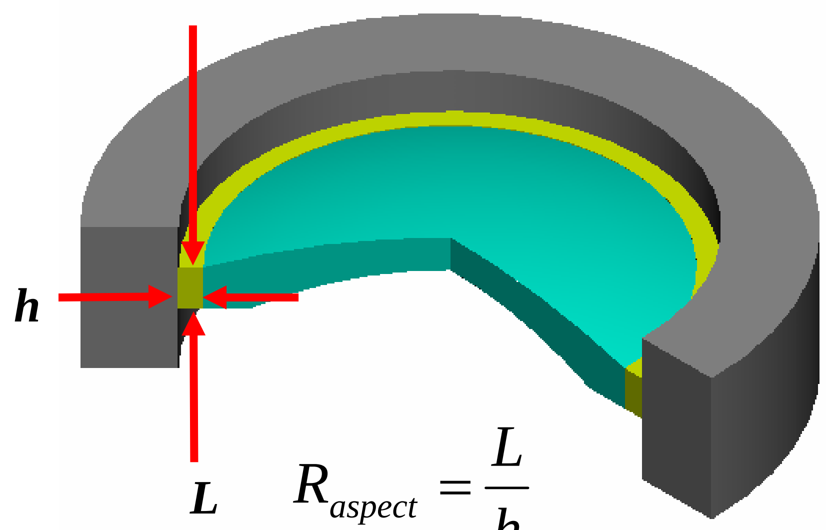
Athermal Bonded Mounts

Incorporating Aspect Ratio into a Closed-Form Solution

Problem and Goal

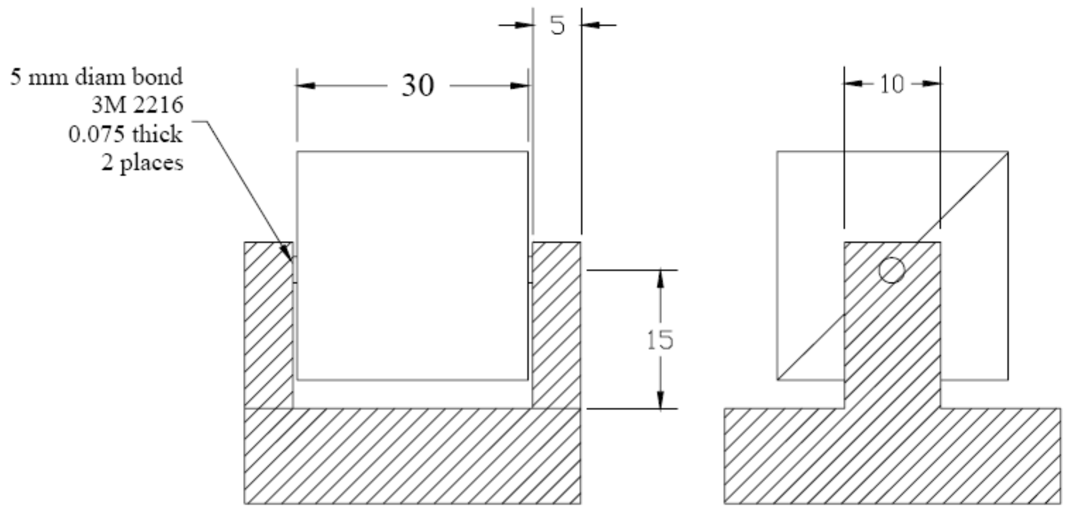
- Problem
  - As a system is heated (or cooled) the CTE mismatch of the optical element, cell, and bond cause radial stress
  - Radial stress on a lens or mirror causes a decrease in optical performance
    - The element may deform
    - Stress birefringence in lenses
- Goal
  - Size the bond thickness so that there is zero *radial* stress when there is a change in temperature



$$R_{aspect} = \frac{L}{h}$$

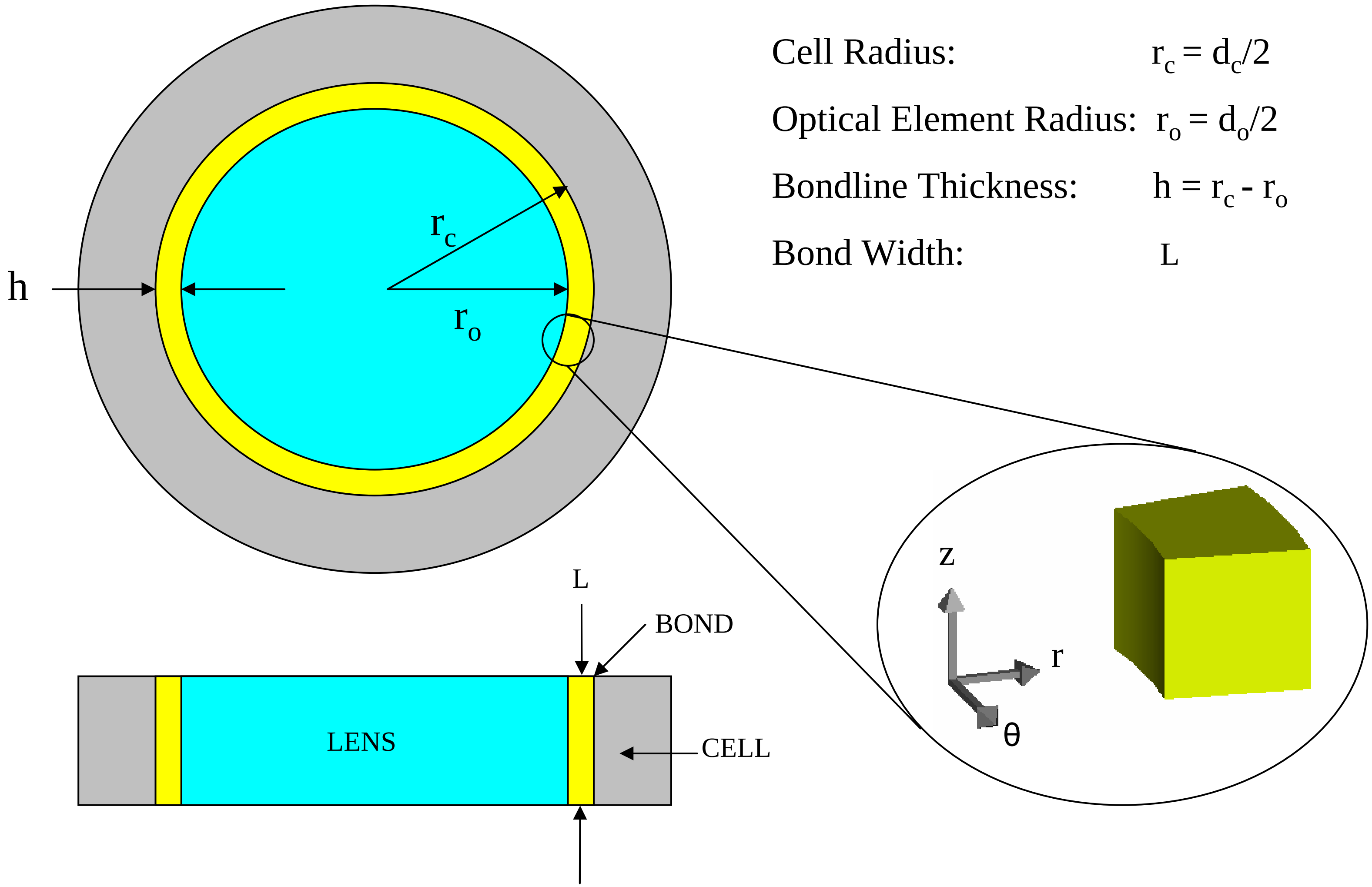
- Existing solutions
  - Different ways of constraining the bond
  - None incorporate the aspect ratio or allow for bulge

Remember this problem?



- We solved the statically indeterminate system to determine the stress in the bond.
- We could have solved for the *bond thickness as a variable* and set the stress to *ZERO.*

System Definition



Cell Radius: $r_c = d_c/2$

Optical Element Radius: $r_o = d_o/2$

Bondline Thickness: $h = r_c - r_o$

Bond Width: L

Hooke's Law

- You know it as:

$$F = kx$$

- The full version:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \begin{bmatrix} \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} & \frac{\nu E}{(1+\nu)(1-2\nu)} & \frac{\nu E}{(1+\nu)(1-2\nu)} & 0 & 0 & 0 \\ \frac{\nu E}{(1+\nu)(1-2\nu)} & \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} & \frac{\nu E}{(1+\nu)(1-2\nu)} & 0 & 0 & 0 \\ \frac{\nu E}{(1+\nu)(1-2\nu)} & \frac{\nu E}{(1+\nu)(1-2\nu)} & \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix}$$

Hooke's Law Continued

- The matrix reduced to one direction – *radial*

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_r + \nu(\varepsilon_z + \varepsilon_\theta)\right]$$

- The goal now is to come up with expressions for the three strains and to solve for zero stress:

$$\sigma_r = 0$$

$$\varepsilon_r = ?$$

$$\varepsilon_z = ?$$

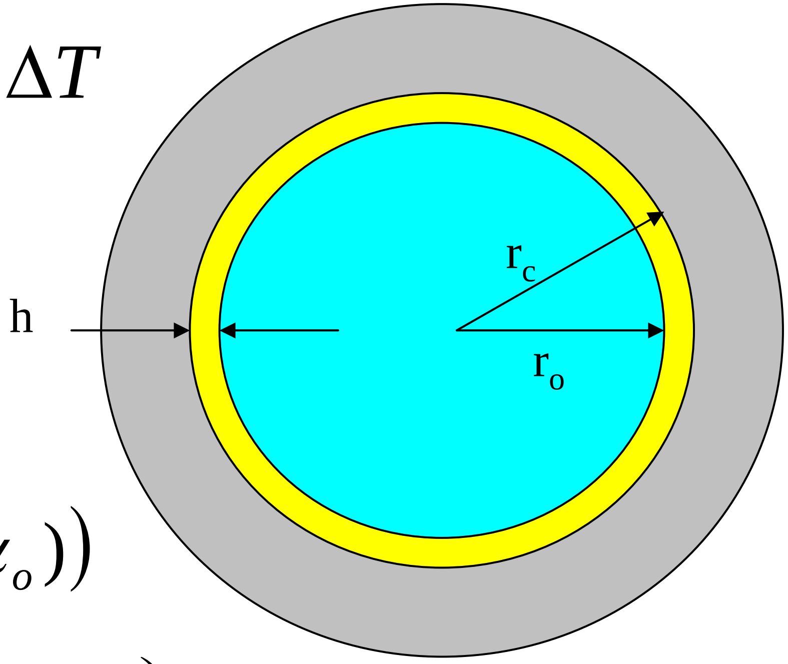
$$\varepsilon_\theta = ?$$

All variables apply to the bond material

Radial Strain

- The bond stuck between the lens and the cell

$$h\alpha_b \Delta T = (r_o + h)\alpha_c \Delta T - r_o \alpha_o \Delta T$$



- Radial Strain

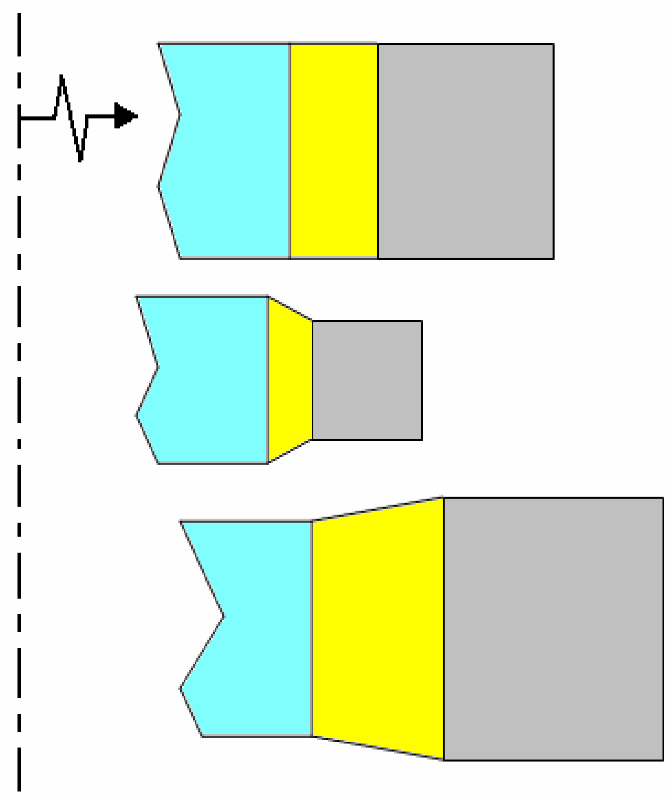
$$\delta h = \Delta T\left(h(\alpha_b - \alpha_c) - r_o(\alpha_c - \alpha_o)\right)$$

$$\varepsilon_r = \frac{\delta h}{h} = \Delta T\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right)$$

Axial and Tangential Strain

- The adhesive is "stuck" to the lens and cell.
- The constraint on the bond can be approximated as the average change in the axial and tangential size of the lens and cell.
- The resulting expressions for strain are:

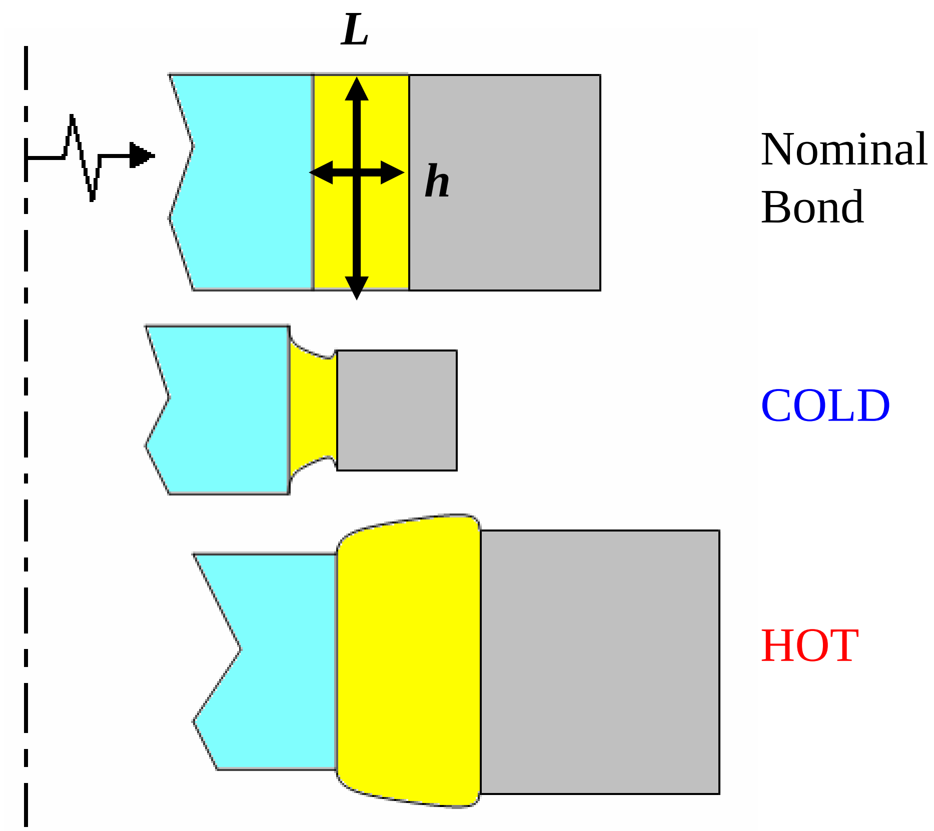
$$\varepsilon_z = \varepsilon_\theta = \Delta T\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)$$



Another Look at Axial Strain

- In reality the bond can bulge a the exposed surfaces
  - A good assumption for a wide bond (large aspect ratio) is that the bond cannot bulge
  - This is a bad assumption for a bond with small width (small aspect ratio)

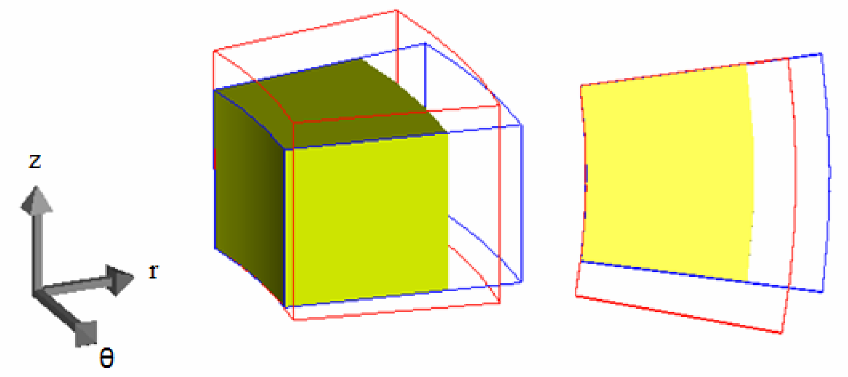
$$R_{aspect} = \frac{L}{h}$$



Constraints

•The red "cube" shows the unconstrained growth of a section of bond material.

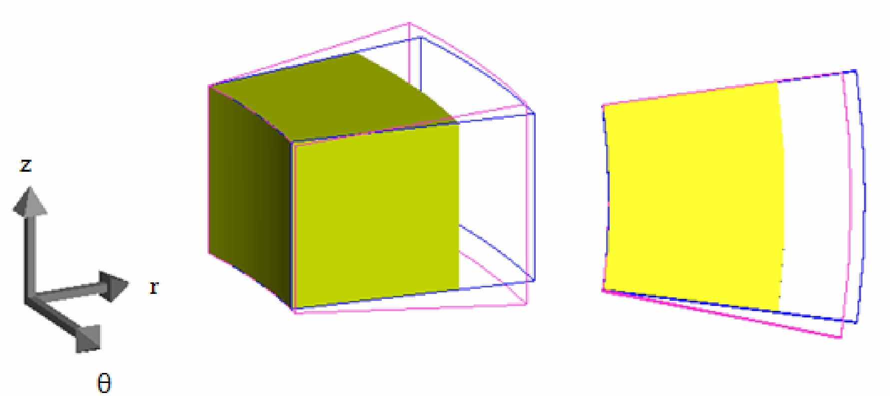
Using this assumption results in the **BARYAR EQUATION**



•The blue volume shows the affect of *fully* constraining the cube to its original size axially and tangentially.

Using this assumption results in the **MODIFIED BAYAR EQUATION**

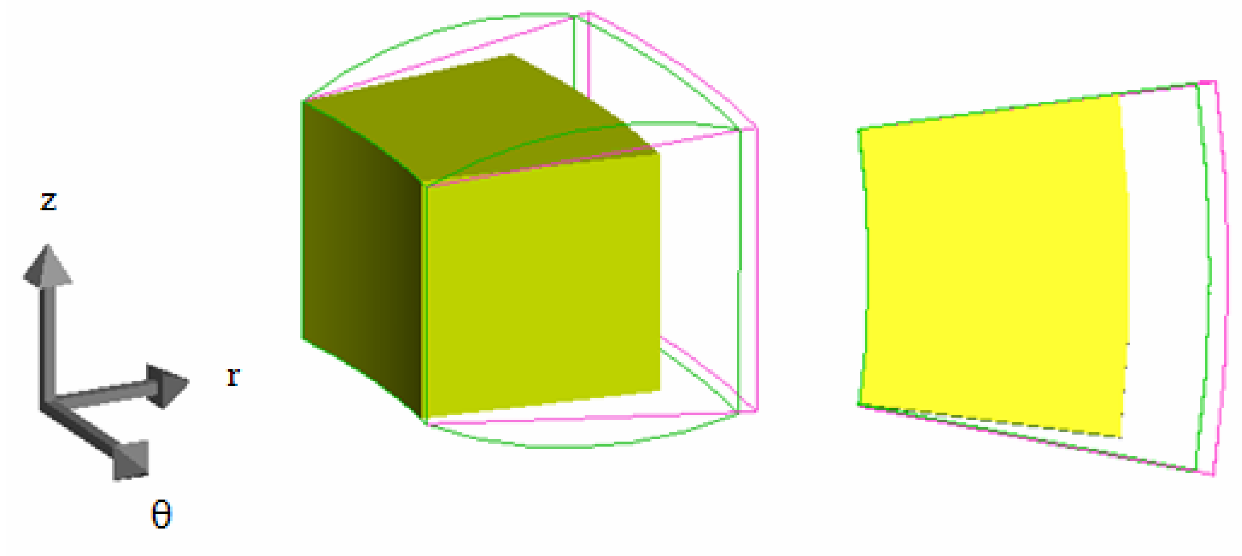
•The pink volume shows the affect of constraining the bond to the lens and cell, which also grow.



Using this (very good) assumption results in the **VAN BEZOOIJEN EQUATION**

Aspect Ratio Approximation

- The bond is not fully constrained in the axial direction



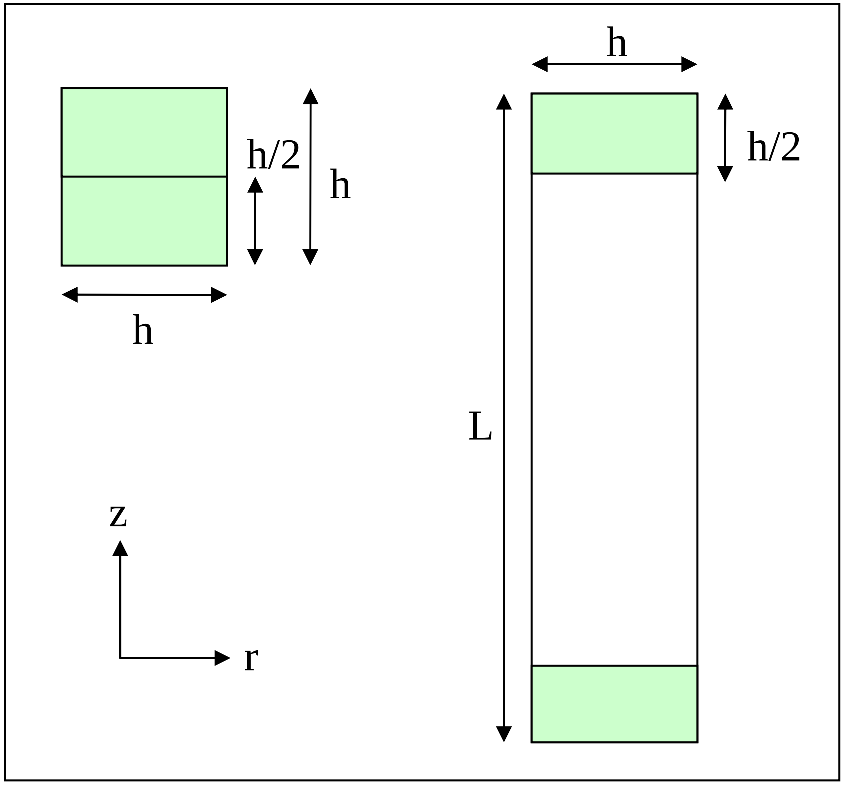
- How constrained is it?

Ratio of constrained bond

- Choose an aspect ratio of 1 to represent a bond that in unconstrained in the axial direction.

$$R_{constrained} = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - R_{aspect}^{-1}$$

The ratio of constrained bond varies between 0 and 1 for aspect ratios between 1 and infinity.



Comparing the Strains

Equation	Radial Strain	Axial Strain	Tangential Strain
Bayar	$\Delta T\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right)$	0	0
Modified Bayar	$\Delta T\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right)$	$\Delta T\alpha_b$	$\Delta T\alpha_b$
Van Bezooijen	$\Delta T\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right)$	$\Delta T\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)$	$\Delta T\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)$
Modified Van Bezooijen	$\Delta T\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right)$	0	$\Delta T\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)$
Aspect Ratio Approx.	$\Delta T\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right)$	$\Delta T\left(1 - \frac{h}{L}\right)\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)$	$\Delta T\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)$

Comparison of the Results

- Bayar

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c}$$

- Modified Bayar

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\frac{1+\nu}{1-\nu} \cdot \alpha_b - \alpha_c}$$

- Van Bezooijen

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{2\nu}{1-\nu}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)}$$

- Modified Van Bezooijen

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)}$$

- Aspect Ratio Approximation

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu}\left(2 - \frac{h}{L}\right)\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)}$$

Problem here:
We haven't
solved for h

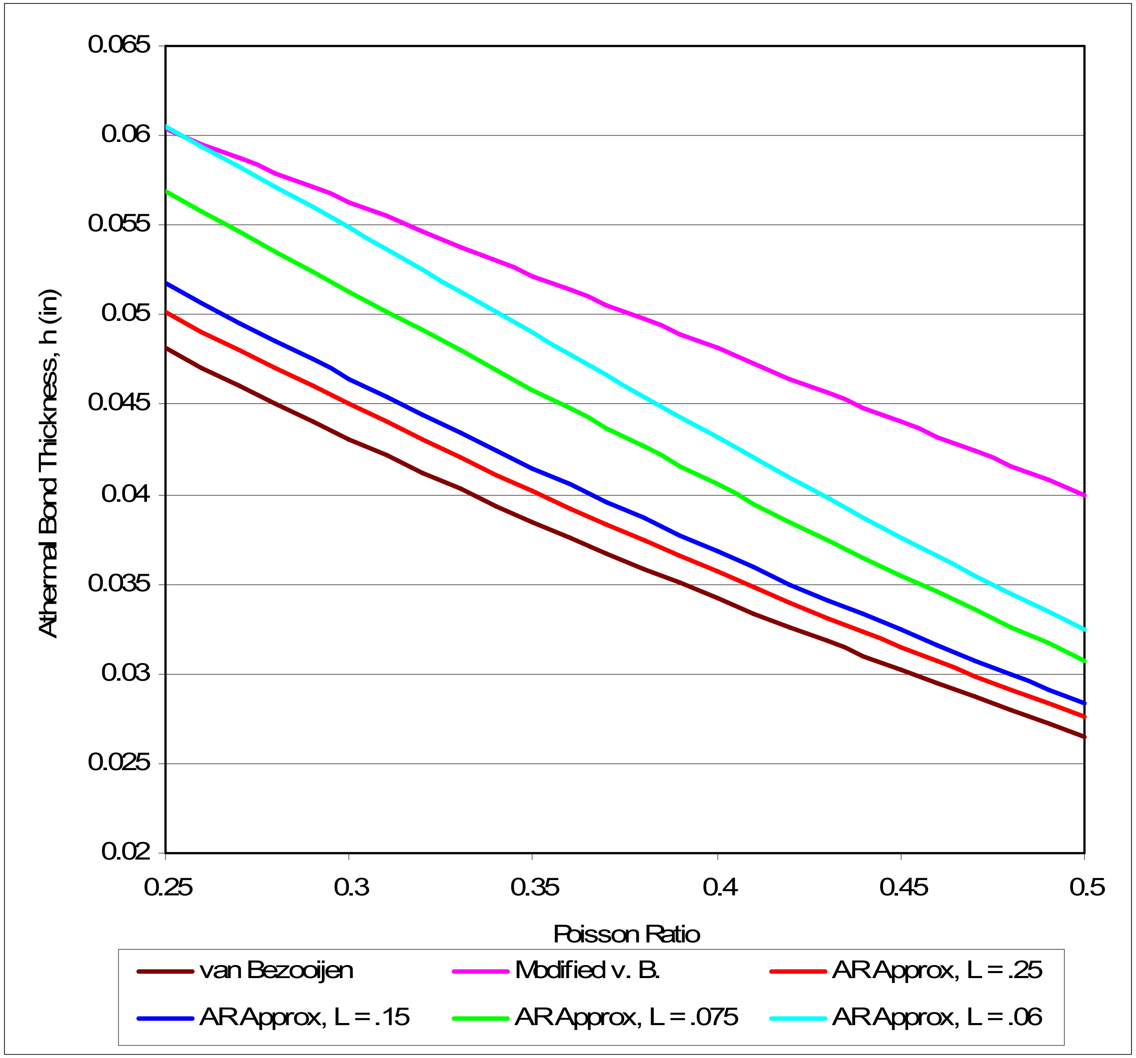
Aspect Ratio Approximation

- A quadratic equation is needed to solve for h

$$0 = h^2\left(\frac{-\nu}{L}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)\right) + h\left((1-\nu)(\alpha_b - \alpha_c) + 2\nu\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)\right) - r_o(1-\nu)(\alpha_c - \alpha_o)$$

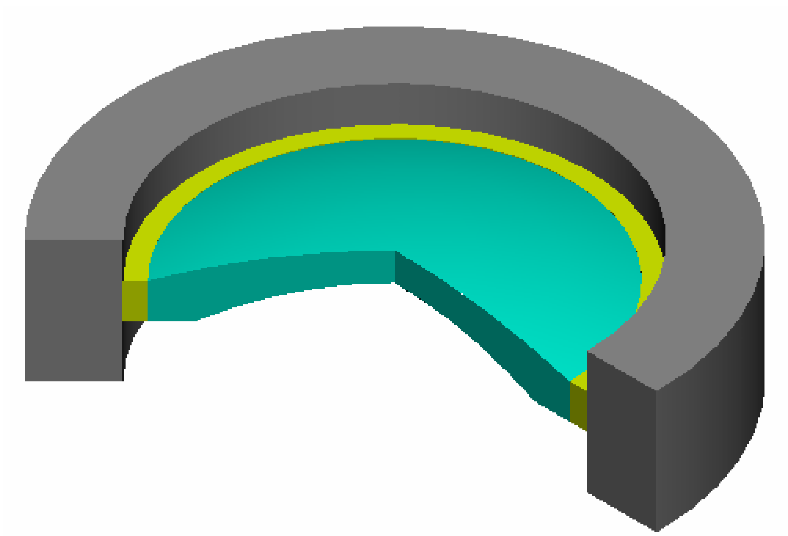
$$\left.\begin{aligned} a &= \frac{-\nu}{L}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right) \\ b &= (1-\nu)(\alpha_b - \alpha_c) + 2\nu\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right) \\ c &= r_o(1-\nu)(\alpha_c - \alpha_o) \end{aligned}\right\} \quad h = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

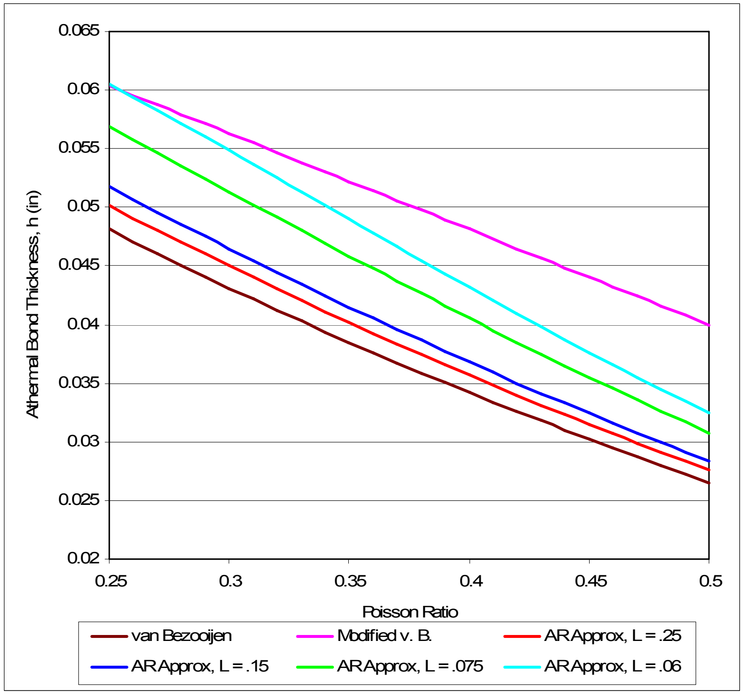
Results for an Example System



Conclusion

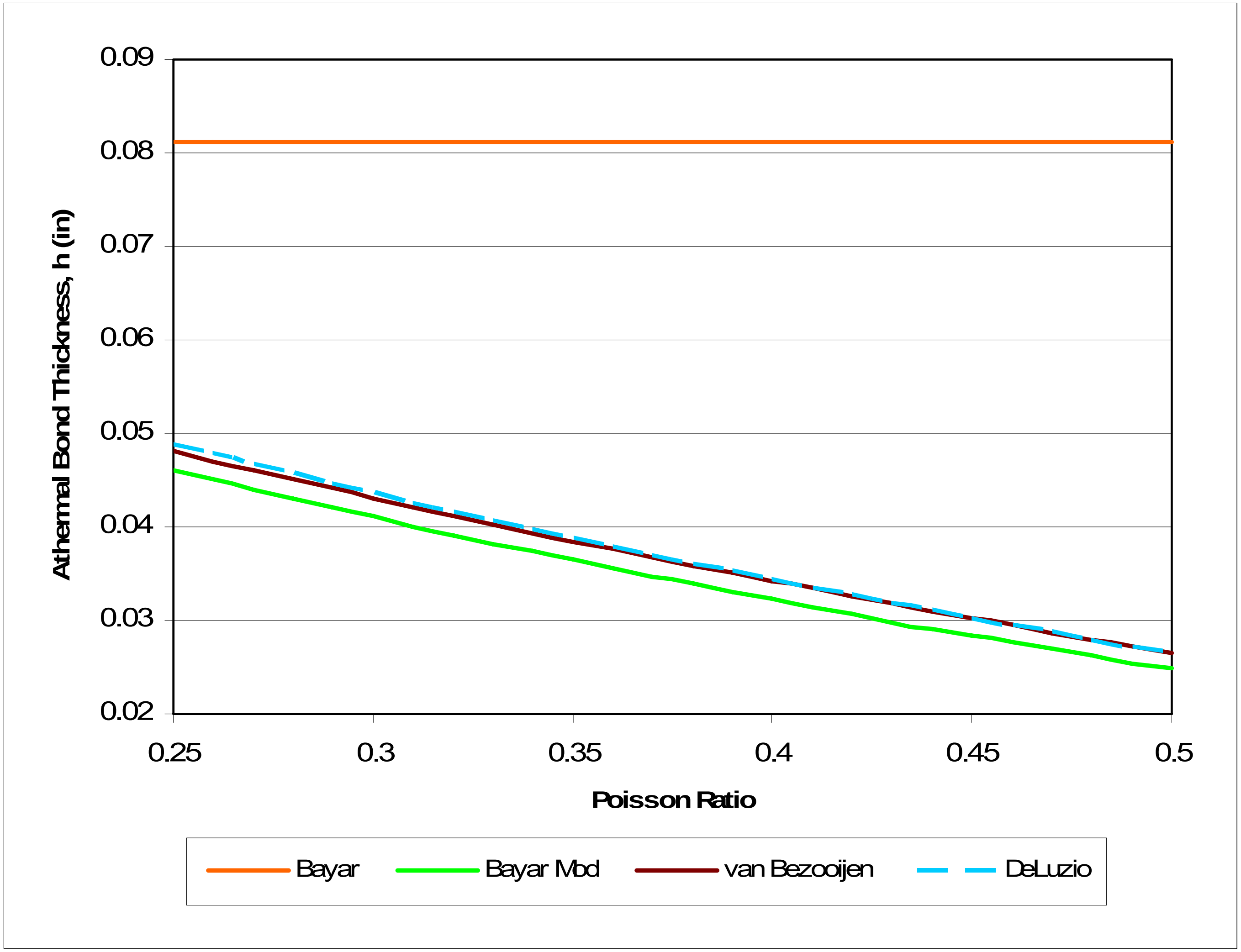
- New limiting equation developed for low aspect ratio bonds
- New equation developed that spans the space between the low and high aspect ratio limits
- New equation compares well to FEA data





Any Questions?

Comparing the Equations



Comparison to FEA data

- The multiplier in the denominator is compared to correction factors from FEA data by another author:

$$\frac{k_{12} + k_{13}}{2 - \frac{h}{L}}$$