

Design and construction of an astrometric astrograph

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ABSTRACT

The Optical Sciences Center, University of Arizona, has designed and constructed a unique "red corrected" astrometric astrograph objective lens for the United States Naval Observatory. A five element design, with an integral Schott OG550 filter, was developed to meet the requirement for a 2060 mm focal length, $f/10$ system. The lens provides a nearly zero distortion flat field of 5 by 5 degrees in the sky. A weight limit of 55 kg led to the use of a titanium lens barrel. Assembly tolerances are satisfied through the use of elastomeric subcell mounting of the individual elements, and an adjustable final element. The lens is hermetically sealed and uses a filter/dessicator system to insure the long term cleanliness of the optics.

1. INTRODUCTION

Large field photographic astrometry involves the use of photographic techniques to determine the positions and proper motions of stars in the sky¹. Photographs of the same area of the sky are periodically taken at long intervals and then compared. Shifts in the image positions of stars from photograph to photograph are measured to determine proper motion. Typically, the shifts in stellar position are no more than a few arc seconds. Current accuracy of photographic astrometric techniques is about ± 0.01 arc-seconds².

A lens used for photographic astrometry must have exceptional long term stability and very low distortion to minimize position error in the focal plane. A large plate scale is also desirable. Large plate scales imply relatively small fields of view, requiring large numbers of exposures to cover a large area of sky during a survey. A careful determination of scale versus coverage is therefore necessary in the design of an astrometric lens.

Photographic astrometric sky surveys are almost as old as the use of photography in astronomy³. Special low distortion long focal length photographic lenses used for astrometric sky surveys are called astrometric astrographs. Despite the long history of photographic sky surveys, surprisingly little information exists about the design of previous photographic astrometric lenses. In 1973, Carl Zeiss, Oberkochen, built an advanced astrometric astrograph for the Hamburg observatory⁴. Spectral range of this lens is 530 to 580 nm, with a 2.065 m focal length and 230 mm aperture. Field distortion for this design is below 0.02 arc-sec/deg³.

In 1990 the United States Naval Observatory (USNO) approached the Optical Sciences Center (OSC), University of Arizona, for assistance in building an advanced astrometric astrograph similar to the Carl Zeiss design described above. An optical design similar to that of the Hamburg lens is used as a starting point for the design of this new lens. Following are some of the important specifications for the design:

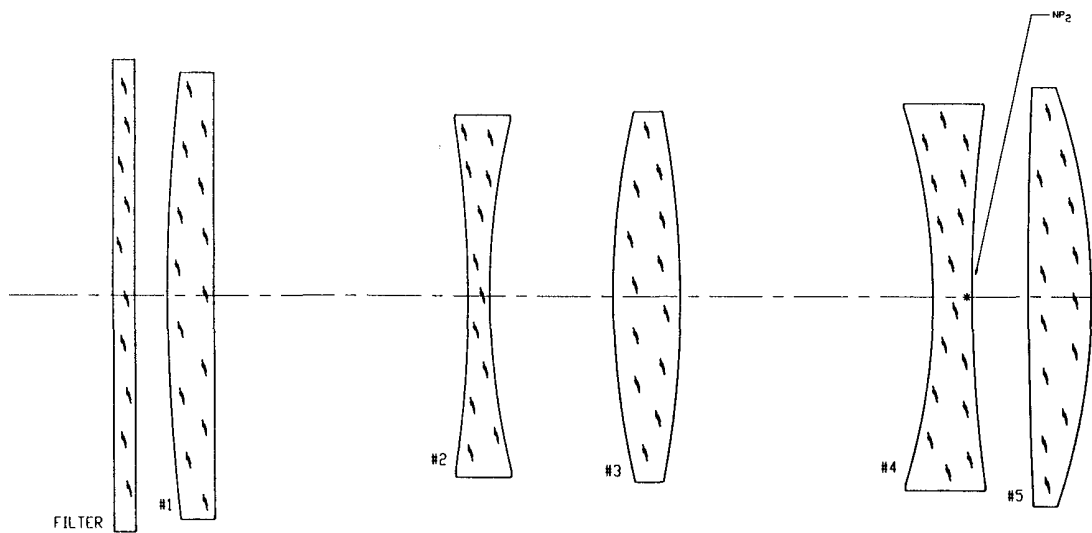
1. Aperture: 203 mm (8 in.)
2. Focal ratio: f/10
3. Pass band: 550 to 710 nm
4. Field of view: At least 8 degrees in diameter
5. Distortion: Mapping shall closely approximate a perfect central projection
6. Image quality: Nearly diffraction limited
7. Weight: 55 kg.

2. OPTICAL DESIGN

Optical design of the lens was performed using both OSLO and CODE V, with the USNO supplied design as a starting point. The high performance required by the design specifications led to the use of two special Schott glass types: LakN16 and KzFSN9. (These glass types are now standard glasses in the Schott catalog.) Other glass types tried in the design were rejected due to poor weathering resistance. The weathering resistance of the KzFSN9 is relatively poor but can be considered acceptable since it is used for internal lens elements. Provided that the lens is hermetically sealed, and the interior kept free of moisture, the KzFSN9 should not be vulnerable to staining with time. An optical filter, made of Schott OG550 glass is positioned in front of the lens.

A variety of configurations were explored during the design in an effort to minimize focus shifts due to temperature changes. The USNO design specification called for a maximum change in focus of 0.1 mm over a 30 minute 3^o C temperature change. This specification could be met by allowing the spacing of the front and rear lens elements to change with temperature. No refocusing is necessary if the spacing is changed by about 1 mm over the 40^o C temperature range. A material with the very high thermal coefficient of expansion of 166 E-6 m/m-K is required to produce the desired change in spacing. In comparison, the thermal coefficient of expansion of aluminum is about 23 E-6 m/m-K. A bi-metallic compensator employing stainless steel and a polyurethane ABS plastic was considered as a means of moving the front and rear elements to eliminate focus shift with temperature. Moving the front and rear elements caused a considerable increase in size, weight and cost of the design. Sealing of the lens became more complex, requiring the use of a special bellows seal at both ends of the lens. Finally, and most important of all, moving the front and rear elements could cause a change in the long term stability of the lens. For these reasons, the final design rejected the use of bi-metallic compensators and accepted the need to refocus the lens as the temperature changes.

The OG550 filter position was another design variable. Front and rear positions were considered. Locating the filter behind the lens affected the color correction. Redesign to minimize the change in color correction is difficult since information about the index and dispersion of filter glasses is not as good as for optical glasses. Locating the filter in front eliminates concern about color correction and allows the filter to be easily removed. Ghost images are a potential problem with a front filter location. Analysis of the ghost image intensity in the focal plane due to the front filter location indicated that a single layer anti-reflection coating would reduce ghost images to an acceptable level of 0.0001 of the intensity of an in focus image. The final, filter in front optical design is shown in figure 1.



DVG. NO.	LENS SURFACE	RAD. OF CURVE	SAG	CLEAR APER.	MEDA. DIA.	EDGE THICK.	CENTER THICK.	VERTEX TO VERTEX	MATERIAL	MELT	TOLERANCES	
											SPACING	CENTRATION
A049104	FILTER	S 1 S 2	-0- -0-	10.200 10.200	11.0000	.500	.500	.7500	DG550		±.001	◎ ±.001
A049105	#1	S 1 cx S 2	43.4900 -0-	.3120 -	9.9213 9.9213	10.4000	.7904	1.1024	LAKN16	B2088/5	±.001	◎ ±.001
A049106	#2	S 1 cc S 2 cc	25.9700 17.0468	.3452 .3306	7.8748 7.8748	8.4400	1.3146	.5118	KZFSN9	B1771/2A	±.001	◎ ±.001
A049107	#3	S 1 cx S 2 cx	19.4327 233.771	.4846 .3978	8.3465 8.3465	8.6258	6.505	1.5328	BAK1	12081/100	±.001	◎ ±.001
A049108	#4	S 1 cc S 2 cc	15.0031 33.7925	.6939 .3023	8.7482 8.7482	9.0200	1.8590	.9068	KZFSN9	B1773/4	±.001	◎ ±.001
A049109	#5	S 1 cx S 2 cx	113.8071 14.9932	.1051 .8198	9.5276 9.5276	9.7800	.5712	1.4961	SK51	B2306/1	±.001	◎ ±.001

*BEFORE FLATS ARE GROUND.

NOTE:

FOCAL LENGTH	81.1024
FRONT FOCAL LENGTH	62.0997
REAR FOCAL LENGTH	78.1880
FIELD OF VIEW	± 4.8 deg.
APERTURE STOP DIAMETER	8.0200
APERTURE STOP POSITION	SURFACE 1
REAR NODAL POINT (NP2)	S1 - #5

* SEE INDIVIDUAL LENS DRAWINGS FOR DETAILED TOLERANCES & SPECIFICATIONS.

SPECIAL NOTES:

WEIGHTS

FILTER	DG550	$\rho = .0995 \text{ lbs./in.}^3$	(4.390 lbs.)
LENS #1	LAKN16	$\rho = .144 \text{ lbs./in.}^3$	(11.565 lbs.)
LENS #2	KZFSN9	$\rho = .108 \text{ lbs./in.}^3$	(5.576 lbs.)
LENS #3	BAK1	$\rho = .115 \text{ lbs./in.}^3$	(7.349 lbs.)
LENS #4	KZFSN9	$\rho = .108 \text{ lbs./in.}^3$	(9.675 lbs.)
LENS #5	SK51	$\rho = .144 \text{ lbs./in.}^3$	(9.889 lbs.)

TOTAL - (48.444 lbs.) ± .1 lbs.

Figure 1. Final optical design of the USNO astrometric astrograph.

Performance of the optical design is very good, with distortion negligible across the field of view, as shown in figure 2. At the edge of a 4.7 degree field, the distortion is about 0.002 percent, or about three micrometers. This amount of distortion is below the 10 micrometer image size at the edge of the field. Both aberrations and diffraction contribute to the final image size. Plots of the actual energy distribution in the focal plane (encircled energy) are given in figure 3. In comparison, a perfect monochromatic lens of $f/10$ at 610 nm would have an 84% energy concentration diameter of about 15 micrometers. This is closely matched for the focus at -0.1 mm, probably because this corresponded to a best focus for the shorter wavelengths.

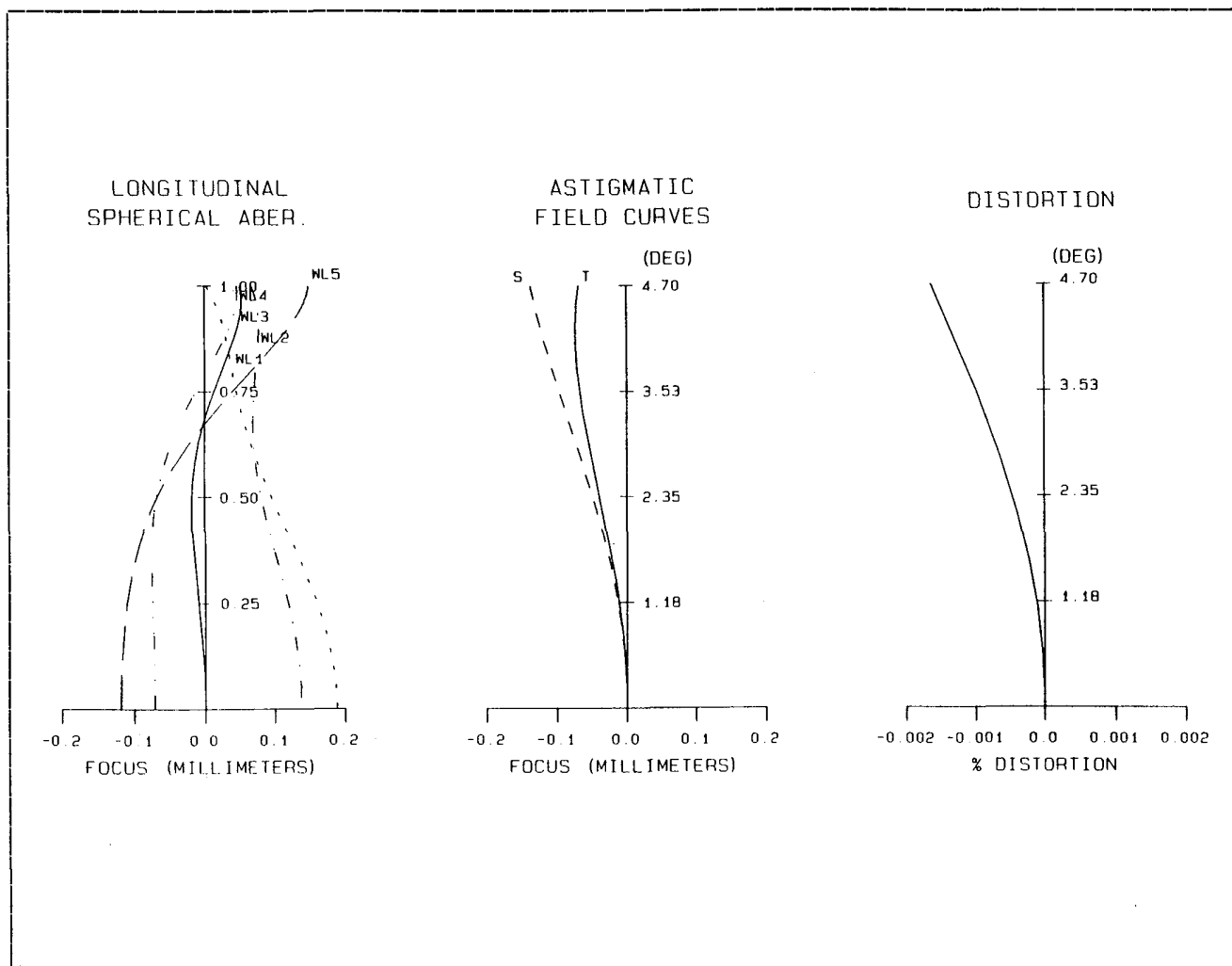


Figure 2. Distortion and other aberrations across the field of view for the final optical design.

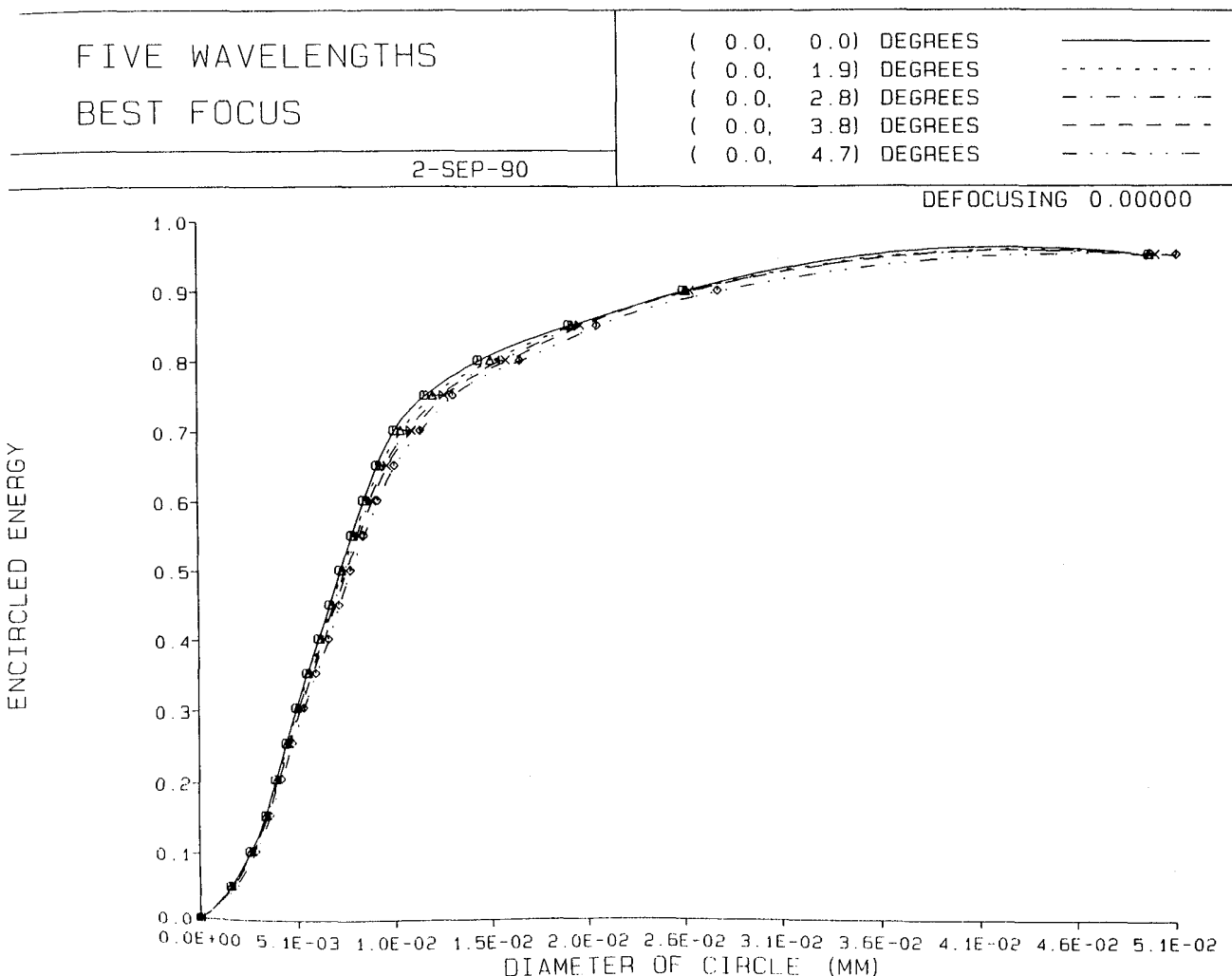


Figure 3. Encircled energy plots at five wavelengths in the final optical design.

Further information about the distortion of the lens design is obtained from a consideration of the Optical Transfer Function. The MTF of the lens for five wavelengths at best focus is shown in figure 4. If the MTF at the edge of the field is evaluated for three wavelengths, and spatial phase shift for some spatial frequency chosen, this value can be used to indicate the actual effect of aberrations and distortion on the centroid of the image. Figure 5 shows a plot of a such a calculation, interpreted in terms of the spatial change in the centroid location by using the spatial phase shift at 72 lines per mm. The change in distortion due to wavelength is quite small. A conventional ray intercept plot is given for the five field points in figure 6.

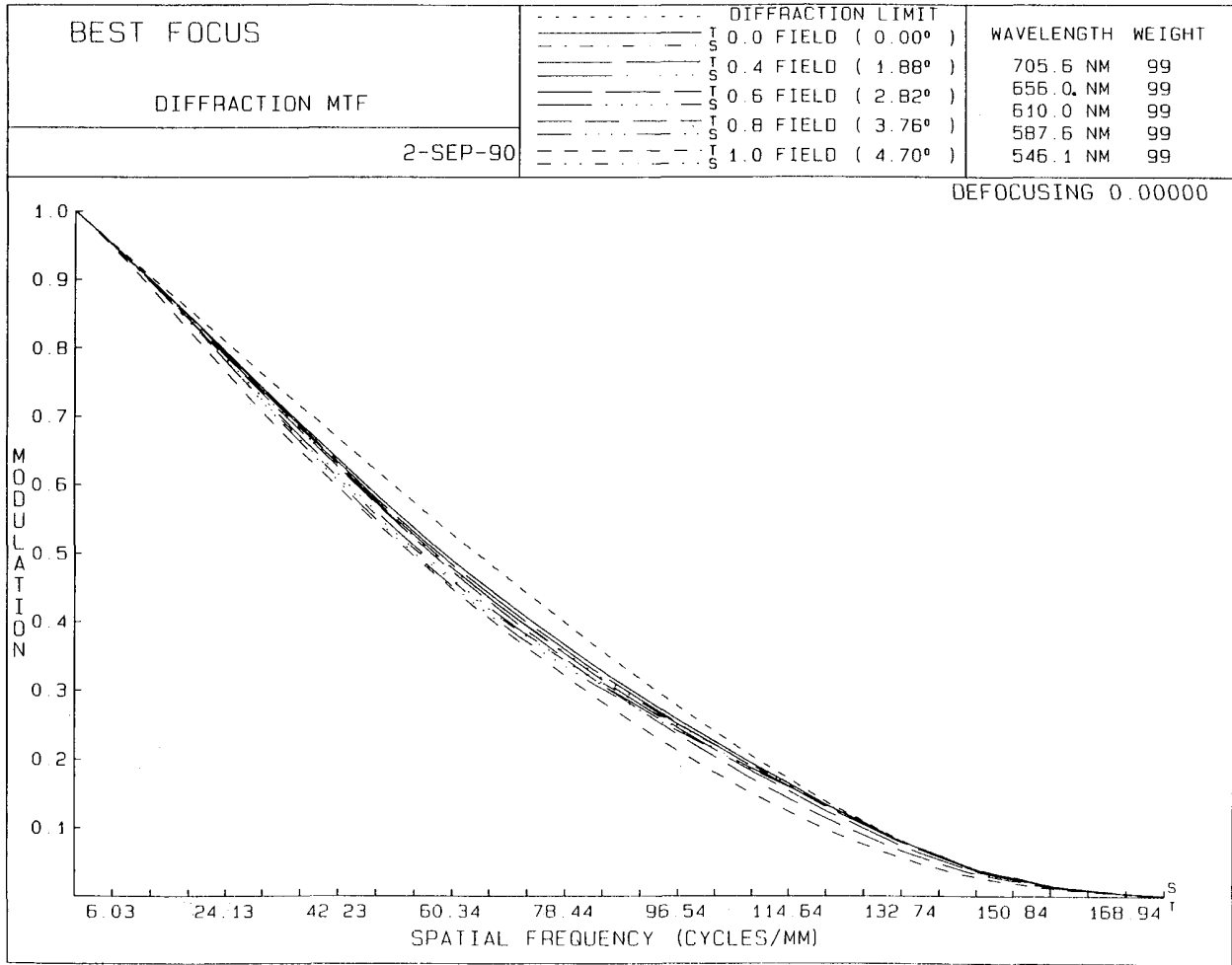


Figure 4. MTF at best focus for five wavelengths in the final optical design.

Distortion versus wavelength From phase shift at 72 l/mm

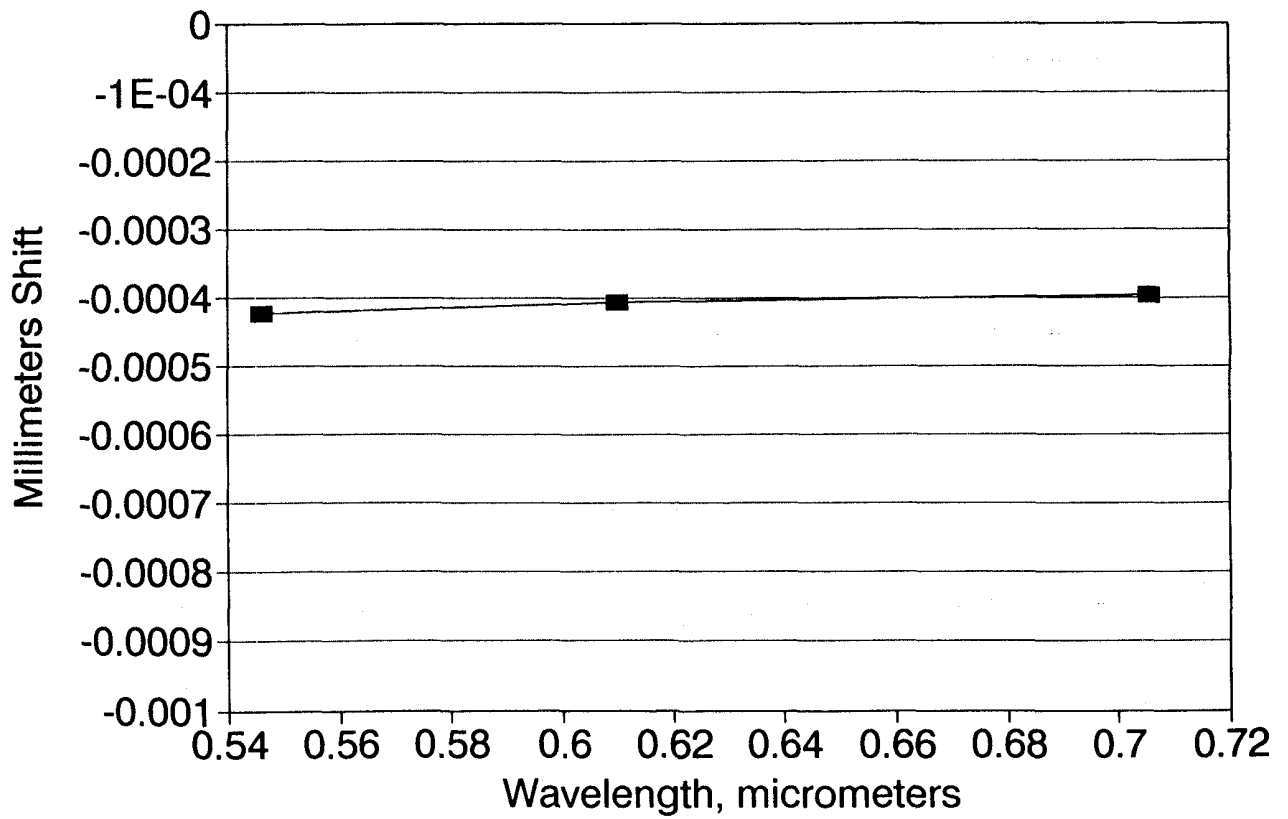


Figure 5. Distortion in the focal plane versus wavelength for the final version of the optical design.

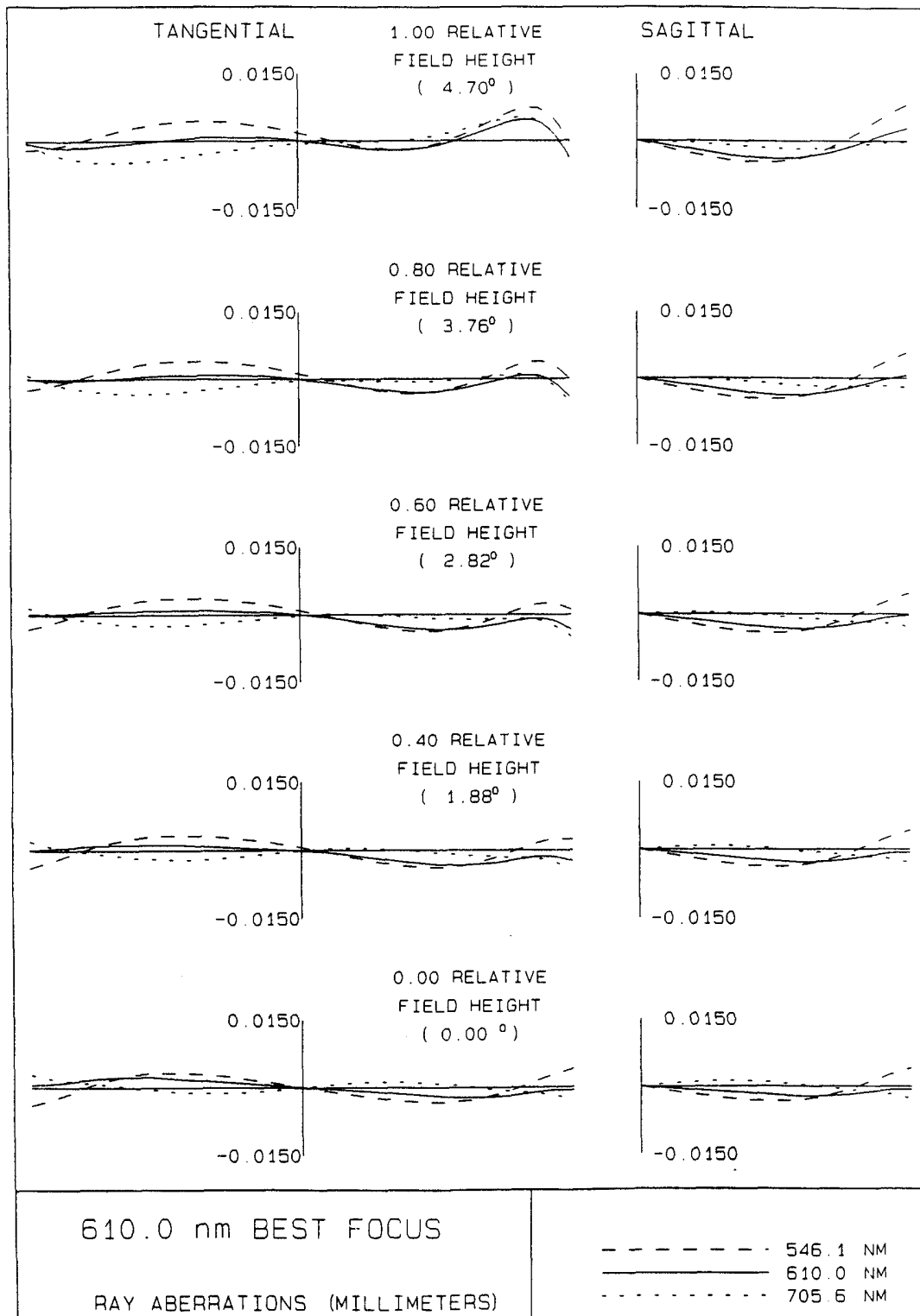


Figure 6. Conventional wave aberration plot at a wavelength of 610.0 nm, for the final optical design of the USNO astrometric astrograph.

Optical tolerances were determined using conventional CODE V techniques. The two most critical tolerances are de-center of the individual elements with respect to the optical axis, and stress birefringence due to mounting stress. De-center specified by the lens tolerances analysis is 25 micrometers. This tolerance is divided between the optical fabrication process, and the mechanical fabricator; each is allowed a tolerance of 12.5 micrometers. Stress birefringence of the entire assembly should not exceed 210 nm. Distributing the stress birefringence tolerance equally among the five elements provides an individual element birefringence tolerance of 42 nm. Vertex to vertex spacing and thickness tolerances are both 25 micrometers.

Initial tolerance analysis indicated that very tight mechanical tolerances would be required to meet the performance specifications. De-center tolerances of about 2.5 micrometers are required. It is difficult and expensive to achieve such tight optical tolerances. Further analysis indicated that the mechanical tolerances could be relaxed if the position of the last element was changed during assembly to compensate for any residual errors.

3. MECHANICAL DESIGN

Mechanical design of the lens barrel assembly of the astrometric astrograph is determined primarily by the 55 kg weight specification and by the optical assembly tolerances. Additional concerns include long term stability, resistance to corrosion, ease of assembly, hermetic sealing, and cost. Another issue is compatibility with an existing tube assembly and mount. The USNO ultimately decided to modify the existing tube assembly when it became apparent that compatibility with the existing tube assembly would increase the weight to over 66 kg. Modifying the tube assembly is lower in cost than building a compatible lens barrel assembly.

Initially, 17-4 PH stainless steel was the only material considered for the lens barrel. This material has superb long term stability⁵, is very resistant to corrosion, and is relatively easy to machine. Use of this material led to a severe overweight condition. Weight constraints led to considering aluminum and titanium, as well as stainless steel as alternate materials.

A simple GIFTS finite element model was used to study barrel deflection due to different materials. The barrel, and lens cell within the barrel were approximated as cantilevered hollow cylindrical beams of varying thickness and diameter. The large, heavy optical lenses were assumed to be non-structural and were approximated as point masses placed at each of the lenses center of gravity. The barrel was supported at the large mounting flange at one end of the assembly. An end load of 45 N was placed at the free end of the barrel to approximate the effects of an end baffle and dew cap assembly. The self weight induced stress and deflection of this finite element model was evaluated for designs employing stainless steel, aluminum, and titanium. Results of a typical finite element run were given both numerically and graphically. Figure 7 is a typical result, and shows the deflected shape of the barrel.

Structural analysis showed that the use of stainless steel produced an overweight design, while the use of aluminum produced an unacceptable deflection in the end of the barrel. Only titanium simultaneously satisfied weight and deflection specifications. Titanium is reputed to have poor long term stability. Dimensional instability in titanium is associated with relatively high stress levels, typically about 0.85 of yield⁶. At the very low stress levels of about 400 kPa found in the barrel, titanium is very stable⁷.

There was concern that the stiffness of the mounted lenses would influence the bending behavior of the barrel. Finite element runs were made with and without the lenses as structural elements. The results, given in table 1, indicate that there is a maximum error of about 4 percent in neglecting the structural effect of the lenses. This error is about the same as the error in a typical finite element analysis, and can be neglected to simplify the model.

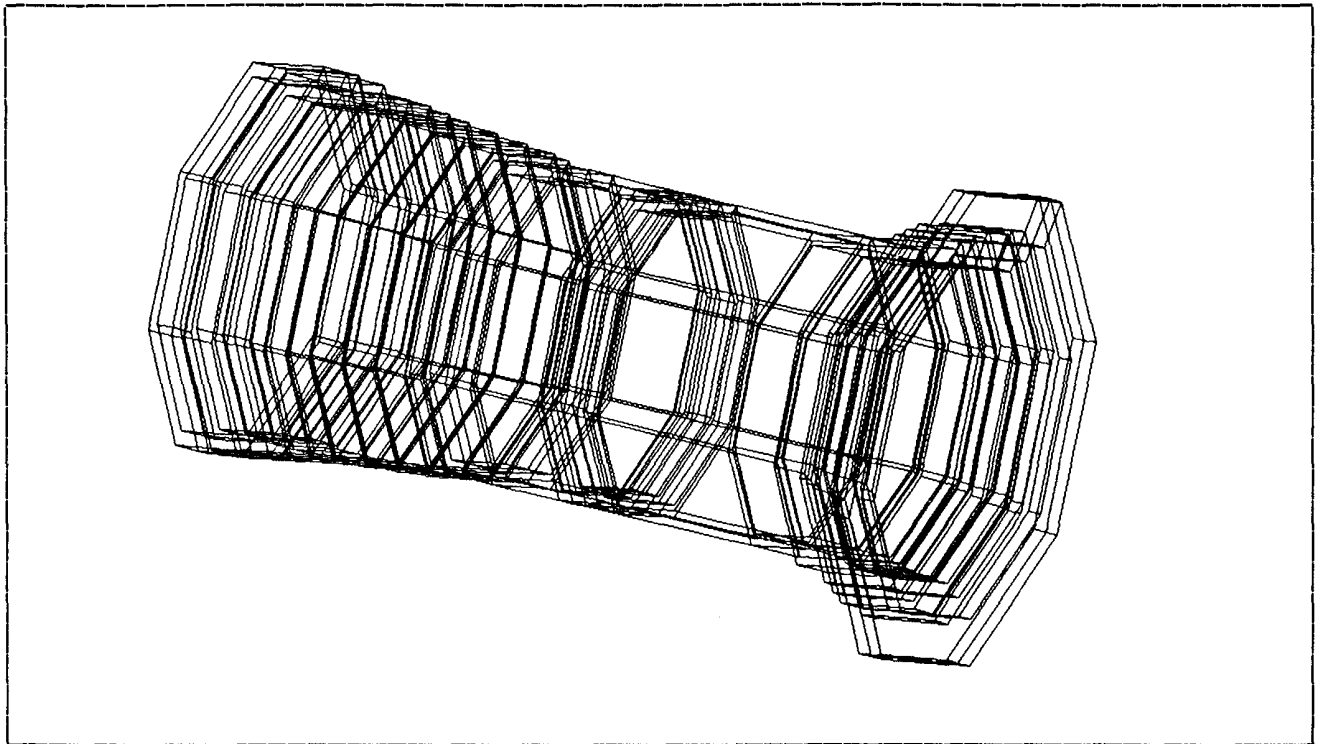


Figure 7. A typical GIFTS finite element model of the USNO astrometric astrograph lens barrel.

CASE	CELL THICKNESS (IN)	WEIGHT (LB)	MAX. DISPLACEMENT (POINT 1) (IN)	MAX. PRINCIPLE STRESS (PSI)
1) Cell without lenses	.15	33	-4.273×10^{-5}	18
2) Cell with lenses as structural elements	.15	91	-1.403×10^{-4}	57
3) Cell with lenses as point loads	.15	91	-1.354×10^{-4}	53
4) Cell without lenses	.25	54	-4.210×10^{-5}	18
5) Cell with lenses as structural elements	.25	112	-9.912×10^{-5}	41
6) Cell with lenses as point loads	.25	112	-9.613×10^{-5}	38

Table 1. Lens barrel deflections for finite element models with and without the use of lens elements as structural components.

Conventional lens barrels employ mechanical seats to locate the lens and retainers (or spacers) used to hold the lens in position. Experience at OSC and elsewhere indicates that a minimum radial clearance of 25 micrometers between the edge of the lens and inner diameter of the cell bore is required to allow assembly of the lens into the barrel. Using conventional fabrication methods, a diameter tolerance of ± 12.5 micrometers can be produced on both lens and barrel bore. Assuming a minimum lens diameter, and maximum cell bore diameter, and adding the required assembly clearance, the maximum clearance between the edge of the lens and cell could be as great as 75 micrometers. This tolerance is the lower limit of what can reasonably be expected, since it is quite difficult to produce lens elements of the necessary size to a diameter tolerance of 12.5 micrometers. Unfortunately, a centering tolerance of 75 micrometers is six times larger than the optical assembly tolerance of 12.5 micrometers.

A spherical lens clamped between two rings will tend to self-center. Self-centering partially compensates for the clearance between the edge of the lens and the inner diameter of the lens barrel. Self-centering accuracy is limited by the friction between lens and seat, and is best when the lens surfaces are strongly curved. A method developed by Zschommler was used to calculate the self-centering tolerance of the each lens element⁸. The radius of curvature of the first element is too large for effective self-centering, elements two, three, and five self-center to 50 micrometers, and element four self-centers to 25 micrometers. All of these self-centering tolerances exceed the desired de-center assembly tolerance⁹.

Inability to achieve the required tolerance using a conventional cell led to the use of an elastomeric mounting technique. In elastomeric mounting, each individual element is centered in a subcell. The subcell is interference fitted into the barrel. Using this method, the error in centering is caused by the residual error in centering the lens element relative to the subcell, and the out of roundness condition of both subcell and barrel. Using precision machining techniques, it is possible to obtain cells and barrels that are round to better than 2.5 micrometers, while interferometric methods of aligning the lens element to the subcell achieve centering tolerances of 2.5 micrometers or better. Normally the lens is held in the subcell with a semi-flexible adhesive. Use of a semi-flexible adhesive isolates the lens from expansion or contraction of the cell, and from stress due to press fitting the subcell into the barrel. Elastomeric assembly is widely used for high precision lens assemblies, and is employed at OSC for projects of comparable tolerance.

A possible source of error in an elastomeric mounting is compression of the elastomer between lens and subcell under the weight of the lens. Both the type of adhesive and required adhesive thickness must be selected to keep self-weight deflection within tolerance. Deflection decreases as the radial thickness of the adhesive is decreased, and as the adhesive stiffness is increased¹⁰.

Although decreasing elastomer thickness decreases self-weight deflection of the lens relative to the subcell, it increases stress induced birefringence. Stress from press fitting the subcell into the barrel is passed through the elastomer into the lens; increasing the elastomer thickness reduces the stress in the lens¹¹. Reducing lens stress reduces stress birefringence. A balance between self-weight deflection and stress birefringence is required in selecting adhesive and radial elastomer thickness.

A standard locational interference fit between subcell and barrel bore of 58 micrometers was assumed. Stress in the lens elements due to the interference fit and self-weight deflections for three different adhesives were calculated. Table 2 shows the results of these calculations. A Dow Corning Adhesive 93-500 Thixotropic was selected, with a radial adhesive thickness of 5 mm to satisfy both stress and deflection requirements. Since the barrel is sealed, a low outgassing ("space qualified") adhesive is required.

Elastomer # 1
Milbond
E = 23E3 psi
v = .43

Elastomer # 2
Dow Corning
E = 500 psi
v = .50

Elastomer # 3
3M-EC2216
E = 100E3 psi
v = .43

Δ = Mechanical deflection of the elastomer (in)

σ = Stress induced in glass by interference fit (psi)

Δs = Optical path difference due to the stress in the glass (in)

Element	Elastomer # 1			Elastomer # 2			Elastomer # 3		
	Δ	σ	Δs	Δ	σ	Δs	Δ	σ	Δs
1	2.1E-6	233	3.3E-6	9.1E-5	5.5	7.7E-8	4.8E-7	1012	1.4E-5
2	1.2E-6	147	1.7E-6	5.4E-5	3.5	4.0E-8	2.8E-7	639	7.4E-6
3	1.4E-6	175	4.8E-6	5.9E-5	4.1	1.1E-7	3.1E-7	759	2.1E-5
4	1.3E-6	233	4.8E-6	5.7E-5	5.5	1.1E-7	3.0E-7	1012	2.1E-5
5	1.8E-6	233	3.5E-6	7.8E-5	5.5	8.3E-8	4.1E-7	1012	1.5E-5

Table 2. Stress induced in the individual lens elements and optical path difference due to interference fit of subcell into barrel for different types of adhesives.

Elastomeric mounting, and the use of subcells is an aid in adjusting the position of the fifth lens element to remove residual error during assembly. As discussed above, the fifth lens element must be moved along the optical axis and in a plane perpendicular to the optical axis to remove errors built up in the other elements. Once this adjustment is made, it must be positively locked. An aerospace adjustment technique called liquid pinning is used in the subcell for the fifth element. The subcell for the fifth element is provided with three axial holes. Each axial hole is a very loose fit to a pin fixed to the main barrel. The radial clearance between pin and hole permits a centering adjustment to be made. After the position of the subcell is adjusted, the space between pin and hole is filled with a hard, high strength epoxy adhesive. The adhesive fixes subcell to barrel, and provides a durable lock for the adjustment.

Maintaining optical quality of the lens over the design lifetime of 25 years requires that the interior of the lens barrel be kept free of dirt and moisture. A hermetically sealed barrel protects the optics from contaminants. A change in external pressure creates a pressure differential between the sealed barrel interior and the outside. A pressure differential induces stress in the lens elements at both ends of the barrel, and may move these elements out of position. A vent is required to eliminate such pressure differential effects. Unfortunately, an air vent may allow moisture and dirt to enter the barrel.

Placing a combined filter and dessicator on the barrel vent allows the barrel to "breathe" and accommodate external pressure changes. At the same time, the filter and dessicator insure that the internal volume of the lens barrel is maintained in a clean and dry condition. Particles of over 0.5 micrometer diameter are trapped by the filter. Regular cleaning of the filter and replacement of the dessicator are required in service. The service intervals depend upon external conditions. Both filter and dessicator were designed and built by AGM Container Controls, of Tucson, Arizona.

A cross section of the final barrel assembly is shown in figure 8.

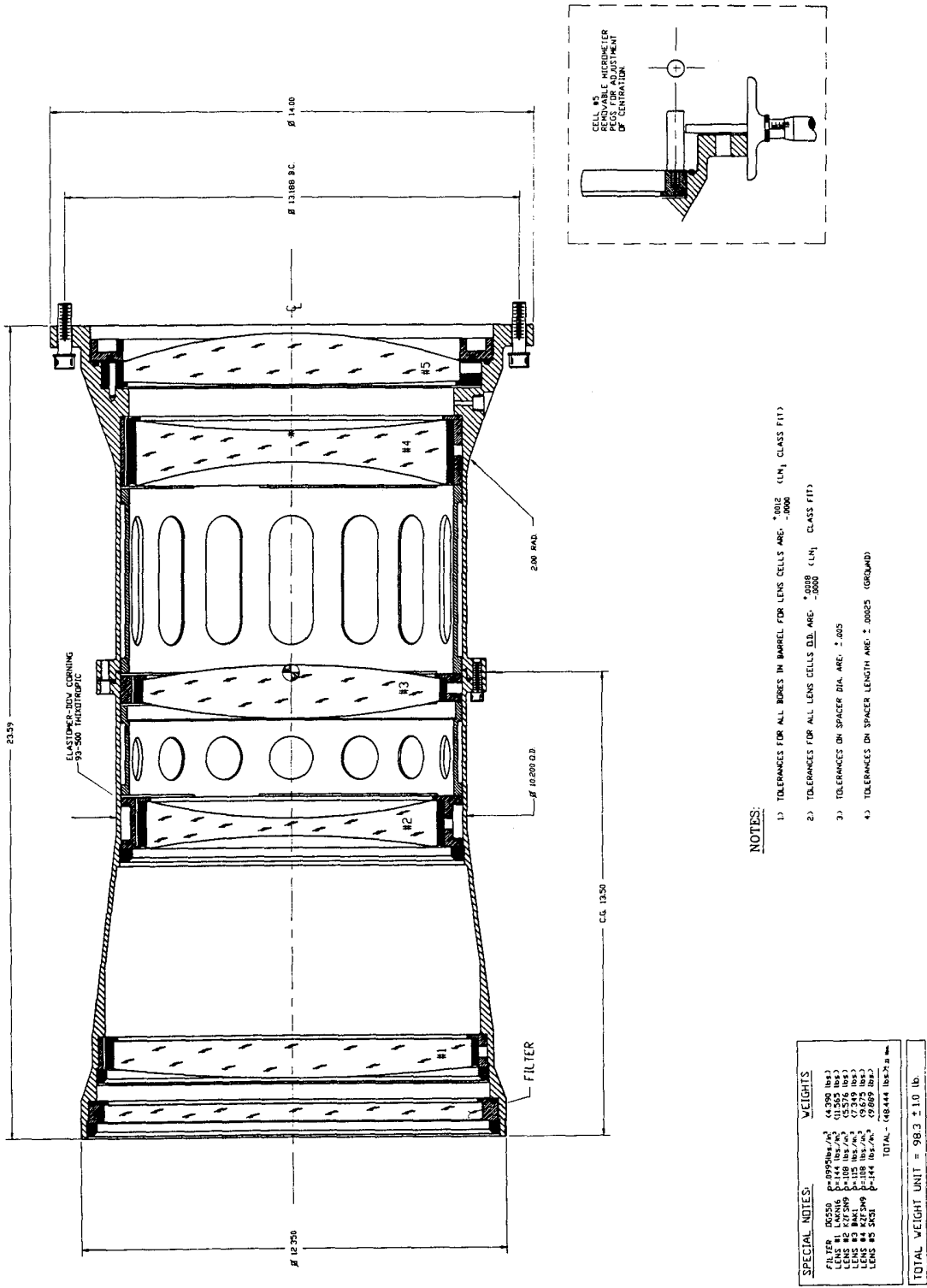


Figure 8. Cross section of the final design of the lens barrel assembly for the USNO astrometric astrograph.

4. FABRICATION

Fabrication of both lens elements and mechanical components followed established practice. As insurance against catastrophe during fabrication of the lens elements, a spare set of glass blanks was held at Schott for a period of one year. No other spares were considered necessary.

All lens elements were tested against test plates. A departure from conventional practice was the use of an interferometer and high precision linear encoder to control the radius of the test plates. This is a much more precise method of controlling test plate radius than a conventional spherometer. Fabrication of the lens elements was performed at the OSC optical shop; while the OG550 filter was made by Zygo, of Middlefield, Connecticut.

Machining the lens barrel was beyond the capacity of the OSC instrument shop, and was sub-contracted to Vonkampen, in Los Angeles, California. The main barrel was machined from a solid billet of 6Al-4V titanium, weighing about 550 kg. The particular titanium alloy used for the barrel was selected on the basis of cost and availability of information on performance. Optimum roundness of the barrel during machining was obtained through the use of modified collet type chucks.

Bare titanium can not be interference fitted to another bare titanium part; the two parts will seize and gall¹². Use of a proprietary titanium coating eliminates this galling and permits free assembly¹³. This special TiodizeTM coating (available from Tiodize Co, Inc., Huntington Beach, California) also provides a dark gray finish for the titanium to help reduce glare due to stray light.

Final assembly of the lens requires adjustment of the fifth lens element. A large parabolic mirror provides collimated light for this adjustment. It is anticipated that further adjustment may be necessary after the lens is placed into use. Precision measurement of stellar images is a possible source of information for this adjustment.

5. CONCLUDING REMARKS

The Optical Sciences Center, University of Arizona is in the final stages of fabrication and assembly of the United States Naval Observatory astrometric astrograph (known by USNO as the "red lens" due to its spectral range). Although it appears that the design is a success, several current problems deserve comment.

Experience at OSC indicates that obtaining even very simple single layer anti-reflection coatings on lens elements of the size used in the astrometric astrograph is extremely hazardous. No U.S. coater is prepared to guarantee against breakage and accept responsibility for damage to a lens element. Insurance for elements of this size costs about half the fabrication cost of the element. Some of the risk and insurance cost could be offset by making two sets of elements. Making a second set of elements at the same time as the first set is usually significantly less than twice as costly as making a single set of elements.

"Space qualified" adhesives are relatively uncommon and expensive. Conventional "low outgassing" semi-flexible adhesives normally have an unacceptable level of contaminant production. Enough special adhesive to assemble a lens barrel of this type can cost as much as one of the lens elements.

The USNO astrometric astrograph is an example of how the optical designer must work closely with the opto-mechanical engineer to produce a viable design. Excellent performance, with a distortion below diffraction across an eight degree field is achieved with this design. Use of titanium reduces the weight of the barrel to about the same weight as the optical elements. Elastomeric subcell mounting coupled with an adjustable element permit very tight assembly tolerances to be met. It is hoped that this brief description may be an aid to future designers.

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