Synopsis and Discussion of "Derivation of Line-of-Sight Stabilization Equations for Gimbaled-Mirror Optical Systems

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Abstract

This paper is a synopsis and commentary on the technical paper by DeBruin (1991) on the derivation of stabilization equations for gimbaled-mirror systems. The relevance of the paper is discussed followed by the basic derivation of the stabilization equations using two distinct algorithmic methods. One method which is completely general but somewhat cumbersome and another that is simplified but restricted to a subset of possible implementations. Examples of stabilized optical systems are discussed.

1 Introduction: Why This Paper is Relevant

This paper provides a synopsis and commentary on the technical paper "Derivation of line-of-sight stabilization equations for gimbaled-mirror optical systems". The paper was written by James C. DeBruin at Control Systems Technology Center, Texas Instruments in 1991. I found this particular paper both interesting and practical for a variety of reasons.

First, the basic equations for gimbal mounted mirror stabilization are still quite valid and give the user significant insight into the dynamics involved in gimbaled mirror systems. In many ways, these equations are more applicable today then they were when the paper was written. As computing power has increased significantly and the cost of sensing and actuating devices has decrease, the past decade has seen a proliferation of smaller, less expensive, stabilized optical systems. A common example of these are image stabilization systems for hand held cameras which would have been unthinkable back in the early nineties.

Another reason why I found this paper quite interesting is that is serves as a benchmark to changes that have occurred in the science of optical system stabilization in the recent past. At the time this paper was written, computing hardware and software were much more restrictive on the size and speed of matrix manipulations required for transformation of reference frames. Many excellent real time software programs now exist which can easily manipulate the matrices for multiple coordinate transformations. Modern sensing and actuating devices have also changed the way stabilization is performed on optical systems. When this paper was written, optical stabilization systems required large and expensive gyroscopic instruments which were only practical to implement on large and expensive programs. Advances in MEMS technology, particularly of accelerometers, has seen tremendous reduction in size and cost of sensing instruments, making optical stabilization systems practical for much smaller and less costly systems.

2 Derivation of the LOS Stability Equations

2.1 Derivation of the LOS Reference Frames

The paper begins by introducing the concept of a *line-of-sight reference frame* and how that is defined. The author defines the LOS reference frame as the combination of a line-of-sight vector and two mutually normal image plane vectors. This is simply an orthonormal triad which has one unit vector along the boresight of the optical system, the other two unit vector orientations must be orthogonal but the rotation is somewhat arbitrary and is usually left as a matter of convenience. The transformation of the incoming LOS vector $\vec{r}_1$ to the outgoing LOS vector $\vec{s}_1$ is given by

$$\vec{s}_1 = \left[\bar{M}\right] \vec{r}_1$$

where $\bar{M}$ is the matrix found from the mirror normal vector components, n_i

$$\left[\bar{M}\right] = \begin{bmatrix} 1 - 2n_1n_1 & -2n_1n_2 & -2n_1n_3 \\ -2n_2n_1 & 1 - 2n_2n_2 & -2n_2n_3 \\ -2n_3n_1 & -2n_3n_2 & 1 - 2n_3n_3 \end{bmatrix}$$

In this paper, the author allows the handedness of the reference frames to change due to parity changes of the mirror. My experience in control algorithm design suggests this is not wise as it will almost certainly cause confusion during implementation and testing of the software. It seems that it would be better to define a conventional right handed coordinate frame at each transformation and live with the fact that the sign of a particular matrix element may be reversed.

Once the LOS reference frame has been defined, the stabilization equations can then be derived and used to relate the gimbal rotational rates to the angular velocity of the platform on which the optical system is mounted. The relations given by the stabilization equations are useful for so-called *feedforward control systems* in which the angular velocities of the platform are measured, then solved for the gimbal angle rates. The computed gimbal angle rates are then used as commanded inputs (feedforward) to the gimbal servo loops which attempt to drive the line-of-sight angular rate error to zero.

2.2 Angular Velocity of an LOS Reference Frame

From basic dynamics, the time rate of change of a rotating coordinate system V relative to a Newtonian reference frame N is given from [2] as

$$\frac{{}^N d\vec{v}}{dt} = \frac{{}^N \delta\vec{v}}{\delta t} +^N \vec{\omega}^V \times \vec{v}$$

and for angular motion relative to the platform on which the optical system is mounted

$$\frac{{}^N d\vec{v}}{dt} =^N \vec{\omega}^V \times \vec{v}$$

Using some vector algebra, this leads to the definition of angular velocity given in the paper

$${}^N\vec{\omega}^V \equiv \left[\vec{v}_1 \cdot \frac{d\vec{v}_2}{dt} \cdot \vec{v}_3\right] + \left[\vec{v}_2 \cdot \frac{d\vec{v}_3}{dt} \cdot \vec{v}_1\right] + \left[\vec{v}_3 \cdot \frac{d\vec{v}_1}{dt} \cdot \vec{v}_2\right]$$

where the author defines the Newtonian reference frame as the *base frame.* It is often desirable to use intermediate reference frames for the computation of angular velocity in which case we can employ the *addition theorem.* For example, if we know the angular velocity of frame N with respect to P and the angular velocity of frame P with respect to V, then we can compute the angular velocity of N with respect to V by simply adding the two known angular velocities.

$${}^N\vec{\omega}^P =^N \vec{\omega}^P +^P \vec{\omega}^V$$

One other useful concept used in the derivation of the stabilization equations is the idea of *simple angular velocity.* If a unit vector $\hat{k}$ is fixed in both reference frames A and B then we can say that A rotates about B along the axis defined by $\hat{k}$ through angle θ given by

$${}^N\vec{\omega}^P = \dot{\theta}\hat{k}$$

2.3 LOS Stabilization

The line-of-sight is often defined as stabilized if the orthogonal components of the LOS angular velocity are zero. The author of this paper uses an alternate definition. He states that the difference between commanded slew rates and actual rates should be zero which includes cases where the optical system is tracking or slewing. This definition is also somewhat problematic in that the difference (e.g. the error signal) is never actually zero as the control system always lags behind the commanded rates. So from this definition, the LOS is never completely stabilized. This is more of a mathematical concept and less of an actual concern to the practicing controls engineer who is more interested in the overall system response.

At any rate (pun intended), this leads to stabilization of the LOS frame if the following two conditions are satisfied

$$ {}^N\vec{\omega}^V \cdot \vec{v}_2 - \Omega_2 = 0 \qquad {}^N\vec{\omega}^V \cdot \vec{v}_3 - \Omega_3 = 0 $$

where $\vec{v}$ forms the line-of-sight and image vectors of the LOS reference frame V, and ${}^N\vec{\omega}^P$ is the angular velocity of V with respect to N. Ω_2 and Ω_3 are the commanded slew rates in N about $\vec{v}_1$ and $\vec{v}_1$ respectively.

2.4 Stabilization Equations

Using the results from the previous sections, we can now derive the equation for line-of-sight stability. The form of the equations follow directly from the stabilization conditions. Let V represent an outgoing LOS reference frame from an optical system attached to platform P. Frame V is steered by a gimbaled mirror on the platform and has the angular velocity ${}^P\vec{\omega}^V$. The platform is in motion with respect to the inertial frame N with angular velocity ${}^N\vec{\omega}^P$. Using the two stability conditions and the addition theorem leads directly to

$$ \left({}^N\vec{\omega}^P +^P \vec{\omega}^V\right) \cdot \vec{v}_2 - \Omega_2 = 0 \qquad \left({}^N\vec{\omega}^P +^P \vec{\omega}^V\right) \cdot \vec{v}_3 - \Omega_3 = 0 $$

Setting the commanded slew rates to zero, we get the stabilization equation given in the paper

$$ \left({}^N\vec{\omega}^P +^P \vec{\omega}^V\right) \cdot \vec{v}_2 = 0 \qquad \left({}^N\vec{\omega}^P +^P \vec{\omega}^V\right) \cdot \vec{v}_3 = 0 $$

The angular velocity of the platform in inertial space ${}^N\vec{\omega}^P$ acts as a perturbation to the system. However, we have control over ${}^P\vec{\omega}^V$ which is some function of the gimbal angle rates $\dot{\theta}_i$. Thus, the stabilization equations are used to control the line-of-sight by changing $\dot{\theta}_i$ as a function of the platform angular velocities which drive the system to satisfy the stabilization conditions.

2.5 Two Different Methods

The paper presents two different methods for formulating the stabilization equations. The methods differ only in the manner in which ${}^P\vec{\omega}^V$ is computed and in there applicability to specific problems. The first method, called *direct differentiation*, uses the direct calculation based on the time rate of change of the vector basis fixed in the platform frame.

The second method of *intermediate reference frames* involves finding adjacent reference frames between the platform and the outgoing LOS frame, each of which moves with simple angular velocity in the adjacent frame. The addition theorem is then employed to compute ${}^P\vec{\omega}^V$. This method considerably simplifies the matrix transformations and, therefore, reduces the required computing power. However, it is only applicable if, for each gimbaled mirror and for all mirror orientations, a vector ($\vec{u}$) exists that is both fixed in the plane of the mirror and orthogonal to the incoming line-of-sight.

3 Examples

Although the examples provided by the author are helpful in understanding the detailed process of deriving the stabilization equations for specific cases, what is most important are the conclusions that the author draws from the two examples provided.. Using the method of direct differentiation results in a somewhat complex 3 by 3 array of sines and cosines. The matrix must be transposed and the matrix elements must be differentiated using

$$^{N}\vec{\omega}^{V} \equiv \left[\vec{v}_1 \cdot \frac{d\vec{v}_2}{dt} \cdot \vec{v}_3\right] + \left[\vec{v}_2 \cdot \frac{d\vec{v}_3}{dt} \cdot \vec{v}_1\right] + \left[\vec{v}_3 \cdot \frac{d\vec{v}_1}{dt} \cdot \vec{v}_2\right]$$

This is, of course, algebraically complex if one is to attempt the calculations by hand. Even for the computers of the day, this would be difficult to implement in a real-time system at a reasonable update rate.

The second example using intermediate reference frames is significantly simpler than the method of direct differentiation in terms of required computation. Even though the simplifications required limit the types of gimbaled system which this method can be used, it would have been much more tractable and easier to implement on the available computers and software of the day. The author therefore concludes that the second method is preferred if at all possible. The author does note that the use of computers using symbolic algebra greatly facilitates the process of direct differentiation.

4 Conclusions

The reviewed paper presented some very interesting results and practical tools for dealing with line-of-sight stabilization of optical systems. The basic stabilization equations are still applicable today and give great insight into the physical processes of more complex stabilization problems.

Recent advances in computers and real time software make manipulation of large matrices and complex trigonometric functions much easier to deal with. Most systems today would have no problem in implementing the method of direct differentiation for solving the stabilization equations, which provides a general solution applicable to any gimbaled system. Advances in MEMS technology have made accelerometers more preferable than gyroscopes as sensing devices for many applications.

With the dramatic increase in computing power, the development of excellent real-time software systems, and the decrease in cost and size of sensing and actuating devices, the use of the line-of-sight stabilization equations are more relevant now than ever. Once reserved for large and expensive optical systems, line-of-sight stabilization is now used in a wide variety of application including hand held cameras, video equipment, laser tracking and metrology, as well as the occasional 6.4 meter telescope.

5 References

[1] Derivation of line-of-sight stabilization equations for gimbaled-mirror optical systems, DeBruin, James C., SPIE Vol.1543, 1991

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[5] OPTI-521 class notes, Burge, J.