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SLIT DIAPHRAGM FLEXURES FOR OPTOMECHANICS

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ABSTRACT: The slit diaphragm is a useful type of flexure for producing small rotations or translations. This type of flexure is easily fabricated using modern techniques such as electro-discharge machining (EDM), and provides a convenient solution for certain types of precision motion problems. There are two types of slit diaphragms used in optomechanics: a two axis gimbal for rotation and a linear flexure for translation. Closed form solutions are discussed for determining rotational and radial stiffness of the two axis gimbal slit diaphragm flexure. Similar equations are given for finding the linear and radial stiffness of the linear translation slit diaphragm flexure. A design example of a focus mechanism employing a pair of linear translation slit diaphragm flexures is discussed.

INTRODUCTION: Many precision motion applications in optical engineering require limited ranges of travel. Often the linear range of travel is less than 1 mm, and the range of rotation is less than 5 degrees. Conventional rolling element bearings are not a good choice for providing motions of this type. The relationship between friction and torque in a rolling element bearing is non-linear for small ranges of travel[1,2]. Hysteresis and rapid wear are other problems associated with rolling element bearings used for small ranges of travel[3,4].

Flexures are elastic elements used to provide guidance of small motions. Flexures provide motion without the hysteresis or wear associated with conventional rolling element bearings[5]. Most flexure designs are based on the simple single strip flexure, with more complex configurations built up of multiple single strip flexures. The most commonly used flexure designs are the parallel spring guide, for linear translation[6], and the two strip cross flexure, for rotation[7].

A significant disadvantage of traditional flexure designs is the large physical size necessary to produce even small ranges of travel. For example, a parallel spring guide flexure used to provide a translation range of 1 mm might occupy a cubical space 100 mm on a side. In comparison, a conventional rolling element bearing translation stage with the same load capacity and range of travel might be 50 mm long, 20 mm wide, and 10 mm thick. Packaging f'exural bearings in space constrained designs is very difficult. Application of conventional flexures is limited by the volume required in most designs.

An alternative to conventional flexures based on the single strip flexure are designs based on diaphragms. Both corrugated and slit diaphragm flexures are used in scientific instruments, usually for measurement applications[8]. Corrugated diaphragms are often employed in pressure gauges[9]. Conventional corrugated or flat diaphragms provide axial motion. The slit diaphragm flexure offers both translation and rotation.

The main advantage of the diaphragm flexure over the traditional "single strip" flexure is compactness. A secondary advantage is the simplicity of the diaphragm, which reduces production cost. Although the pattern of slits cut into a diaphragm flexure may be complex, methods such as electro-discharge machining can produce the slit pattern at low cost.

Despite the attractive qualities of the diaphragm flexure, there is relatively little information available on design.. What is available is information on design of corrugated diaphragms[10]. The corrugated diaphragm flexure is expensive to produce, and although useful for instrument applications, is relatively restricted in utility for optomechanics. This paper is an attempt to remedy this deficiency and contains design equations for the two most useful types of slit diaphragm flexures: the two axis diaphragm flexure and linear translation slit diaphragm flexure.

THE LINEAR TRANSLATION SLIT DIAPHRAGM FLEXURE: Figure 1 shows the linear translation slit diaphragm flexure. Diaphragm flexure stiffness is calculated by summing the stiffness of the individual flexures in the diaphragm. Each small flexure is assumed to consist of two single strip flexures connected together to make a series (not parallel) combination. Each flexure is considered as a beam in bending, with the ends constrained against rotation. One end of each flexure is translated with respect to the fixed end. For a single flexure, the translation is one half that of the full range of travel. The axial force required to translate an individual flexure is then given by:

$$F_I = \frac{12\, E\, I\, \frac{\delta}{2}}{L^3}$$

where:	F_I	is the axial force required to translate the flexure
	E	is the elastic modulus of the flexure material
	I	is the cross section moment of inertia of the flexure
	δ	is the translation
	L	is the flexure length

The flexure length is given by the circumference of the diaphragm at the point of the flexures minus the connecting length, divided by the number of flexures, or:

$$L = \frac{2\, \pi\, r_f - n\, c}{n}$$

where:	L	is the flexure length
	r_f	is the radius of diaphragm at the point of the flexures
	n	is the number of flexures in the diaphragm
	c	is the tangential distance between flexures

The moment of inertia of the cross section of one of the flexures is given by:

$$I = \frac{b\, h^3}{12}$$

where:	I	is the cross section moment of inertia of an individual flexure
	b	is the width of the flexure in the radial direction
	h	is the thickness of the flexure in the axial direction

Combining the above equations, the force needed to translate the center of the flexure relative to the edge is given by:

$$F_A = \frac{n^4 \, E \, b \, h^3 \, \delta}{(n \, c - 2 \, \pi \, r_f)^3}$$

where: F_A is the force required to translate the center of the flexure relative to the edge by the distance δ

The above equation indicates a strong dependence of force on the number of flexures; fewer flexures provide lower axial forces for a given range of motion. Axial force is also strongly dependent on the axial thickness of the diaphragm. This equation assumes that the flexures do not carry any radial loads.

Translating the center of the diaphragm with respect to the edge introduces stress into the flexure. The stress due to the bending in the individual flexures is given by:

$$\sigma_A = \frac{3 \, E \, \delta \, n^2 \, h}{2 \, (n \, c - 2 \, \pi \, r_f)^2}$$

where: σ_A is the stress due to a translation of the center of the flexure relative to the edge by the distance δ

The flexural stress equation indicates that stress is strongly dependent on the number of flexures. Fewer flexures provide lower stress. There is a linear dependence of stress with axial diaphragm flexure. Analysis of the equations for both stress and motion shows the importance a material parameter called the reduced tensile modulus, the ratio of elastic modulus (E) to allowable stress (σ_{YS}). The larger the value of the reduced tensile modulus, the larger the range of travel for a given flexure geometry. Materials with a high yield stress and low elastic modulus are desirable for diaphragm flexures. Table 1 gives the reduced tensile modulus for some selected flexure materials.

Diaphragm flexures are often subject to radial loads. The radial stiffness of the diaphragm is calculated using an approach developed by Richard[11]. The radial spring rate of the diaphragm is given by:

$$k_R = \frac{n_i}{2} \, (k_{fr} + k_{ft})$$

where:
- k_R is the radial spring rate
- n_i is the number of flexures, and must be a multiple of 2 or 3: $n_i = 2i$ or $n_j = 3j$, where i is an integer ≥ 2, j is an integer ≥ 1
- k_{fr} is the radial spring rate of one individual flexure
- k_{ft} is the tangential spring rate of one individual flexure

The spring rates of the individual flexures are found using a method similar to that used to find the axial stiffness of the diaphragm. The radial spring rate for an individual flexure is given by:

$$k_{fr} = \frac{n^4 \, E \, b^3 \, h}{(2 \, \pi \, r_f - n \, c)^3}$$

Similarly, the tangential spring rate for an individual flexure is given by:

$$k_{ft} = \frac{2\ b\ h\ E}{2\ \pi\ r_f - n\ c}$$

Compliance is the reciprocal of the spring rate. Combining the equations for the radial and tangential spring rates with the equation for the overall spring rate gives the compliance of the slit diaphragm flexure in the radial direction:

$$S_R = \frac{2\ (nc\ -\ 2\pi r_f)^3}{nbhE\left(n^4b^2\ +\ 8\pi^2 r_f^2\ -\ 8\pi r_f nc\ +\ 2n^2c^2\right)}$$

where: S_R is the radial compliance of the flexure

Typically the linear translation flexure is not used by itself due to its relatively poor rotational stiffness. Two linear translation flexures are employed in tandem, connected at their centers, to produce an axial translation. This assembly geometry is suited for applications such as focus mechanisms in lens barrels.

As indicated by the above equations, the axial stiffness decreases as the number of flexures in the diaphragm is increased. Increasing the number of flexures decreases the radial stiffness of the diaphragm. In the design of a linear translation diaphragm flexure the number of individual flexures in the diaphragm will be determined by the balance between radial and axial stiffness.

THE TWO AXIS GIMBAL ROTATION DIAPHRAGM FLEXURE: Calculation of the stiffness of the two axis gimbal flexure is performed by summing the torsional stiffness of the individual flexures. The stiffness of each individual flexure is found by treating the flexure as a beam in torsion[12]. Figure 2 shows a two axis gimbal diaphragm flexure. The angle of rotation due to an applied torque in a thin rectangular cross section beam is given by:

$$\theta = \frac{T_i L}{KG}$$

where:
- θ is the angle of rotation
- T_i is the torque applied to the individual flexure
- L is the length of the individual flexure
- K is a factor dependent on the cross section of the flexure
- G is the shear modulus of the flexure material

The angle of rotation produced for the two axis gimbal diaphragm flexure is then:

$$\theta = \frac{TL}{2Gbh\left(\frac{1}{3}\ -\ 0.21\ \frac{h}{b}\left[1\ -\ \frac{1}{12}\left(\frac{h}{b}\right)^4\right]\right)}$$

where:
- θ is the angle of rotation
- T is the torque applied to the diaphragm flexure
- L is the length of the individual flexure
- G is the shear modulus of the flexure material
- b is the width of the individual flexure
- h is the axial thickness of the individual flexure

The maximum stress in the diaphragm flexure is given by:
where: τ is the shear stress in the flexure

$$\tau = \frac{T\,(3b + 1.8h)}{2b^2h^2}$$

Radial compliance of the two axis gimbal flexure is found using an approach similar to that used for the linear translation flexure. The radial and tangential stiffness of the individual flexures are found, and then combined. The radial compliance of the two axis gimbal flexure is given by:

$$S_{TR} = \frac{1}{2\,E\left(\frac{bh}{L}\right)\left[1 + \left(\frac{b}{L}\right)^2\right]}$$

where: S_{TR} is the radial compliance of the two axis gimbal flexure diaphragm

The material property determining the performance of the two axis gimbal diaphragm flexure is the reduced shear modulus. The reduced shear modulus is the ratio of the shear modulus (G) of the material to the allowable shear stress in the material (τ_{YS}). Larger values of the reduced shear modulus produce larger angles of rotation in the flexure.

The two axis gimbal diaphragm flexure is normally employed by itself. Compactness in the axial direction is the main virtue of this type of flexure. The two axis gimbal diaphragm flexure is also simple, and relatively low in cost.

DESIGN EXAMPLE: An example of a focus mechanism employing a pair of slit diaphragm linear translation flexures in shown in cross section in figure 3. Figure 4 is an isometric of the cell and diaphragm. Figure 5 shows a "12 spoke spring," a slit diaphragm linear translation flexure.

Assume that stainless steel is used in the 12 flexure slit diaphragm, and define:

E	=	200 Gpa	Elastic modulus of stainless steel
h	=	0.5 mm	Diaphragm thickness
b	=	1.65 mm	Width of flexure in radial direction
r_f	=	35 mm	Mean radius of flexure
c	=	2.5 mm	Tangential distance between flexures
δ	=	1 mm	Axial stroke or travel

The axial stiffness of this flexure is then 135 N/m, and the radial stiffness is 19 MN/m. At the maximum axial stroke of 1 mm, the stress in the flexure is 624 MPa. Although high, the stress is below the fatigue limit for a high strength stainless steel such as 17-4 PH. Decreasing the number of flexures to 8 reduces the axial stiffness to 22 N/m, and the radial stiffness to 7.7 MN/m. The 8 flexure diaphragm has a maximum stress for a 1 mm stroke of 250 Mpa.

CONCLUSIONS: The two types of flexures discussed in this paper provide solutions for many optomechanical motion problems. The linear translation diaphragm flexure provides a low cost and compact means of producing translations. This type of flexure is particularly well suited for applications with circular symmetry, such as a focus mechanism for a lens barrel. The translation of a linear translation diaphragm flexure is linear with respect to applied force. The two axis gimbal diaphragm flexure provides a low cost means of producing two orthogonal rotations. Like the linear translation diaphragm flexure, the rotation of the two axis diaphragm flexure is linear with respect to applied torque.

The expressions in this paper are approximations, and should serve to start the design process. Certain aspects of diaphragm behavior, such as buckling under load, are not easily calculated using closed form expressions. More sophisticated methods, such as finite element analysis, or in some cases model tests, may be necessary to analyze diaphragm flexures in greater detail.

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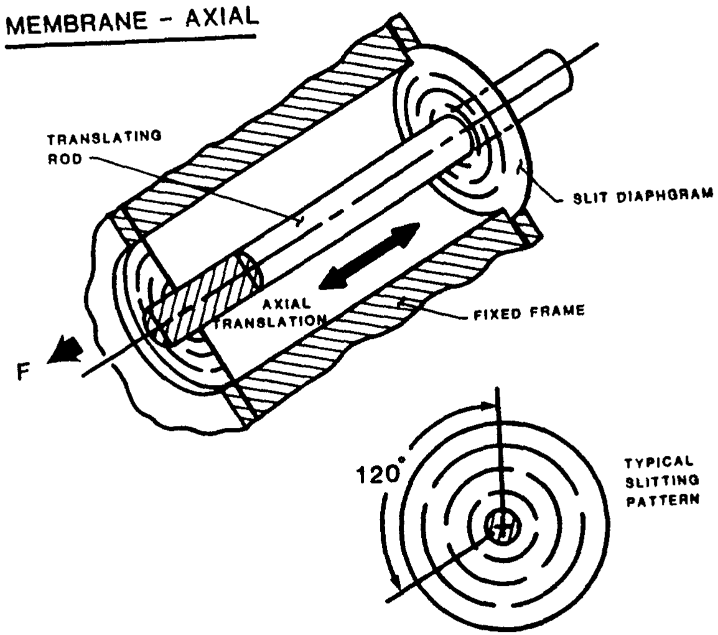

Fig. 1: Linear translation slit diaphragm flexure

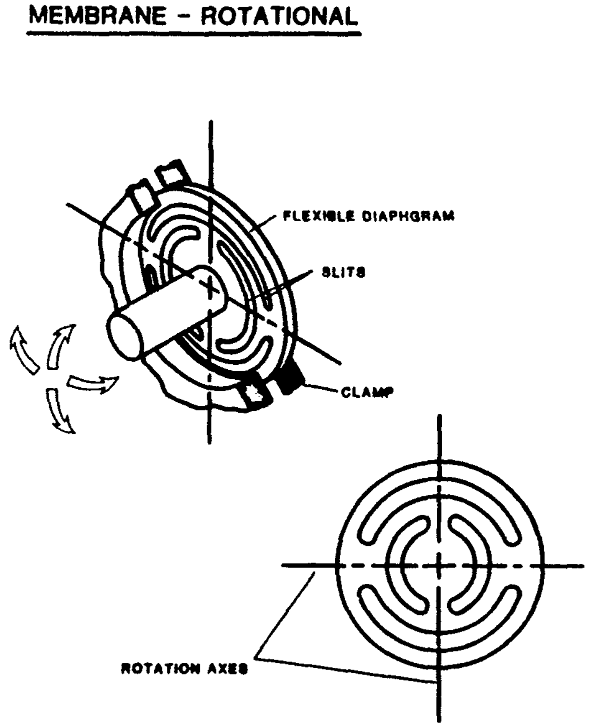

Fig. 2: Two axis gimbal slit diaphragm flexure

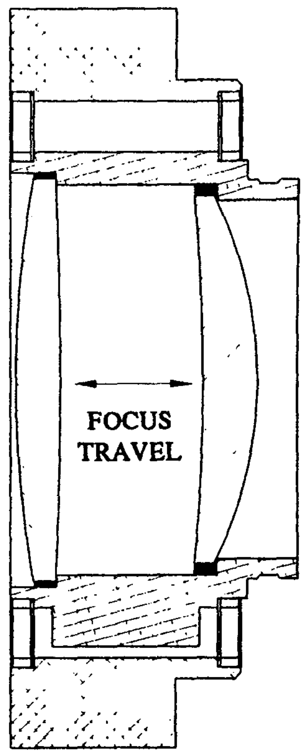

Fig. 3: Cross section through lens cell with two linear translation slit diaphragm flexures providing guidance for focus motion

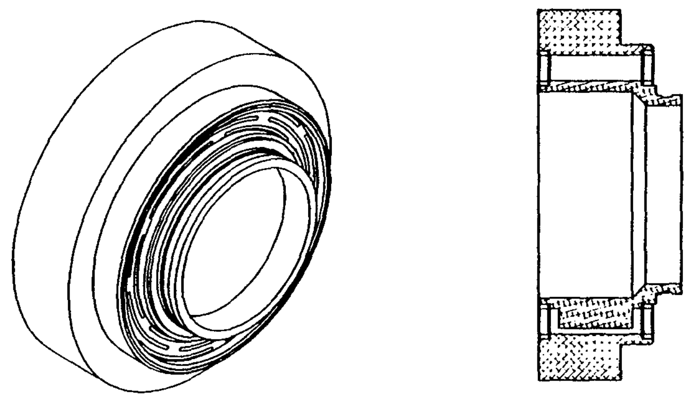

Fig. 4: Isometric and cross section of lens cell using linear translation slit diaphragm flexures

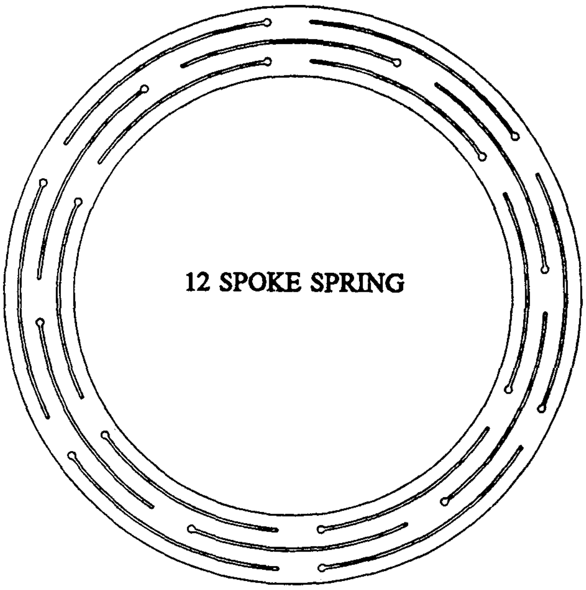

Fig. 5: "12 spoke spring" -- a linear translation slit diaphragm flexure

FLEXURE MATERIAL	$\sigma_{YS}/E \times 10^{-3}$
ALUMINUM ALLOY 1100-H12	1.40
STAINLESS STEEL 17-4 PH COND. H1150-M	4.39
STAINLESS STEEL TYPE 304	1.25
TITANIUM 6AL-4V ELI	7.27
TITANIUM 5AL-2.5Sn ELI	5.94
MARAGING STEEL 18 Ni (250)	9.25
INVAR 36 Ni	4.75
BERYLLIUM COPPER 1/2 HARD	7.14

Table I: Reduced tensile modulus of flexure materials