Synopsis of "Predicting the vibration characteristics of elements incorporating Incompressible and Compressible Viscoelastic Materials"

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Abstract

A method to predict the value of Poisson's ratio was developed and validated for viscoelastic elements. Chan, Oyadiji, Tomlinson and Wright reasoned that while several techniques exist for determining the frequency dependent complex modulus of viscoelastic material, none have been developed to determine the Poisson ratio. To that end, they present a method to predict the Poisson ratio of viscoelastic material using the Finite Element Method (FEM).

Introduction

The complex modulus and Poisson's ratio are frequency dependent parameters used in the prediction of the vibration characteristics of viscoelastic materials. Viscoelastic elements are used in a wide variety of vibration isolating equipment so understanding these material properties is critical to predicting performance of the equipment. Chan, Oyadiji, Tomlinson and Wright argue that while the methods to predict the complex modulus of viscoelastic material are well developed the same cannot be said for Poisson's ratio. To that end they establish a method using FEM to predict Poisson's ratio for viscoelastic material. The method is implemented by comparing test data and FE predictions for two types and two geometries of viscoelastic samples. The method is validated by on an actual anti-vibration mount.

Theory of Elasticity – the basics

Using elasticity theory as the basis for the experiment a relationship between Poisson's ratio, ν, and the Shear and Bulk modulii (G and K respectively) is given by the authors as follows:

$$E = 2G(1 + \nu)$$

and

$$E = 3K(1 - 2\nu)$$

Yields,

$$\nu = \frac{3 - {}^{2G}/_{K}}{6 + {}^{2G}/_{K}}.$$

Viscoelastic materials can exist in different regions or phases over a range of temperature and frequency in which they may be used. These regions are typically referred to as the glassy, transition, and rubbery regions. The primary intended use of these materials is for vibration isolation and so they are typically used in the rubbery region. In this region, K is several orders of magnitude larger than G and so as the ratio G/K tends to zero, Poisson's ratio tends to the commonly accepted value of 0.5for these materials.

This is considered to be an incompressible viscoelastic material. If Poisson's ratio is less than 0.5 it is considered compressible.

Compressible materials occur typically in the transition or glassy regions, when the values of G and K are of the same order and thus Poisson's ratio is less than 0.5. The authors include a viscoelastic material that has air voids within as their compressible sample. The effect of the air voids is to reduce Poisson's ratio to less than 0.5.

Laying the Groundwork

For the purposes of developing this methodology two types of viscoelastic materials were prepared in two different configurations. The types of material are homogeneous and "composite" (filled with air voids). The configurations were generically, "long" and "short". A long sample is defined as that with a diameter to length ratio less than unity and small is defined as a diameter to length ratio greater than unity. The specific dimensions are provided in Table I.

Table I. Dimesions of prepared viscoelastic samples.

Sample	Homogeneous		Composite	
	Long	Short	Long	Short
Diameter (mm)	30	30	30	30
Length (mm)	40	15	40	15

Each cylindrical sample was bonded at both ends to steel plates and mounted to an electrodynamic shaker table as shown in Figure 1. A 0.135 kg mass placed at the free end. The transmissibility characteristics were determined by taking the ratio of the output to the input accelerations measured after exciting the material over a frequency range of 0 – 500 Hz at a temperature of +20 °C.

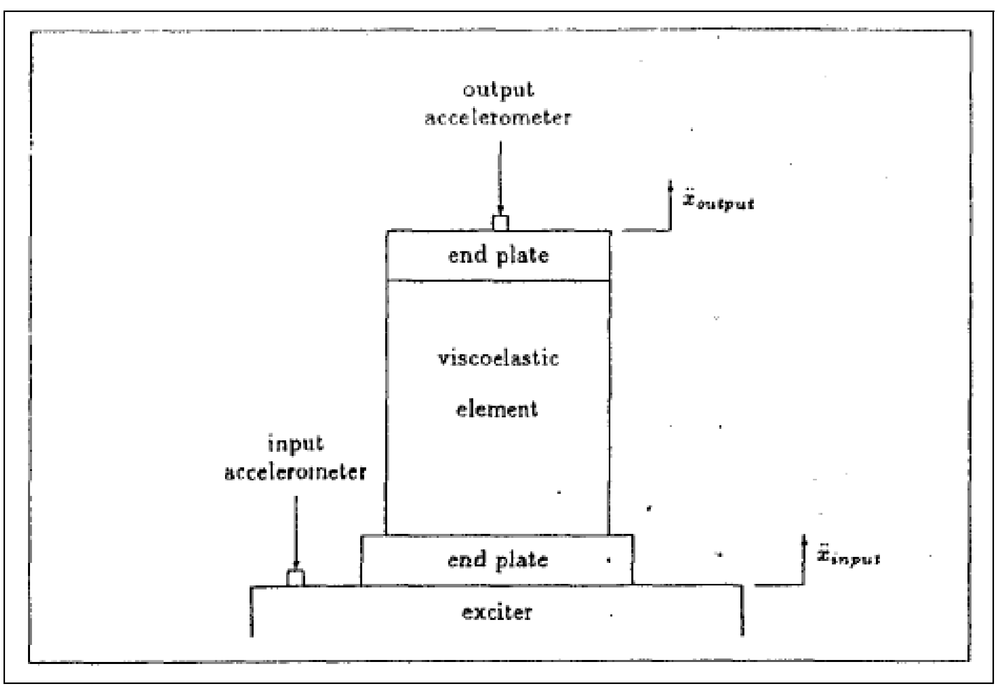

Figure 1. Experimental setup of viscoelastic samples.

The steel plates act as a constraint to the material and the Poisson ratio effect is prevented at those ends. The underlying theory used by the author's is that for the long samples the effect of the constraint is negligible and exact knowledge of Poisson's ratio is not required. For the small sample however, the effect is significant and an accurate value of Poisson's ratio is critical.

This assertion was verified via finite element modeling of the materials to predict the static stiffness of the long and short configurations. Axisymmetric elements were used to model the samples and predictions were made for three values of Poisson's ratio, 0.3, 0.4 and 0.499999. The results for $\nu = 0.3$ and 0.4 were compared the accepted material value of 0.499999. Table II repeats the author's results for static stiffness and resonant frequency measurements and shows that the value of Poisson's ratio has a far greater effect on the predicted results of the short configuration than for the long configuration.

Table II. Predicted static stiffness and first resonant frequency.

Static stiffness		
	% difference w.r.t. $\nu = 0.499999$	
Poisson's ratio	Long sample	Short sample
0.3	10.6	43.2
0.4	6.4	25.6
Resonant frequency		
	% difference w.r.t. $\nu = 0.499999$	
Poisson's ratio	Long sample	Short sample
0.3	4.5	18.3
0.4	2.8	10.9

Poisson's Ratios Predicted

The experimental procedure was to experimentally determine the vibration transmissibility characteristics of each sample, including the complex Young's modulus as described above. The transmissibility characteristics are then predicted using the FEM incorporating the experimentally measured complex Young's modulus. Due to the relative insensitivity of the long sample to the end constraints, the predicted transmissibility characteristics of the long sample were used to indicate the accuracy of the measured Young's modulus. Using this same value of Young's modulus, the authors iterated the value of Poisson's ratio for the short sample until agreement with the experimental data was obtained.

The procedure was performed first for the long sample of the homogeneous material. The measured transmissibility curve was compared to predictions using values of 0.3 and 0.499999. Poisson's ratio of 0.499999 provided a more accurate prediction of the measured results but the predicted resonant frequency using $\nu = 0.3$ showed only a 5% difference. Therefore the authors conclude that the measured

Young's modulus was accurate and was used to predict the transmissibility of the short sample again with $\nu = 0.3$ and 0.499999.

Again the predicted results match the measured results well using 0.499999 as Poisson's ratio. It was concluded that this material agrees with the "typical" value of Poisson's ratio for viscoelastic material in the rubbery region and is a good representation of incompressible material.

The same procedures were followed for a composite material (viscoelastic epoxy resin with 20% by volume air voids) although this time the long sample was tested over a frequency range of 0 to 1600 Hz and the short sample was tested over a frequency range of 0 to 2500 Hz. Predictions were also made using Poisson's ratio of 0.4 in addition to the 0.3 and 0.499999 used in the homogeneous material test.

Again for the long sample, the results showed a relative insensitivity to the value of Poisson's ratio used in the prediction. The measured value of Young's modulus was accepted as accurate and used in the predictions for the short sample. For both samples, $\nu = 0.4$ provided the most accurate prediction compared to measured results and was accepted as the value for the composite material.

Method Validation

To validate the method on an actual device, the authors selected a marine vessel anti-vibration mount that employed both a viscoelastic rubber block and a viscoelastic epoxy resin damping compound. The rubber block acts as a high frequency attenuator and the epoxy is used as a damping material between U-shaped leaf springs riveted to a stainless steel mount. Only half of the device was modeled due to symmetry as shown in Figure 2.

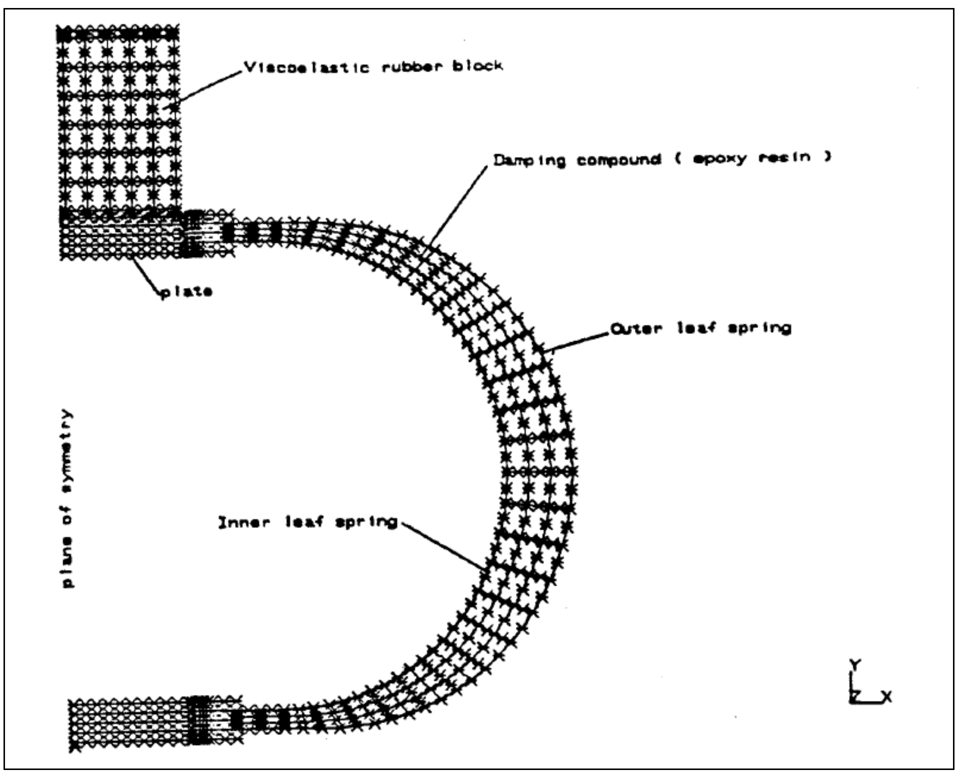

Figure 2. Finite element model of the anti-vibration mount.

The mount with and without a 0.5 kg load attached was excited from 0 to 500 Hz and the transmissibility of the mount was measured. Using values of Poisson's ratio of 0.4 and 0.499999 for the epoxy resin and the rubber block respectively, the transmissibility of the mount was predicted and the results compared to the measured data. Figure 3 shows the measurement and prediction results for the loaded case and demonstrates the accuracy of the model and the employed values of Poisson's ratio.

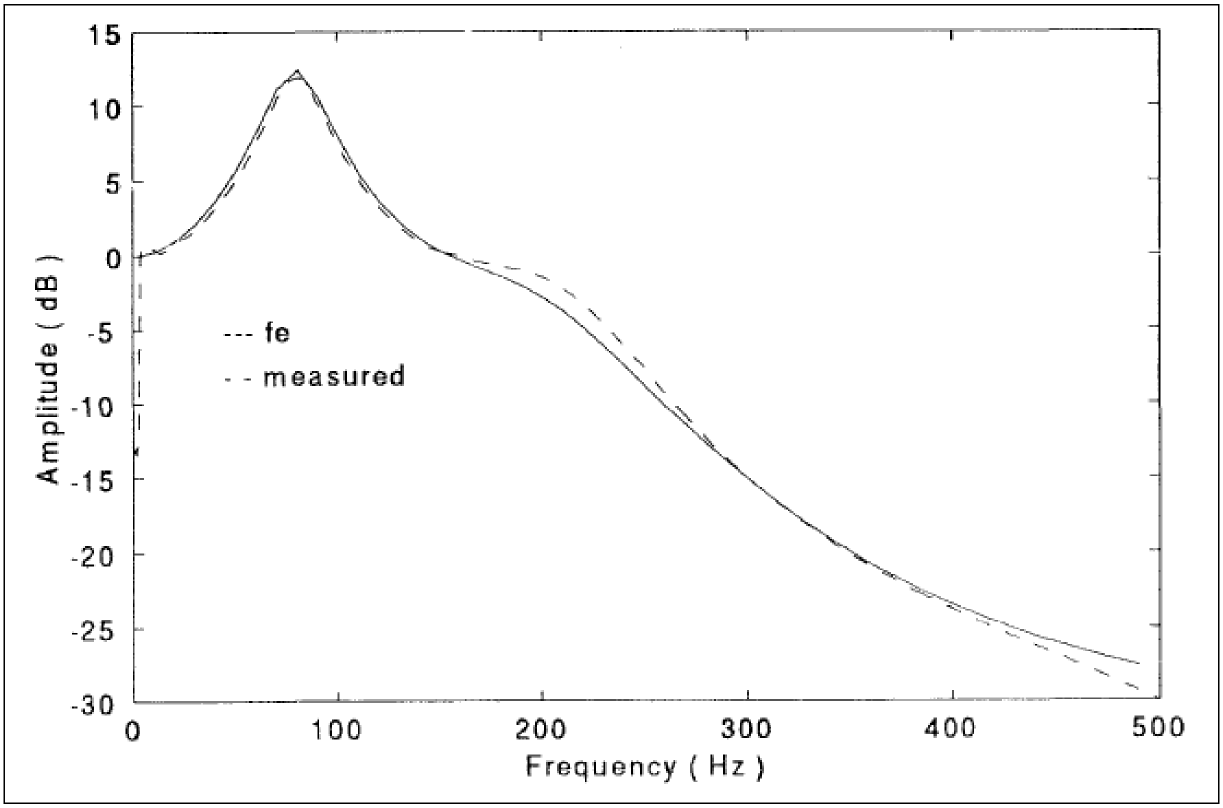

Figure 3. Predicted and measured transmissibility curves for the vibration mount with 0.5 kg load.

Conclusion

The authors' method demonstrates not only the importance of an accurate value of Poisson's ratio for predicting the vibration characteristics of viscoelastic elements, but also a valid method for determining this value. The results further indicate that the method is accurate for both compressible and incompressible materials.

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