

Methods for measuring a lens focal length

December 2011

Sophie Morel

OPTI - 521: Tutorial

Introduction

The focal length is one of the most important parameter of a lens. Sometimes the focal length is slightly different from the datasheet value given by the manufacturer, having an impact on the predicted optical system performances. Many methods have been elaborated to measure with more or less accuracy a lens' focal length. Some methods are easy to implement, other require more material and analyses, with more accurate measurements.

- By definition, the effective focal length EFL is the distance between the rear principal point P' , and the rear focal point F' of the lens.
- The back focal length BFL is the distance between the rear vertex V' of the lens, and the rear focal point F' .
- The front focal length FFL is the distance between the front focal point F and the front vertex V .

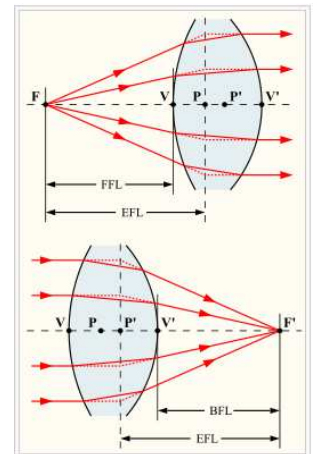


Figure 1 : focal length definition

Measuring the back focal distance is usually easy. Determining the distance between P' and V' is a bigger issue. Besides, the result accuracy depends on several parameters, such as the glass index inhomogeneity, surfaces errors, radiuses of curvature error... The lens application defines the accuracy needed on its focal length knowledge.

This tutorial first describes in detail methods which are easy to implement in student labworks sessions. The last methods are usually more accurate, but require more equipment and analyses.

1. Quick estimation

The easiest and fastest way to roughly evaluate a lens focal length is to hold it under a ceiling light. A projection screen (the floor, or the surface table) is under the lens. The optical element is moved vertically back and forth until a sharp image of the ceiling light is projected on the screen. The distance between the lens and the projection surface gives an idea of the focal length. Of course, there is no precision with this technique, but it can be used to check the results obtained with other methods.

2. $y'/\tan(\theta)$ method

a. Principle

This method uses the paraxial optic formula which relates the focal length with the image height and the angle of incidence:

$$y' = -f \cdot \tan(\theta) = f' \cdot \tan(\theta')$$

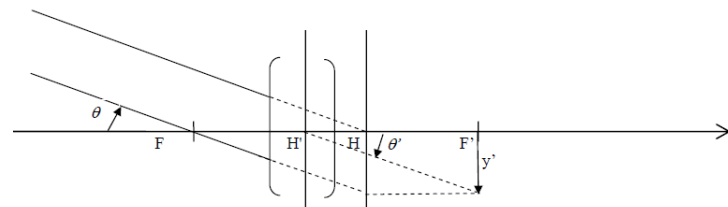


Figure 2: $y' = f' \cdot \tan(\theta)$

Assuming that the lens is in the air, its nodal points are coincident with its principal points, and the angle in the image space is equal to the angle of incidence in the object space: $\theta = \theta'$.

b. Measurement

Here is the setup drawing:

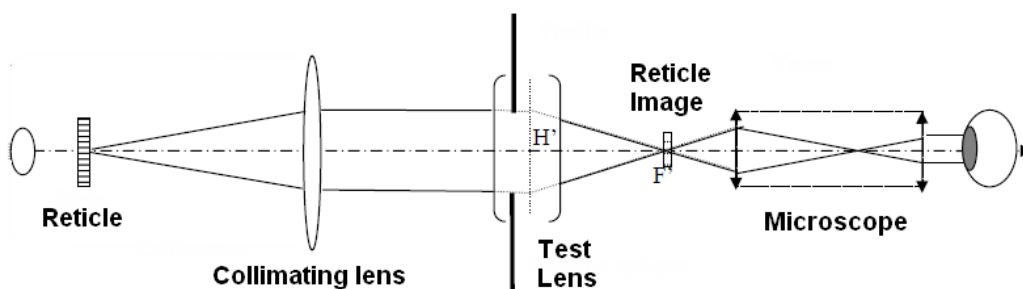


Figure 3: $y'/\tan(\theta)$ setup

The reticle is located at the front focus of the collimated lens. It is illuminated by a light source. Thus, the reticle appears to be at infinity for the test lens, and its image is located at the rear focal plane of the lens. The angular size of the reticle is known with high accuracy.

The observer measures the magnified reticle image size y'' with the microscope. The reticle image size is y''/m , where m is the microscope magnification. It depends on the objective and eyepiece characteristics.

The focal length is deduced from: $f = -\frac{y'}{\tan(\theta)} = -\frac{y''}{m.\tan(\theta)}$

c. Uncertainty analysis

The result uncertainty depends on the accuracy with which the angular size of the reticle is known. It is also related to the reading uncertainty. Measurements should be repeated several times, and the standard deviation Δx of the set of measurement is computed. Assuming that all sources of error are independent, error estimation is obtained with the root sum square technique:

$$\Delta f' = f' \sqrt{\left(\frac{\Delta y''}{y''}\right)^2 + \left(\frac{\Delta \theta}{\theta}\right)^2 + \left(\frac{\Delta gy}{gy}\right)^2}, \text{ and } \Delta gy = gy \sqrt{\left(\frac{\Delta y''}{y''}\right)^2 + \left(\frac{\Delta y'}{y'}\right)^2}$$

3. Nodal slide method

a. Principle

Let be assumed that the lens is in the air. Thus, its rear principal point is superimposed with its rear nodal point.

The nodal slide method uses the property that the image is not shifted when the lens is rotated about its rear nodal point.

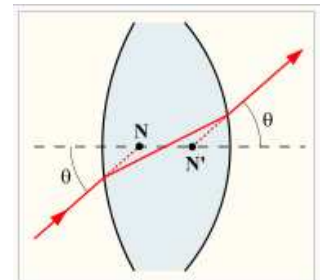


Figure 4: nodal points

b. Measurement

Here is a setup example:

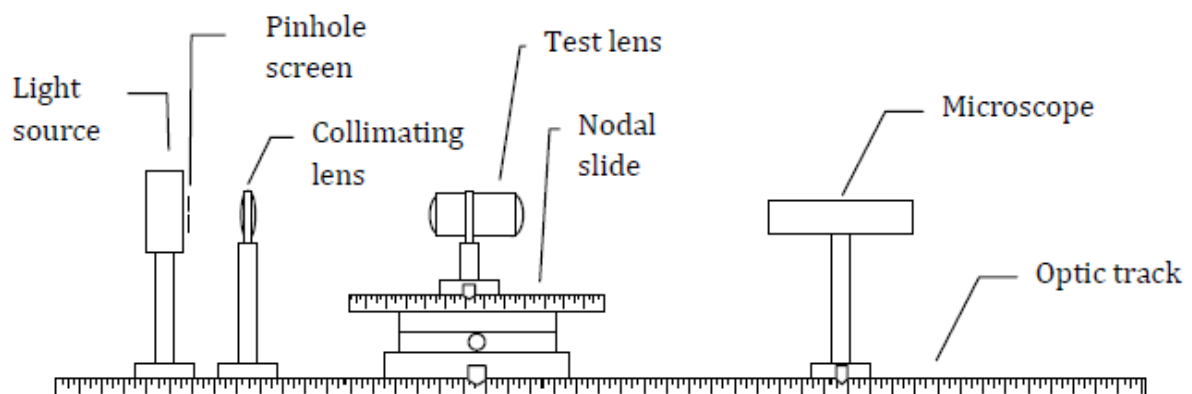


Figure 5: nodal slide setup

The lens focal length is deduced from two configurations:

- The lens rear vertex is on the rotation axis, and the microscope is positioned such that it focuses on this rear vertex.
- The lens nodal point is on the rotation axis, and the microscope focuses on the rear focal point. The lens is at the right position if the image does not shift when the lens is rotated.

The microscope position is marked for both configurations, and the lens focal length results from the difference in position of the microscope.

c. Uncertainty analysis

The uncertainty of this measurement is mainly due to the microscope and lens position error. It also depends on the accuracy of the position reading device. The depth of focus depends on the microscope objective aperture, because of diffraction:

$\Delta z_{diff} \sim \frac{\lambda}{NA^2}$, where $NA \sim \frac{1}{2F/\#}$ is the microscope objective's numerical aperture.

A short focal length lens has a short depth of focus, and position reading is more precise than with a longer focal length lens. The observer should repeat the measurements many times in order to evaluate the positioning error. A root sum square of the different sources of errors also gives an estimation of the measurement uncertainty, since all sources of error are independent.

4. Cornu method

a. Principle

This technique is based on Newton's formula: $f \times f' = \overline{FA} \times \overline{F'A'} = \overline{F'B} \times \overline{FB'}$, Where A is the front vertex of the lens, A' is the image of A through the lens, B is the rear vertex, and B' is the image of B through the lens.

b. Measurement

- The first step is to measure the reticle image position which is located at the test lens focal plane. Then the microscope is translated in order to focus on the lens rear vertex B, and on the image of the front vertex A'.

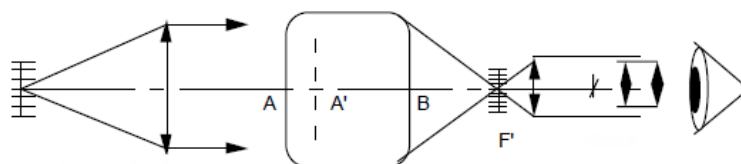


Figure 6: Cornu method first measurement.

The microscope position is marked for each measurement. At this stage, $\overline{F'B}$ and $\overline{F'A'}$ are known.

- The lens is flipped, and similar measurements are performed, in order to locate the front focal point F, the front vertex A, and the image of the rear vertex B'.

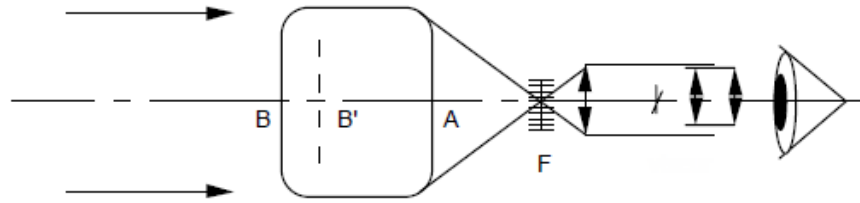


Figure 7 : Cornu method second measurement

Now \overline{FA} and $\overline{FB'}$ are known, and Newton's formula can be used to calculate the lens focal length.

c. Uncertainty analysis

Sources of error are the same as with the nodal slide method:

- Diffraction
The depth of focus is related to the diffraction such that: $\Delta z_{diff} \sim \frac{\lambda}{NA^2}$, where NA is the microscope objective's numerical aperture.
- Reading error

5. Moiré deflectometry

Interference between two waves diffracted by two different gratings creates a Moiré pattern.

For a converging lens, two gratings are placed between the lens and its rear focal plane.

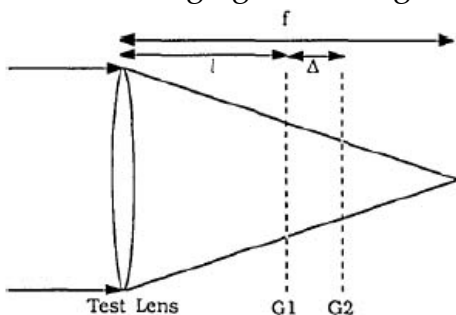


Figure 8: Moiré deflectometry

It can be shown that the focal length is functions of the second grating pitch p , the aperture size a , the distance between the two gratings Δ , and the number of observed fringes N : $f = \frac{a\Delta}{pN}$

The error measurement comes from the counting of fringes. Assuming that $\frac{1}{2\pi}$ of a fringe is be resolved, the focal length can be measured with about 0.8% accuracy, according to the authors of this method.

6. Talbot interferometry

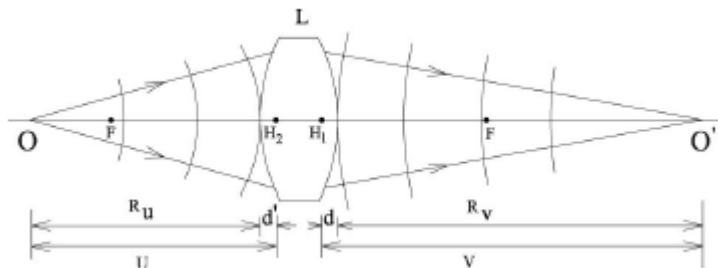
This technique uses the Talbot effect: the image of a coherent wave with 2D periodic amplitude distribution, incident upon a diffraction grating, is regularly repeated. The distance between each self-image is the Talbot length Δ . If the wave interferes with another wave diffracted by a grating with a different spatial period, it creates a Moiré pattern. This is Talbot interferometry.

The self-images of a diffracted wave are imaged by the test lens. Another diffraction grating is placed in the lens image space, creating a Moiré pattern for each image with a different spatial period of the fringes.

It can be shown that the lens focal length is related to the distance between different Moiré images. According to the author, this method gives the focal length value with less than 1% accuracy.

7. Fizeau interferometry

The length focal length is calculated using the following formula:



$$\frac{1}{U} + \frac{1}{V} = \frac{1}{f}$$

U and V are measured from the principal planes. R_u and R_v are measured from the lens vertices.

Figure 9: Fizeau interferometry method: definition of the parameters of interest

If the object O is close to the front focal point, the beam coming out of the lens is almost collimated and R_v is very large. Thus, $d \ll R_v$, and one can approximate $V \sim R_v$.

R_v is measured for two positions U_1 and U_2 , using a Fizeau interferometer.

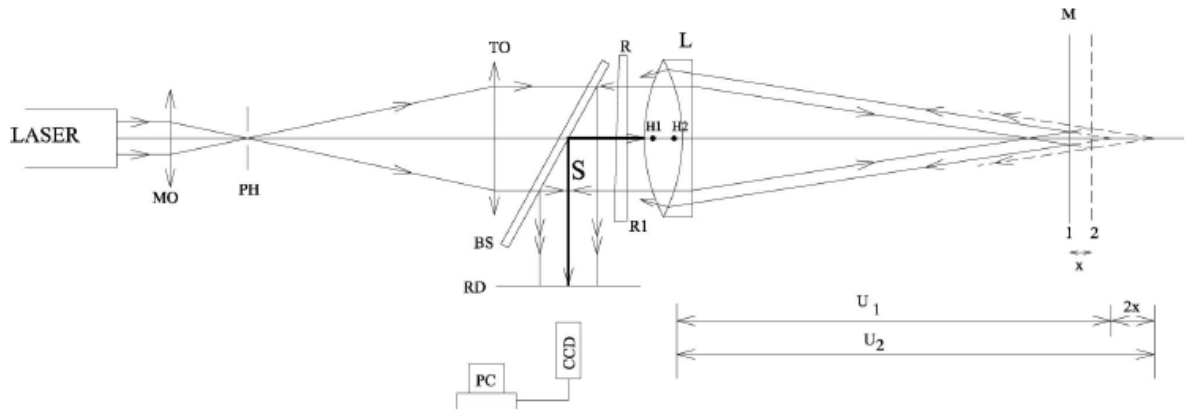


Figure 10 : Fizeau interferometry setup

The reference beam reflects at R, L is the test lens, and the mirror M is placed slightly away from the lens focal point, at distance U_1 . Measurements are made for two positions of the mirror. The mirror displacement x is known. It results in $2x$ displacement of the virtual object: $U_2 - U_1 = 2x$

The Gaussian lens formula is applied for these two positions:

$$\frac{1}{U_1} + \frac{1}{Rv_1} = \frac{1}{f}$$

$$\frac{1}{U_1 + 2x} + \frac{1}{Rv_2} = \frac{1}{f}$$

$$\text{And } U_1 = -x + \sqrt{x^2 + 2x \frac{Rv_1 \cdot Rv_2}{Rv_1 - Rv_2}}$$

Circular fringes are observed. The wavefront radius Rv_1 is related to the spacing between the n^{th} and $(n+p)^{\text{th}}$ fringes, and the distance S between the lens vertex and the screen:

$$Rv_1 = \frac{D_{n+p}^2 - D_n^2}{8p\lambda} + S$$

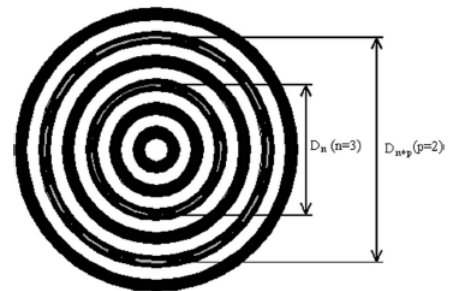


Figure 11 : interference pattern

Knowing U_1 and Rv_1 , the focal length can be computed using the Gaussian lens formula.

The measurement error is about 2.5% according to the authors. It depends mainly on the error in measuring dU and the fringes diameter.

8. Shack-Hartmann wavefront analysis

The Shack-Hartmann wavefront sensor uses a lenslet array to measure the wavefront slope coming from the optical system under test. Polynomial expansion can be used to retrieve the corresponding wavefront curvature. For this method, a Shack-Hartmann sensor is used to determine the radius of curvature R of the wavefront coming out of the test lens.

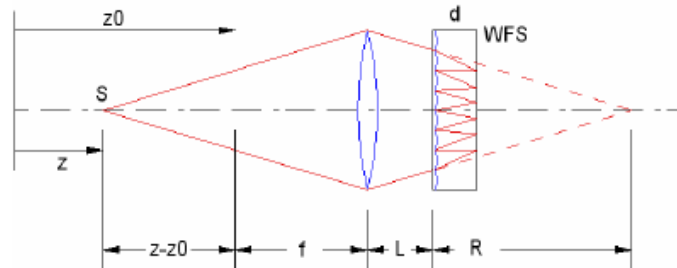


Figure 12 : Schack-Hartmann wavefront sensor

Z is the source position, Z_0 is the source position for a collimated beam, L is the distance between the wavefront sensor and the principal plane of the lens. The focal length is deduced from the following equation:

$$\frac{1}{f} = \frac{1}{f + Z - Z_0} + \frac{1}{R + L}$$

The lens power is measured for different positions of the source.

The author of this method claims that routinely measurements are made with less than 0.5% accuracy.

Conclusion

This tutorial describes methods for measuring a lens focal length. Most of them use paraxial optic formula (Gaussian equation, Newton equation,...) Others use diffraction theory with gratings. Measurements are performed with a microscope translated on an optical rail, or by using interferometers for more precision. All techniques can be applied to positive or negative lenses, although it may require some adjustments.

This list of methods is not all-comprehensive, and probably many other techniques have been and will be developed.

References:

- « Polycopié de Travaux Pratique d'Optique 1^{ère} année 1^{er} semestre », Lionel Jacubowicz, Institut d'Optique
- « Measurement of lens cardinal points and planes using a nodal slide », OPTI 515 lab report from Byron Cocilovo
- “Evaluation of the focal distance of a lens by Talbot interferometry”, Luis M. Bernardo, Oliverio D. D. Soares, Applied Optics, Vol. 27, No 2. January 1988
- “Measurement of lens focal length using multi-curvature analysis of Shack-Hartmann wavefront data”, Daniel R. Neal, James Copland, David A. Neal, Daniel M. Topa, Phillip Riera, Proc. of SPIE Vol. 5523
- “Technique for the focal length measurement of positive lenses using Fizeau interferometry”, Yeddanapudi Pavan Kumar, Sanjib Chatterjee, Applied Optics, Vol. 48, No. 4, February 2009