot be avoided. Given these two assumptions, the following equation approximates the cost versus tolerance of any parameter or dimension:

$$\$_i = A_i/T_i + B_i \qquad (3)$$

Here, $\$_i$ is the contribution to overall cost of the tolerance $T_i$ on a given dimension. $A_i$ is the tolerance-related constant and $B_i$ is the base cost of producing that dimension irrespective of tolerance. $A_i$ and $B_i$ depend on the type of dimension involved. They represent, in practice, a data base that can be developed for various types of dimensions and can be used in any situation where they are applicable. Figure 2 illustrates an example of Equation 3.

The data points on the figure represent the cost versus tolerance estimates for various fabrication processes. This data is primarily for metal working, but is generally believed to be of the same form for the working of optical components. The data points in the figure were taken from Martin Marietta internal engineering design manuals and similar documents, and they are generally consistant with the above referenced works. The squares result from the authors' investigations on certain milling and boring operations. The solid curve represents the mathematically simple form which we have chosen as a compromise generic representation of all points.

The total system cost that is influenced by the n dimensions in question will be:

$$\$_T = \$_1 + \$_2 + \$_3 \ldots . \$_n = \sum_{i=1}^{n} \$_i \tag{4}$$

Tolerance distribution to minimize cost

The objective of the exercise is to minimize the total cost $\$_T$ while satisfying all the requirements $E_j$. The values of all $A_i$, $B_i$, and $C_i$ are given. In the case of a single performance requirement $E_j$ that must be met, the equation for each $T_i$ reduces to:

$$T_i = E_j \sqrt{\frac{A_i}{C_{ij}}} \quad \frac{1}{\sum_{i=1}^{n} \sqrt{A_i C_{ij}}} \tag{5}$$

In this case, there is only one $E_j$, which is $E_1$ or simply E. Similarly, the subscript j could be removed from the $C_{ij}$ to give $C_i$. The derivation of this equation and the more complex problem of multiple $E_j$ will be described in a future paper. The purpose of this paper is to describe a pragmatic tool for the designer. In the more general case, an iterative procedure using Lagrange Multipliers appears to yield a solution where two of the $E_j$ are exactly satisfied. The remaining allowable performance errors $E_j$ are greater than the total error contributions from the tolerances, that is, they are more than satisfied.

Therefore, it is only necessary to determine the values of the A, B, C, and E's in order to find the T's that will yield the minimum cost and satisfy all of the requirements.

## The method and an illustration

The steps used to solve for the tolerance distribution include the following:

Determine the performance requirement $E_j$.
Identify the parameters that effect performance and for which Ti can be determined.
Evaluate the sensitivity coefficients $C_{ij}$ for each $T_i$.
Find the cost versus tolerance coefficients $A_i$ and $B_i$.
Use the method to determine the values for the $T_i$.
Use the $T_i$ to verify that the requirements are satisfied.

Using the design in Figure 1 and Table 1, a mounting design that allows focus and boresight adjustment is assumed. This implies that the spacing between the two lenses can be adjusted. The angle of the entering laser beam as well as the aiming of the entire system can be used to achieve boresight. The performance requirements thereby are reduced to a single $E_j$ that we will call E. The requirement is that the increase in the size of the image at 200 meters does not exceed 13mm. It is further assumed that the design has been adjusted to the glass melt so that the index of refraction tolerance can be ignored. Table 1 shows 15 T variables and the A and C values used in this illustration. The above assumptions allow eliminating $T_6$, $T_8$, $T_9$, and $T_{13-15}$ from consideration.

The $C_{ij}$ (equal to $C_i$ in this case) are found by varying each parameter in turn from the nominal design by a small amount to find $dE_j/dT_i = C_{ij}$. This is actually only an approximation and assumes that the function is linear over the region of use. Table 2 shows the $C_{ij}$ found for the illustration case by ray trace evaluation.

The cost versus tolerance coefficoemts $A_i$ and $B_i$ represent a data base that should be evolved and maintained from actual experience. It has been somewhat surprising to find a dearth of firm data in this area. It appears that good estimators, for cost of mechanical and optical components, quickly make a great many "intuitive" calculations by an overall examination of a production drawing. Their ability is often highly to be praised, but appears to be more art than science. Owing to the inherent variability in the cost to produce a given part, any estimates are at best "engineering approximations." The $A_i$ and $B_i$ should be estimated on the basis of whatever data and experience is appropriate. One should keep in mind that good approximations are desired, but that rigor might tend toward wasting more effort in one area than would be saved in another. Since the $B_i$ cannot be effected by the tolerance and, therefore, do not change, they do not enter into the cost minimization process. Therefore, the only concern is the $A_i$. Table 2 shows the $A_i$ that have been assumed for this illustration. Note that they are not necessarily representative of any actual values.

Table 2. Coefficients associated with illustration beam expander

| Parameter # $i$ | Cost Sensitivity $A_i$ | Performance Sensitivity $C_i$ | Tolerance Allowance $T_i$ | Parameter Variable Cost $\$_i$ |
|---|---|---|---|---|
| 1 | 25 $/% | .006 | 2.70% | $ 9.25 |
| 2 | 25 $/% | .0095 | 2.15% | 11.64 |
| 3 | 25 $/Fringe | .00014 | 17.7 Fringes | 1.41 |
| 4 | 25 $/% | .0024 | 4.27% | 5.85 |
| 5 | 50 $/Micron | .00026 | 18.4 Microns | 2.72 |
| 6 | 50 $/Micron | 0 | 0 | 0 |
| 7 | 50 $/Micron | $2 \times 10^{-7}$ | 662 Microns | 0.08 |
| 8 | 5 $/.001 | 0 | 0 | 0 |
| 9 | 5 $/.001 | 0 | 0 | 0 |
| 10 | 50 $/Micron | .00034 | 16.1 Microns | 3.12 |
| 11 | 50 $/Micron | .00012 | 27.0 Microns | 1.85 |
| 12 | 50 $/Micron | $4.9 \times 10^{-5}$ | 42.5 Microns | 1.18 |
| 13 | 50 $/Micron | 0 | 0 | 0 |
| 14 | 50 $/Micron | 0 | 0 | 0 |
| 15 | 50 $/Micron | 0 | 0 | 0 |
| | | Total of tolerance variable costs = $37.10 | | |

All of the data needed to calculate the $T_i$ is now at hand. First, find each of the roots $\sqrt{A_i C_i}$ and the sums of those roots to be used as in Equation 5. The following equations can be used for the single E, when we remove the constant $B_i$ from $\$_i$:

$$\$_i = \sqrt{A_i C_i} \frac{\sum_{i=1} \sqrt{A_i C_i}}{E} \qquad (6)$$

$$T_i = A_i / \$_i \qquad (7)$$

It is then easiest to compute the $\$_i$ using Equation 6 and then the $T_i$ using Equation 7. The sum of the $\$_i$ then represents the total cost contributed to the system production cost by those tolerances. It does not include the $B_i$ related cost and other factors such as material cost. The above method is all that is needed when only one requirement E must be satisfied. Table 2 lists the values of $T_i$ and $\$_i$ for the illustration case. The resulting $\$_T$ is $37.10.

As another case, no adjustments were assumed. This implied that the components were simply fabricated and assembled and would be in satisfactory focus and alignment without any adjustment. Although the details are not belabored here, the comparable $\$_T$ was $412.42.

An engineering method for multiple requirements

When there is more than one requirement $E_j$ to be satisfied, an engineering approximation that has evolved has proved useful. The concept is to form a single equation for E that is composed of the most dominant $C_{ij}$ factors from each $E_j$ equation. The $T_i$ are then all scaled up by the factor to make the largest $E_j$ just satisfied. The result is an engineering approximation of the minimum cost solution.

To clarify and illustrate the method for approximate solution, the following detailed example is offered. The data of this illustration has no direct relation to the earlier example, but is simpler for ease of explanation.

The $E_j$ and $C_{ij}$ for a particular four-tolerance example are as follows:

$$E_j = C_{1j}\,T_1 + C_{2j}\,T_2 + C_{3j}\,T_3 + C_{4j}\,T_4 \qquad (8)$$

| $j$ | $E_j$ | $C_{1j}$ | $C_{2j}$ | $C_{3j}$ | $C_{4j}$ |
|---|---|---|---|---|---|
| 1 | 1. | 3. | 4. | 5. | 2. |
| 2 | 3. | 6. | 9. | 12. | 9. |
| 3 | 2. | 4. | 12. | 8. | 4. |

Each equation is divided by $E_j$ to normalize to each $E_j = 1$. This gives the following modified set of equations:

$$E_j/E_j = C_{1j}\,T_1/E_j + C_{2j}\,T_2/E_j + C_{3j}\,T_3/E_j + C_{4j}\,T_4/E_j \qquad (9)$$

| $j$ | $E_j/E_j$ | $C_{1j}$ | $C_{2j}$ | $C_{3j}$ | $C_{4j}$ |
|---|---|---|---|---|---|
| 1 | 1 | $\underline{3}$ | 4 | $\underline{5}$ | 2 |
| 2 | 1 | 2 | 3 | 4 | $\underline{3}$ |
| 3 | 1 | 2 | $\underline{6}$ | 4 | 2 |

The underscored values show the next step, which is to find the largest $C_{ij}$ for a given $i$ and to compose a new single E case that uses the most sensitive normalized values. This gives:

$$E = 1 = 3T_1 + 6T_2 + 5T_3 + 3T_4 \qquad (10)$$

Using the following values for $A_i$:

$$A_1 = 10 \qquad A_2 = 15 \qquad A_3 = 25 \qquad A_4 = 20 \tag{11}$$

Solve this system, using the single E method, to get:

$$T_1 = .0539 \qquad T_2 = .0467 \qquad T_3 = .0660 \qquad T_4 = .0762 \tag{12}$$

which gives:

$$\$_T = 1148.56 \qquad E_1 = .8309 \qquad E_2 = 2.2215 \qquad E_3 = 1.6088 \tag{13}$$

Dividing the $E_i$ by the maximum allowed, $E_1$ is found to be closest to its limit. Therefore, an increase of tolerance $T_i$ by 1/.8309, will just satisfy the $E_1$ limit and will not exceed $E_2$ and $E_3$. This results in:

$$T_1 = .0649 \qquad T_2 = .0562 \qquad T_3 = .0794 \qquad T_4 = .0914 \tag{14}$$

which gives:

$$\$_T = \$953.94 \qquad E_1 = 1.000 \qquad E_2 = 2.673 \qquad E_3 = 1.936 \tag{15}$$

This represents a reasonable engineering soluction to a "minimized" cost for the system tolerances.

An exact numerical method for multiple requirements

Suppose that the cost function for some assembly can be represented by Equation 4, where $\$_i$ is the cost component of each assembly component. Also suppose that the cost of each component can be represented by Equation 3, where $A_i$ is the cost sensitivity coefficient, $T_i$ is the tolerance allowance for the component, and $B_i$ is the base cost of the tolerance regardless of the tolerance value. It is desirable to find the set of $T_i$'s which result in the minimized cost of $\$_T$ and will meet system performance requirements which can be represented by Equation 2. Let $L_j$ be Lagrange's undetermined multipliers. Then $\$_T$ is minimized when

$$\frac{\partial \$_T}{\partial T_i} + \sum_{j=1}^{n} L_j \frac{\partial E_j}{\partial T_i} = 0, \quad \text{for } i = 1 \text{ to } n \tag{16}$$

Substituting Equations 2, 3, and 4 into Equation 16;

$$-A_i T_i^{-2} + \sum_{j=1}^{n} L_j C_{ij} = 0, \quad \text{for } i = 1 \text{ to } n \tag{17}$$

or

$$T_i = \sqrt{\frac{A_i}{\sum_{j=1}^{n} L_j C_{ij}}} \tag{18}$$

Substituting this value of $T_i$ into equation 5,

$$E_j = \sum_{i=1}^{n} C_{ij} \sqrt{\frac{A_i}{\sum_{k=1}^{n} L_k C_{ik}}} \tag{19}$$

This equation is the jth L-equation which has to be solved for $L_j$ before least-cost component tolerances can be evaluated. This is done by an iterative process. Least-cost tolerances $T_j$'s are then evaluated by substituting the values of $L_j$ obtained into Equation 18.

Equations 19 are called the L-equations and are solved by an iterative method. This method is basically to determine three successive values of $L_k$'s to use in Wegsteins's accelerated iterative process. The new set of values for $L_i$'s are then used to determine three successive values of $L_k$'s. Then the process is repeated. This process is repeated until two successive values of $L_k$ do not differ by more than a specified amount. This method is described in the literature[3]. Least-cost tolerances $T_i$'s can then be evaluated by substituting the values of $L_1, L_2, \ldots, L_m$ into Equations 18.

It is expedient to check the results of the method by using the tolerances developed in an evaluation of the system with everything at its limits. If this test were to fail to satisfy the requirements, it would indicate an error in the sensitivity coefficients. This might result from an error in analysis or possible nonlinearities in the performance versus the parameters of the tolerances.

## Discussion and conclusions

The experience in the use of these methods has indicated that the most difficult part of the work is to get the values for the sensitivity coefficients $C_{ij}$ of a particular system. The $A_i$ (and $B_i$) are also difficult to obtain with much confidence because they are not currently a normal detail used by estimators or recorded by accountants or cost study engineers. Both difficulties are surmountable, however, and can be obtained to a practical degree in any optical design and manufacturing organization. Cost versus tolerance data $A_i$ can be developed for each facility and could even be pooled across the industry and published for the benefit of design engineers. The sensitivity coefficients, $C_{ij}$ and requirements $E_j$ are necessarily each designers own task.

Once the coefficients are available, the optimization is straightforward. The engineering solution can be accomplished by hand or a one page computer program. The exact numerical method is a reasonably straightforward computer program.

Note that the absolute values of the sensitivity coefficients $C_{ij}$ have been used. The implications are that one is finding a worst case set of tolerances wherein every tolerance would go in the worst direction. In extending the present work, it would probably be appropriate to consider statistical factors such as the works of Rimmer,[5] Koch[6], and Hilbert [7] that would allow the tolerances to be relaxed even further. Some caution is in order here, however, since it is known that certain dimensions tend to crowd in one direction for some processes, such as axial thickness of lenses, etc. Random errors are probably too simple a model to assume in such a case.

The solution with the exact numerical method was optimized until $E_3$ reached the target value of 2.0. The cost was reduced even further to $944.72. This is a minor improvement over Equation 15, but is probably the result of the particular data leading to a close solution by the "Engineering Method". In one complex case, it was found that a further twofold reduction by the exact numerical method occurred over the engineering method.

It is believed that the optical community is entering an era where the economics of design and manufacture will receive increased emphasis. To hold to unnecessarily tight tolerances is a waste of resources. Experienced designers often do well at assigning practical tolerances by intuition, and this method might still be expedient in the case where only one system is to be produced. However, in today's environment of readily available computer power, there is seldom a valid reason to neglect the opportunity to reduce production cost by thorough design cost analysis. The method described makes tools available to the designer that can make his efforts more valuable.

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