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CHAPTER 8

Design of Components

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8.1 Introduction

Elastomers have found use in a wide range of applications, including hoses, tires, gaskets, seals, vibration isolators, bearings, and dock fenders. In most applications, the performance of the product is determined by the elastomer modulus and by details of the product's geometry. Chapter 3 discusses the elastomer modulus and many factors that may cause it to change. This chapter addresses some simple geometries common to applications for elastomeric products. In particular, sample calculations are given for products in which the elastomers are bonded to rigid components and are used for motion accommodation, vibration isolation, or shock protection. Section 8.5 identifies potential design resources for elastomeric products in general.

Depending on the application, many different characteristics of an elastomer may be of interest to the designer. Some of these characteristics are:

- Ultimate conditions; both force and deflection
- Sensitivity to changes in temperature
- Sensitivity to changes in strain
- Resistance to fluids and other contaminants
- Compatibility with mating material (adhesives, metals, etc.)
- Resistance to "set" and "drift" (dimensional stability under load)
- Internal damping

In some cases, these properties are related. Examples include:

- Minimizing "drift" usually requires a material with very low damping. As damping increases, the amount of drift typically increases.
- Selecting a properly compounded silicone material minimizes sensitivity to temperature and strain, for example, but because of the lower strength of silicone rubber, it results in lower allowable values of stress and strain in the design.
- Fatigue performance depends on both the material properties and the detailed product design. The fatigue performance of any given design can be seriously compromised by environmental factors such as fluid contamination and heat.

The first step in most product design is selecting the material to be used, based on the above characteristics. The material selected in turn defines the design allowables. This chapter uses conservative, generic design allowables. Although most manufacturers consider their design criteria proprietary, the information sources noted in Section 8.5 provide some general guidelines.

For most designs, the spring rate (sometimes called stiffness) is a key design parameter. The units for stiffness or spring rate are Newtons per meter (N/m). In terms of function, the spring rate of a part is defined by Eq. (8.1) as the amount of force required to cause a unit deflection:

$$K = \frac{F}{d} \tag{8.1}$$

where F is the applied force (N) and d is the deflection (m). The spring rate K can be for shear, compression, tension, or some combination of these, depending on the direction of the applied force with respect to the principal axes of the part.

The spring rate K of a part is defined by Eq. (8.2) in terms of geometry and modulus:

$$\text{Shear}: K_s = \frac{AG}{t} \tag{8.2a}$$

$$\text{Compression}: K_c = \frac{AE_c}{t} \tag{8.2b}$$

$$\text{Tension}: K_t = \frac{AE_t}{t} \tag{8.2c}$$

where A is effective load area (m^2), t is thickness (m) of the undeformed elastomer, and G, E_c, and E_t represent the shear, compression, and tension moduli (kPa or kN/m^2) of the elastomer. Figure 8.1 (Section 8.2.1) defines area and thickness.

Using Eq. (8.1) and (8.2), the product designer can relate forces and deflections to the design parameters of area, thickness, and material modulus. It is important to use the correct modulus to calculate the appropriate spring rate.

Usually design allowables, such as maximum material strength, are given in terms of stress σ and strain ε. Stress is applied force divided by the effective elastomer load area, and strain is deflection divided by the undeflected elastomer thickness.

$$\sigma = \frac{F}{A} \tag{8.3}$$

$$\varepsilon = \frac{d}{t} \tag{8.4}$$

For the applications considered in this chapter, the usual design aims include the abilities to maximize fatigue life, provide specific spring rates, minimize set and drift, and minimize size and weight. Maximizing fatigue life means minimizing stress and strain, which translates to large load areas and thicknesses. The load area and thickness are also limited by the available range of elastomer modulus. Therefore, the final design represents a tradeoff among size, fatigue life, and spring rate.

The design examples in this chapter assume that operation remains in the linear range of the elastomer modulus. Typically this is less than 75 to 100% strain for shear and less than 30% strain for tension and compression.

Also, the design examples are limited to fairly simple geometries. Typically the shear modulus G is not a function of geometry. However, compression modulus (E_c) is strongly affected by the geometry of a design. If the two designs in Fig. 8.1 have the same modulus elastomer, load area, and thickness, the shear spring rate K_s is equivalent for both designs, while the different shapes have different compression spring rates. The equations used here are good for close approximations of performance and are generally adequate for design purposes. The basic equations must be modified to account for more complex geometry effects and/or nonlinear elastomer properties.

For all the sample problems in the sections that follow, the international system (SI) of units is used. The equations presented are independent of the unit system. Section 8.5 includes some common conversion factors from SI to English units.

8.2 Shear and Compression Bearings

The ability to mold rubber into a wide variety of shapes gives the design engineer great flexibility in the selection of stress and strain conditions in a finished part. This section outlines some fundamental equations for static stress-strain relationships in bonded rubber components of various geometrical configurations. The sample problems are intended to show how these basic equations can be applied to component design.

8.2.1 Planar Sandwich Forms

Simple flat rubber and metal sandwich forms are the basic geometric shapes for a large number of rubber mounts and bearings. Typical shapes are shown in Fig. 8.1. Equations are presented for the two principal modes of loading, simple shear and compression.

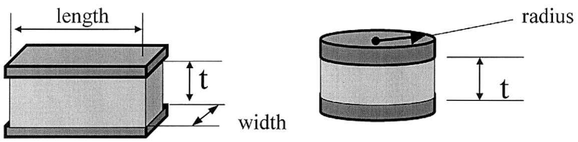

Figure 8.1 Rectangular and circular shear pads.

Combining Eq. (8.1) and (8.2a) yields a basic equation relating shear spring rate and product design variables.

$$K_s = \frac{F_s}{d_s} = \left(\frac{AG}{t}\right) \tag{8.5}$$

where K_s is shear spring rate, F_s is applied force in the shear direction, d_s is shear displacement, A is load area, G is shear modulus, and t is rubber thickness.

Equation (8.5) can be applied to the majority of simple shear calculations for flat "sandwich" parts. The equations are valid only when shear deformation due to bending is negligible. When the ratio of thickness to length exceeds approximately 0.25, the shear deformation due to bending should be considered. The effect of bending is shown in Fig. 8.2 and can be quantified by the following equation:

$$K_s = \frac{F_s}{d_s} = \left(\frac{AG}{t}\right)\left(\frac{1}{1 + t^2/36r_g^2}\right) \qquad (8.5a)[3]$$

where $r_g = (I_b/A)^{1/2}$ is the radius of gyration of the cross-sectional area about the neutral axis of bending and I_b is the moment of inertia of the cross-sectional area A about the neutral axis of bending. Chapter 3 includes other methods for estimating bending.

Two other factors that can affect shear spring rate are mentioned here; because of the limited scope of this chapter, however, no detailed derivation is given. When the shear strain d_s/t exceeds approximately 75%, or when the part has an effective shape factor of less than 0.1, the effect of rubber acting in tension may also need to be considered. At this point, the rubber is no longer acting in simple shear only; there is now a component of tensile force in the rubber between the two end plates. Another important consideration for some components may be a change in shear spring rate due to an applied compressive stress on such components [1]. With components of high shape factor, the shear spring rate increases as a compressive load is applied. This effect increases with increasing shape factor and increased compressive strain, as is shown graphically in Fig. 8.3. The shape factor is defined in Eq. (8.9).

Combining Eq. (8.1) and (8.2b) yields a basic equation relating compression spring rate and product design variables:

$$K_c = \frac{F_c}{d_c} = \frac{AE_c}{t} \tag{8.6}$$

where K_c is compression spring rate, F_c is applied force in compression direction, d_c is compression displacement, A is load area, E_c is effective compression modulus, and t is thickness.

Successful use of this equation depends on knowing the effective compression modulus E_c. The value of E_c is a function of both material properties and component geometry, as discussed in Chapter 3. Many different analytical techniques can be used to calculate E_c, and the method described here yields reasonable approximations for compression spring rates of many simple components. More advanced analytical solutions as well as finite element analyses are available for more accurate spring rate calculations. For a more rigorous analytical solution of compression, bending and shear in bonded rubber blocks, see Gent and Meinecke [2].

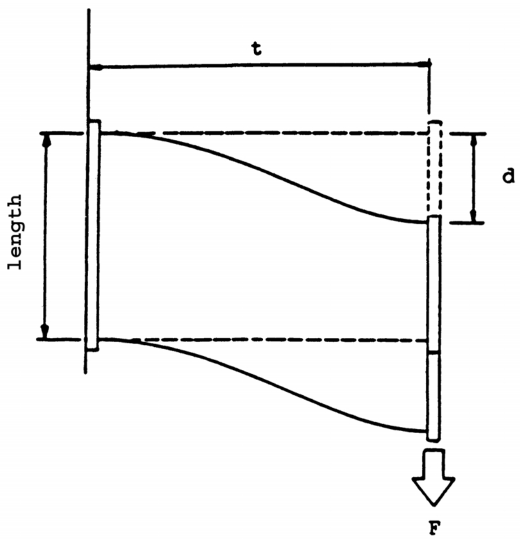

Figure 8.2 Shear pad with shear bending deflection [3].

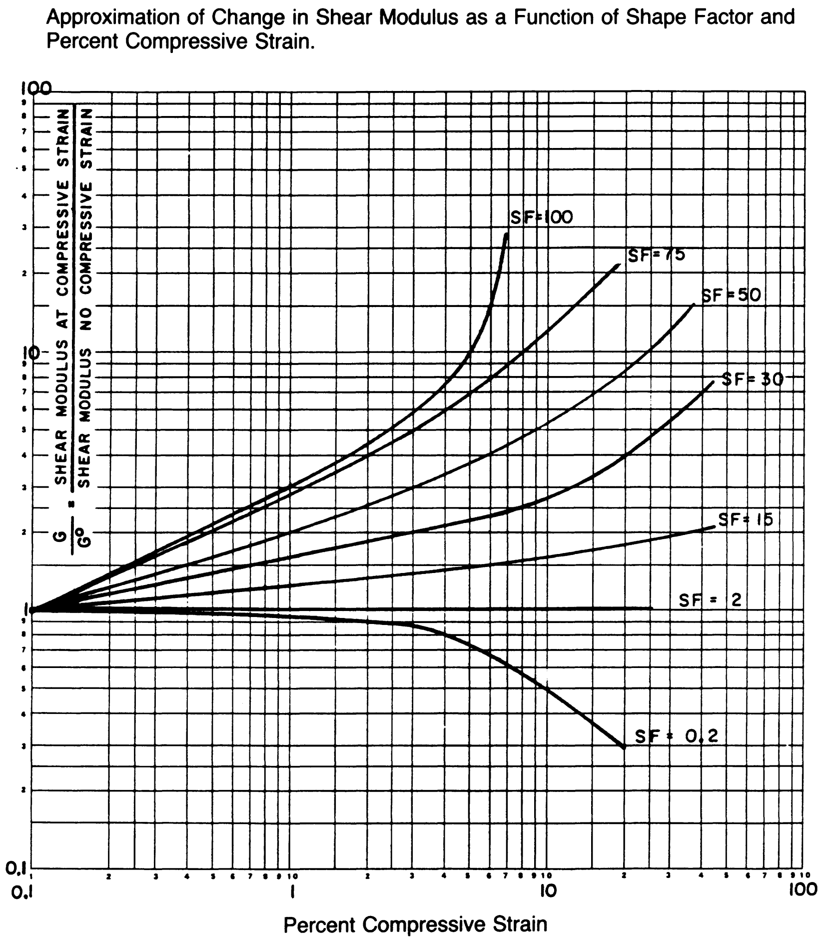

Figure 8.3 Influence of compressive strain and shape factor (SF) on shear modulus [3].

The effective compression modulus for a flat sandwich block is given by the equation

$$E_c = E_0\,(1 + 2\phi S^2) \text{ for bidirectional strain (blocks)} \tag{8.7}$$

or

$$E_c = 1.33\,E_0\,(1 + \phi S^2) \text{ for one-dimensional strain (long, thin compression strips)} \tag{8.8}$$

where E_0 is Young's modulus (see Table 8.1), ϕ is elastomer compression coefficient (see Table 8.1), S is shape factor (defined below) [4].

The coefficient ϕ is an empirically determined material property, which is included here to correct for experimental deviation from theoretical equations. Table 8.1 gives values

Table 8.1 Material Properties [3]

Shear modulus, G (kPa)	Young's modulus, E_0 (kPa)	Bulk modulus, E_b (MPa)	Material compressibility coefficient, ϕ
296	896	979	0.93
365	1158	979	0.89
441	1469	979	0.85
524	1765	979	0.80
621	2137	1,007	0.73
793	3172	1,062	0.64
1034	4344	1,124	0.57
1344	5723	1,179	0.54
1689	7170	1,241	0.53
2186	9239	1,303	0.52

for ϕ for varying elastomer moduli. The modulus and material compressibility values in Table 8.1 are used to solve Problem 8.2.1.1 (below) as well as other numbered problems in this chapter.

The shape factor S is a component geometry function that describes geometric effects on the compression modulus. It is defined as the ratio of the area of one loaded surface to the total surface area that is free to bulge:

$$\text{shape factor } S = \frac{\text{load area}}{\text{bulge area}} = \frac{A_\mathrm{L}}{A_\mathrm{B}} \tag{8.9}$$

For example, the shape factor for the rectangular block of Fig. 8.1 would be:

$$S = \frac{A_L}{A_B} = \frac{(\text{length})\ (\text{width})}{2t(\text{length}) + 2t(\text{width})}$$

$$= \frac{(\text{length})\ (\text{width})}{2t(\text{length} + \text{width})}$$

Results from this method of calculating compression modulus are summarized graphically in Fig. 8.4. Given shape factor and shear modulus, this graph can be used to find effective compression modulus for a component.

Generally, rubber can be regarded as incompressible. In some cases, bulk compressibility makes an appreciable contribution to the deformation of a thin rubber pad in compression [3, 4]. The reason is that a thin pad offers great resistance to compression, and the apparent compression modulus approaches the bulk modulus in magnitude. To account for this decrease in spring rate, the calculated compression modulus should be multiplied by the following factor:

$$\frac{1}{1 + E_o/E_b} \tag{8.10}$$

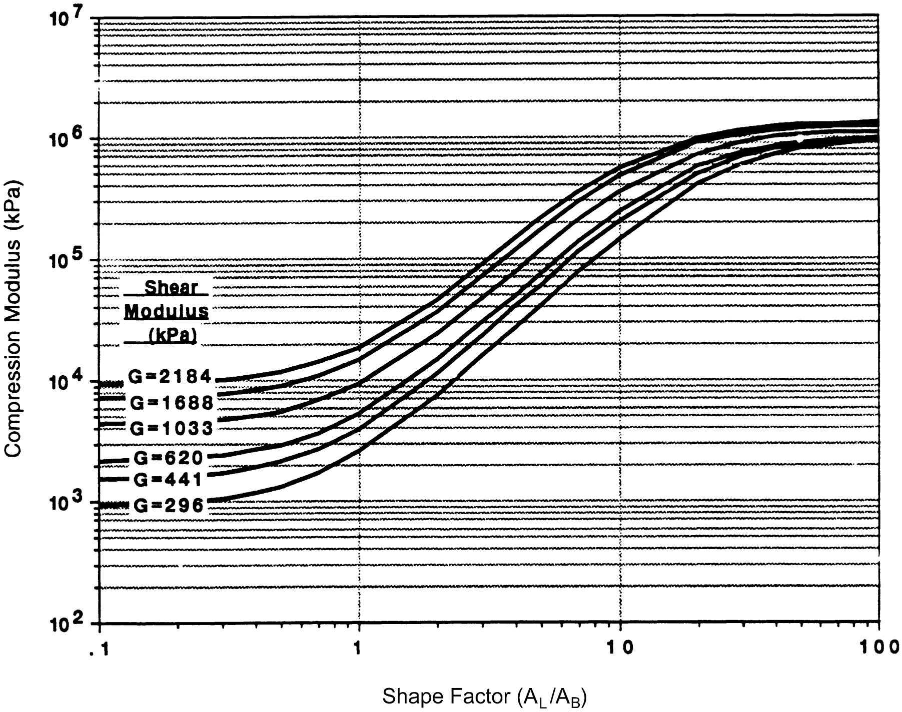

Figure 8.4 Compression modulus E_c versus shape factor S for various shear moduli [3].

where E_b is the modulus of bulk compression, about 1.1 GPa (approximation from Table 8.1).

Problem 8.2.1.1

Calculate compression and shear spring rates and their ratio K_c/K_s for a rectangular block measuring 50 mm × 75 mm × 10 mm thick, made with rubber of shear modulus $G = 793$ kPa. Assume bulk compressibility effects to be negligible.

The solution is as follows:

$$K_s = \frac{(0.050 \text{ m} \times 0.075 \text{ m}) \times (793 \text{ kN/m}^2)}{0.01 \text{ m}} \quad \text{(per Eq. 8.2a)}$$

$$K_s = 297 \times 10^3 \text{ N/m}$$

$$S = \frac{50 \text{ mm} \times 75 \text{ mm}}{(2 \times 10 \text{ mm}) \times (50 \text{ mm} + 75 \text{ mm})}$$

$$S = 1.5$$

$$E_c = 3172\text{ kPa}\,[1 + 2(0.64)(1.5)^2] \qquad \text{(per Eq. 8.7 and Table 8.1)}$$

$$E_c = 12{,}300\text{ kPa}$$

$$K_c = \frac{(0.050\text{ m} \times 0.075\text{ m})(12.3\text{ MPa})}{0.01\text{ m}} \qquad \text{(per Eq. 8.2b)}$$

$$K_c = 4.61 \times 10^6\text{ N/m}$$

$$K_c/K_s = \frac{4.61 \times 10^6}{297 \times 10^3} = 15.5$$

8.2.2 Laminate Bearings

The introduction of rigid shims into the elastomeric section of a bearing is often used to increase compression spring rate while maintaining the same shear spring rate. In the case of a laminate bearing, the compression spring rate equation becomes

$$K_c = \frac{F_c}{d_c} = \frac{AE_c}{tN} \tag{8.11}$$

where N is the number of identical elastomer layers, t is individual layer thickness, and E_c is the individual layer compression modulus. The shear spring rate remains unchanged, assuming the total elastomer thickness is unchanged.

Problem 8.2.2.1

Calculate the shear and compression spring rates and their ratio K_c/K_s for the bearing in Problem 8.2.1.1 if the elastomer is divided into five equal thickness sections by rigid shims (Fig. 8.5). The load area and total elastomer thickness are the same, so the shear spring rate remains unchanged.

$$K_s = 297 \times 10^3\text{ N/m}$$

The compression spring rate becomes

$$K_c = \frac{AE_c}{tN}$$

where

$$S = \frac{50 \text{ mm} \times 75 \text{ mm}}{2 \times 2 \text{ mm} \times (50 \text{ mm} + 75 \text{ mm})}$$

$S = 7.5$ (per Eq. 8.7)

$E_c = 3172 \text{ kPa } [1 + 2(0.64)(7.5)^2]$

$E_c = 232 \text{ MPa}$

Including the bulk correction factor, we write

$$E_c = 232\text{MPa} \times \frac{1}{1 + 3172\text{kPa}/1062\text{MPa}}$$

$E_c = 231 \text{ MPa}$

$$K_c = \frac{(0.050 \text{ m} \times 0.075 \text{ m})(231 \text{ MPa})}{(0.002 \text{ m})(5)} \quad \text{(per Eq. 8.11)}$$

$K_c = 87 \times 10^6 \text{ N/m}$

$$K_c/K_s = \frac{87 \times 10^6}{297 \times 10^3} = 293$$

Problem 8.2.2.1 illustrates the ability to increase the ratio of compressive to shear spring rate significantly, while keeping the overall volume of the part essentially unchanged; ignoring the thickness of the rigid shims.

If the part is simultaneously subjected to both compression and shear loading, the shear spring rate may increase with increasing compression load, as shown in Fig. 8.3.

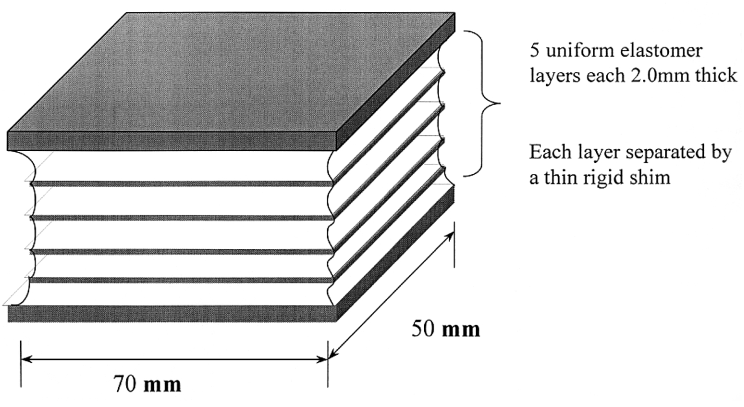

Figure 8.5 Laminate bearing (5 layers).

8.2.3 Tube Form Bearings and Mountings

Tube form mountings and rubber bushings are widely used as they offer flexibility in torsion, tilt (cocking), axial and radial directions. They provide high load carrying capacity in a compact shear isolator. In the torsion and axial directions, the rubber is used in shear and provides relatively soft spring rates. In the radial direction, the rubber is used in compression and tension, which provides much stiffer spring rates and hence, greater stability. When used as spring elements, the torsion and/or axial shear spring rates are generally the key design parameters. However, the radial and cocking spring rates also affect the behavior of the design. Determination of these spring rates is often necessary to ensure that excessive forces or deflections not occur, or that resonant frequencies will not fall within the operating range of a machine.

A general equation for torsional stiffness of a tube form elastomeric bearing with plane ends given in slightly different form in Chapter 3, Eq. (3.9) is:

$$K_{tor} = \frac{T}{\theta} = \frac{\pi G L}{1/(d_i)^2 - 1/(d_o)^2} \tag{8.12}$$

where K_{tor} = torsional spring rate
G = shear modulus
L = bearing length
d_i = bearing inner diameter
d_o = bearing outer diameter
T = applied torque
θ = angular deflection

Problem 8.2.3.1

A spring-loaded arm is used to maintain wire tension on a wire winding machine. The wire passes over several pulleys, two of which are on the hinged arm, which allows for wire slack take-up by arm rotation (see Fig. 8.6). The arm should move $\pm 5^\circ$ with $\pm$ 3 N tension variation in the wire. Assume the effect of the weight of the arm on wire tension to be negligible.

1. What size of elastomeric bearing is required at the arm pivot point? Assume an elastomer modulus of G = 1034 kPa, a bearing inside diameter of 10 mm, and an axial length of 20 mm.

To find the bearing size, begin by summing torques on the arm about the pivot point due to 3 N of wire tension.

$T_{bearing}$ = 3 N (0.030 m) + 3 N (0.050 m) + 3 N (0.060 m) + 3 N (0.080 m)
T = 0.66 N · m

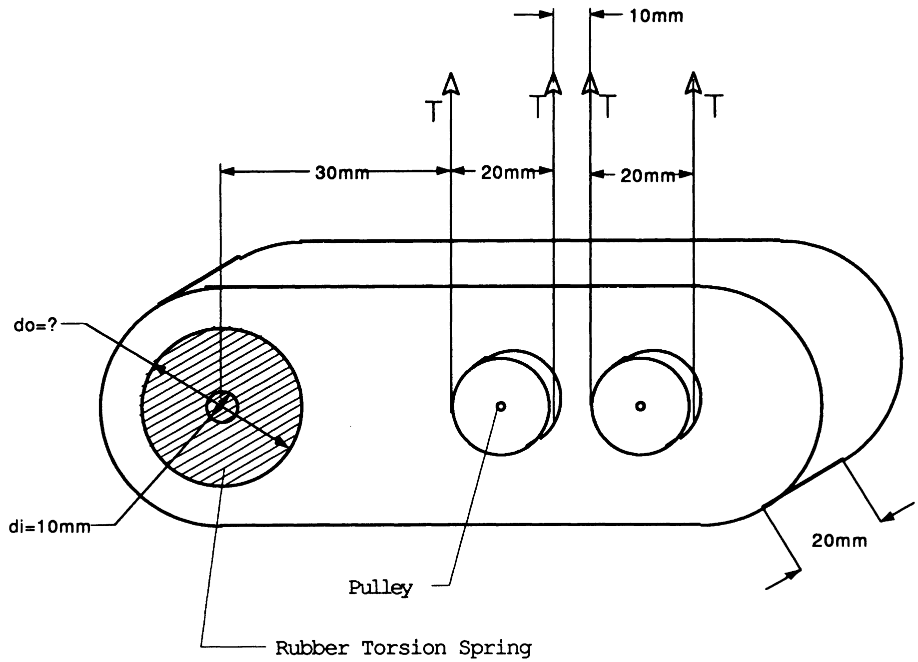

Figure 8.6 Wire tension arm.

Now calculate the desired torsion spring rate for the $\pm$ 5° motion desired.

$$K_{tor} = \frac{T}{\theta} = \frac{0.66}{5^\circ} = 0.13\text{N} \cdot \text{m/deg} = 7.6\text{N} \cdot \text{m/rad}$$

Knowing the desired spring rate, solve Eq. (8.12) for d_o.

$$7.6 = \frac{\pi(1034\text{kPa})(0.02\text{m})}{1/(0.01)^2 - 1/(d_o)^2}$$

$$\frac{1}{(d_o)^2} = \frac{1}{0.01^2} - 8590$$

$$d_o = 27 \text{ mm}$$

2. How much does the arm translate in the radial direction (vertical in Fig. 8.6) as a result of the imposed radial force caused by the wire tension?

A free body diagram on the arm shows a radial force F_r of 12 N at the bearing. The effective shape factor can be calculated as follows (see Section 8.2.4):

$$S = \frac{A_L}{A_B} = \frac{\text{load area}}{\text{bulge area}} \cong \frac{(d_o - d_i)L}{2\pi[(d_o)^2/4 - (d_i)^2/4]}$$

$S \cong 0.34$

The radial direction in a tube form acts in the compression/tension direction on the rubber. Radial deflection can be calculated using the basic equation

$$K_t = \frac{AE_c}{t} = \frac{F_r}{d_r}$$

or

$$d_r = \frac{F_r t}{AE_c}$$

where $t = (d_o - d_i)/2 = 8.5$ mm

$A \cong (d_o - d_i)\ L = 340\ \text{mm}^2$

Knowing the elastomer modulus and shape factor, we can find E_c from Fig. 8.4.

$$E_c = 5000\ \text{kPa}$$

$$d_r = \frac{12\ \text{N}(0.0085\ \text{m})}{(340 \times 10^{-6}\ \text{m}^2)(5.0 \times 10^6\ \text{Pa})}$$

$$d_r = 0.06\ \text{mm for 3N of wire tension}$$

Another method of finding tube form mount or bearing shape factors accepted in the rubber industry is to convert the tube form to an equivalent block bonded with equivalent size parallel plates (see Fig. 8.7). For the present problem, the length is 20 mm and the width can be approximated by the following expression [5].

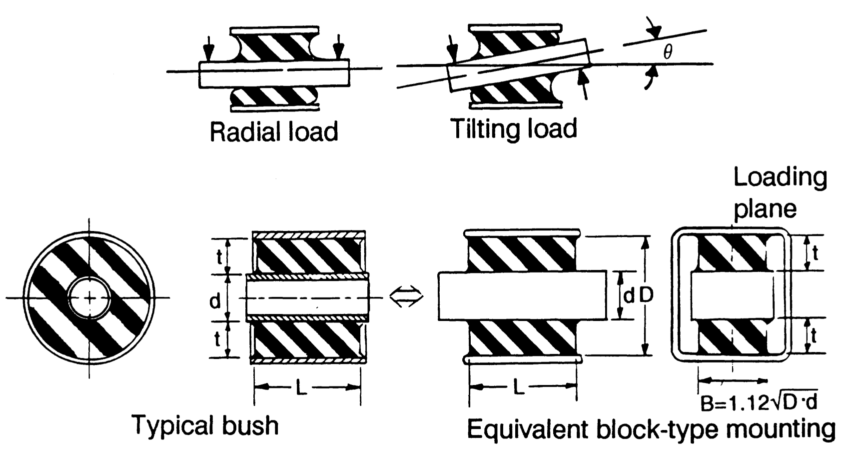

Figure 8.7 Bush and equivalent block.

$$B = 1.12\sqrt{d_o d_i} = 0.018\text{m} \tag{8.13}$$

Now, shape factor $S = A_L/B_L$

$$S = \frac{0.020(0.018)}{2(0.020 + 0.018)[(0.027\text{–}0.010)/2]}$$

$$S = 0.56$$

The compression modulus E_c is now 5900 kPa (Fig. 8.4 or Eq. 8.7), and the radial deflection is:

$$d_r = \frac{12\ \text{N}(0.0085\ \text{m})}{(340 \times 10^{-6}\ \text{m}^2)(5.9 \times 10^6\ \text{Pa})}$$

$$d_r = 0.051\ \text{mm for 3 N of wire tension}$$

This alternative method of equivalent block-tube form calculation can be used for more complicated combined radial and cocking loading conditions.

Problem 8.2.3.2

A wind tunnel diverter door (Fig. 8.8) needs to open 10° when an equivalent force of 300N aerodynamic force is exerted as shown. How large in diameter does a full-length door hinge need to be if the hinge post is 20 mm in diameter? Assume using an elastomer with $G = 621$ kPa.

From the known force and deflection, start by calculating the required spring rate.

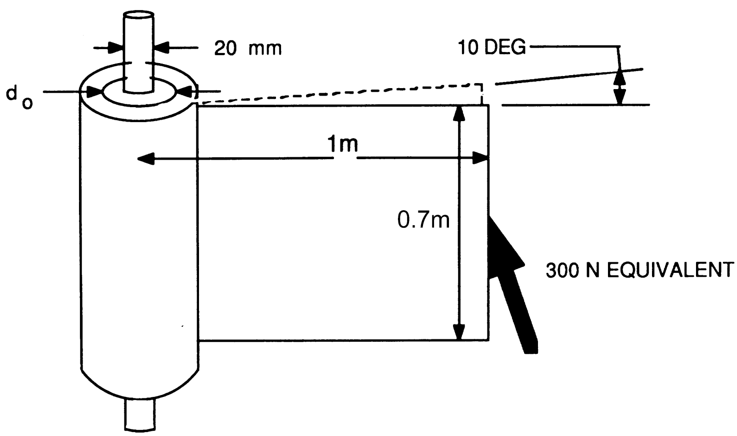

Figure 8.8 Wind tunnel hinged diverter door.

$$K_{tor} = \frac{T}{\theta} = \frac{(300\text{N})(1\text{m})}{10} = 30\text{N} \cdot \text{m/deg}$$

$$= 1719 \text{ N} \cdot \text{m/rad}$$

Now, go back and determine the unknown geometry.

$$K_{tor} = \frac{\pi G L}{1/(d_i)^2 - 1/(d_o)^2}$$

Solve for d_o:

$$1719 N \cdot \text{m/rad} = \frac{\pi(621\text{kPa})(0.7\text{m})}{1/(0.02\text{m})^2 - 1/(d_o)^2}$$

$$\frac{1}{(d_o)^2} = \frac{1}{(0.02 \text{ m})^2} - \frac{\pi(621\text{kPa})(0.7 \text{ m})}{1719\text{N} \cdot \text{M/rad}}$$

$$d_o = 24 \text{ mm}$$

8.2.4 Effective Shape Factors

The effective loaded area of tube form mountings described in Section 8.2.3 is dependent on the load or deflection of interest. Loaded areas in torsion or axial deflection are obvious, but the values used for effective loaded areas in radial or cocking deflections are only estimated approximately. In Problem 8.2.3.1, the radial load areas were taken as (d_o–d_i) L and 1.12 L$\sqrt{d_o d_i}$. It is not clear which of the various projected areas or effective areas should be used in radial and cocking stiffness calculations (see Fig. 8.9). To further complicate the calculation of stiffness for tube form mountings, secondary processes are often used to induce pre-compression to enhance performance and fatigue life. These secondary processes include

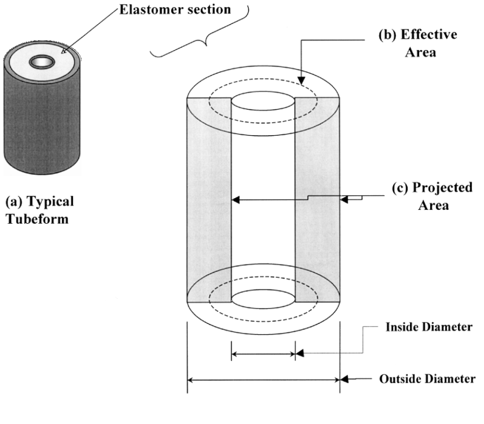

Figure 8.9 Radial and cocking load areas for a tube form.

- Spudding or enlarging the diameter of the inner member after bonding
- Swaging or reducing the outer member diameter after bonding
- Molding at pressures high enough to cause residual pre-compression in the elastomer

Figure 8.10 shows how swaging, spudding, and high pressure molding affect elastomer behavior. In a tube form mounting without pre-compression, one side of the elastomer works in tension and the other works in compression. Elastomers in tension have low modulus and can be damaged at relatively low loads due to internal cavitation. Elastomers in compression, on the other hand, particularly in high shape factor designs, have an effective modulus that can approach the modulus of bulk compression. Pre-compressing a tube form mounting effectively makes the "tension side" work in compression. This means that the radial and cocking stiffness can be nearly twice as high. This effect holds until the deflection reaches a point where the initial pre-compression is relieved. When pre-compression is used, the effective shape factor increases as a result of an apparent increase in loaded area. It also could be said that the shape factor does not change, but the loaded area increases. In either case, some compensation is made to the numerator of the expression: $K_r = AE_c/t$ in calculating radial or cocking stiffness. For most engineering designs, the radial stiffness of a tube form mounting is calculated both with and without pre-compression to bound the expected performance.

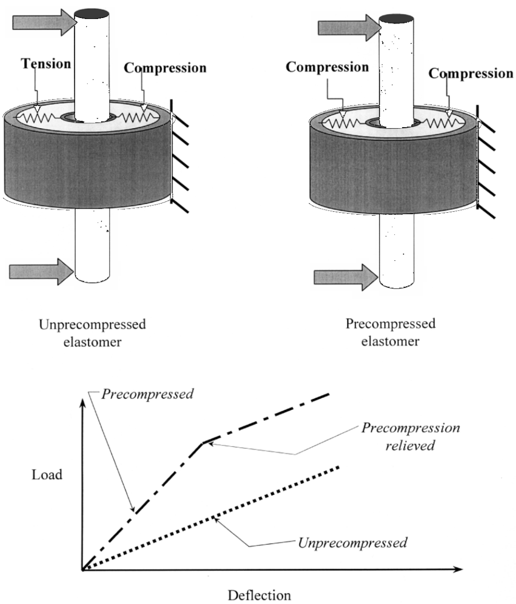

Figure 8.10 Tubeform models for pre-compressed and unpre-compressed elastomer.

8.3 Vibration and Noise Control

The examples in this section show how to establish design requirements from a basic problem statement and then relate these requirements to the design of an appropriate elastomeric product.

8.3.1 Vibration Background Information

Solving vibration and noise control problems with elastomeric products requires understanding basic product design concepts and vibration theory. Developing the basic equations from vibration theory is outside the scope of this chapter. There are a number of excellent references that deal with this subject.

For the purposes of this section, all systems are assumed to be represented by a damped, linear, single degree of freedom system as shown in Fig. 8.11. The functions of the spring and damper in the mechanical system of Fig. 8.11a are replaced by a single elastomeric part in Fig. 8.11b, which works as both spring and damper.

Two basic formulas are needed to work vibration isolation problems. The first defines the natural frequency of vibration for the isolation system and the second defines the transmissibility of the system as a function of frequency.

$$f_n = \frac{1}{(2\pi)}[(K'g)/W]^{1/2} \tag{8.14}$$

which reduces to

$$f_n = 15.76(K'/W)^{1/2} \tag{8.15}$$

where f_n = system natural frequency of vibration (Hz)
K' = dynamic spring rate (N/mm)
g = gravitational constant = 9800 mm/s^2
W = weight of the system (N)

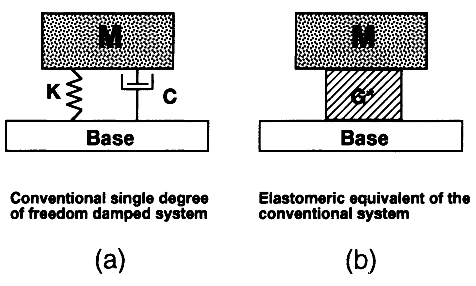

Figure 8.11 Damped linear single degree of freedom model.

The second formula is

$$T_{ABS} = \left[\frac{1 + (\eta r)^2}{(1 - r^2)^2 + (\eta r)^2}\right]^{1/2} \tag{8.16}$$

which reduces to

$$T_{ABS} = \frac{1}{r^2 - 1} \text{ for } r > \sqrt{2} \text{ and } \eta \approx 0 \tag{8.17}$$

where T_{ABS} = transmissibility of input vibration at f
r = frequency ratio = f/f_n (dimensionless)
f = vibration input frequency (Hz)
η = dimensionless loss factor, defined in Eq. (8.19)

Equation (8.16) for transmissibility is shown in graphical form in Fig. 8.12. Note that isolation of input vibrations begins at a frequency of roughly $f/f_n = \sqrt{2}$, above which T_{ABS} is less than 1.0. Also note that isolation at high frequencies decreases as the damping in the system increases. Finally, for elastomeric products the actual isolation at high frequencies is slightly less than predicted by the classical spring damper analysis used to create Fig. 8.12; this is a result of deviation from a single degree of freedom model. The actual isolation depends on the type of elastomer, the type and magnitude of input, the temperature, and the amount of damping present.

It is appropriate to note here that the elastomer shear modulus is actually a complex number G * consisting of a real and complex part (see Chapter 4):

$$G^* = G' + iG'' \tag{8.18}$$

or

$$G^* = G' (1 + i\eta) \tag{8.19}$$

where η is called the loss factor (tan δ), given by $\eta = G''/G'$, G'' is the damping modulus (MPa), and G' is the dynamic elastic modulus (MPa).

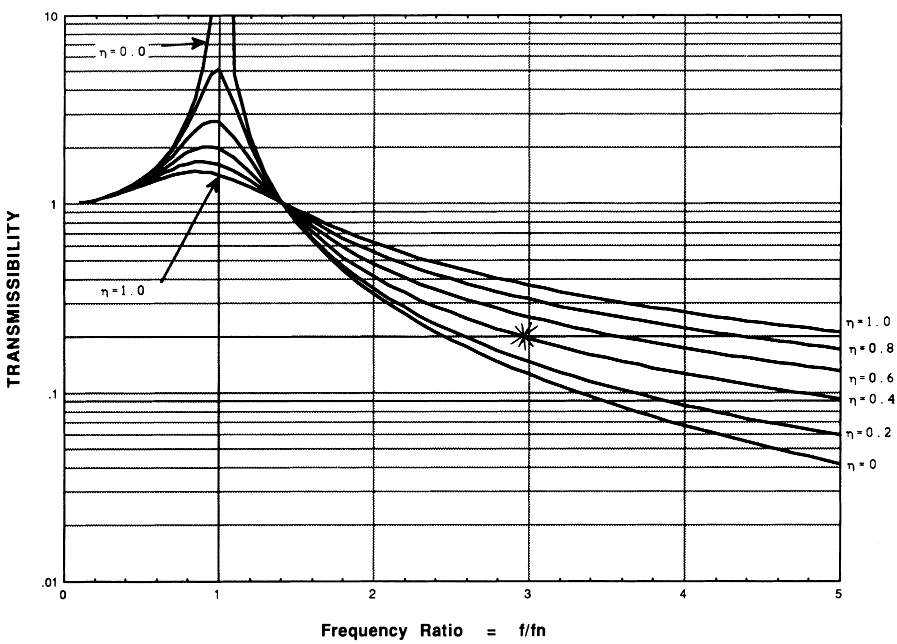

Figure 8.12 Transmissibility function.

8.3.2 Design Requirements

The basic equations from Section 8.1 and equations for vibration performance given above can be combined to solve product design problems. The starting point for any design is understanding the basic requirements. The most important factors include:

- Specifications for the equipment to be isolated (typically: weight, size, center of gravity, and inertias)
- Types of dynamic disturbance to be isolated (sinusoidal and random vibration, frequency and magnitude of inputs, shock inputs, etc.)
- Static loadings other than weight (e.g., a steady acceleration in many aircraft applications)
- Ambient environmental conditions (temperature ranges, humidity, ozone, exposure to oils and other fluids, etc.)
- Allowable system responses (What are the maximum forces the isolated equipment can withstand? What is the maximum system deflection allowed?)
- Desired service life

8.3.3 Sample Problems

Problem 8.3.3.1

A sensitive piece of electronic equipment is to be mounted on a platform that is subjected to a 0.4 mm SA sinusoidal vibration at 50 Hz. SA means single amplitude, so the peak-to-peak motion is 0.8 mm. The input vibration (disturbance) is primarily in the vertical direction. The task is to design a vibration isolator that provides 80% isolation while minimizing the clearances necessary to allow this level of isolation. The equipment has a mass of 5.5 kg and a weight of 53.9 N. Assume use of a very low damped elastomer with a loss factor $\eta = 0.02$ for this design.

To begin the design, solve Eq. (8.17) to find the system natural frequency that will provide $T_{\mathrm{ABS}} = 0.2$, which is equivalent to 80% isolation. Equation (8.17) is acceptable because the material has a low loss factor and r exceeds $\sqrt{2}$ for isolation:

$$T_{\mathrm{ABS}} = \frac{1}{f^2/(f_n)^2 - 1} \quad \text{(solve for } f_n\text{)}$$

$$(f_n)^2 = \frac{f^2 T_{\mathrm{ABS}}}{1 + T_{\mathrm{ABS}}}$$

where $f = 50$ Hz and $T_{\mathrm{ABS}} = 0.2$

$(f_{\mathrm{n}})^2 = (50)^2\ (0.2)/1.2$

$f_{\mathrm{n}} = 20.4$ Hz

Therefore, the isolation system must be designed to have a natural frequency of 20.4 Hz to isolate 80% of an input vibration at 50 Hz.

The isolation system dynamic spring rate can now be found using Eq. (8.15) and solving for K':

$$K' = \frac{(f_n)^2 W}{248.4}$$

where $f_n = 20.4$ Hz and $W = 53.9$ N
$K' = (20.4)^2\ (53.9)/248.4$
$K' = 90.3$ N/mm

Now some assumptions and decisions about the isolation system need to be made to design the individual isolators. A key assumption is that the structural components of the system are infinitely rigid, so all deflections occur in the isolator. For stability, a four-isolator system is used. Each isolator is located symmetrically about the equipment center of gravity as shown in Fig. 8.13. Since the input is primarily in the vertical direction, the isolators are oriented such that vertical deflections cause shear deflections. To provide the desired isolation, a system dynamic spring rate of 90.3 N/mm is required. The dynamic spring rate for each isolator is then 22.6 N/mm. The static load per isolator due to the equipment weight is 13.5 N.

To find the appropriate isolator size, some design limits need to be applied. For a starting point, limit the static stress on the isolator to 0.069 N/mm^2. Knowing the static

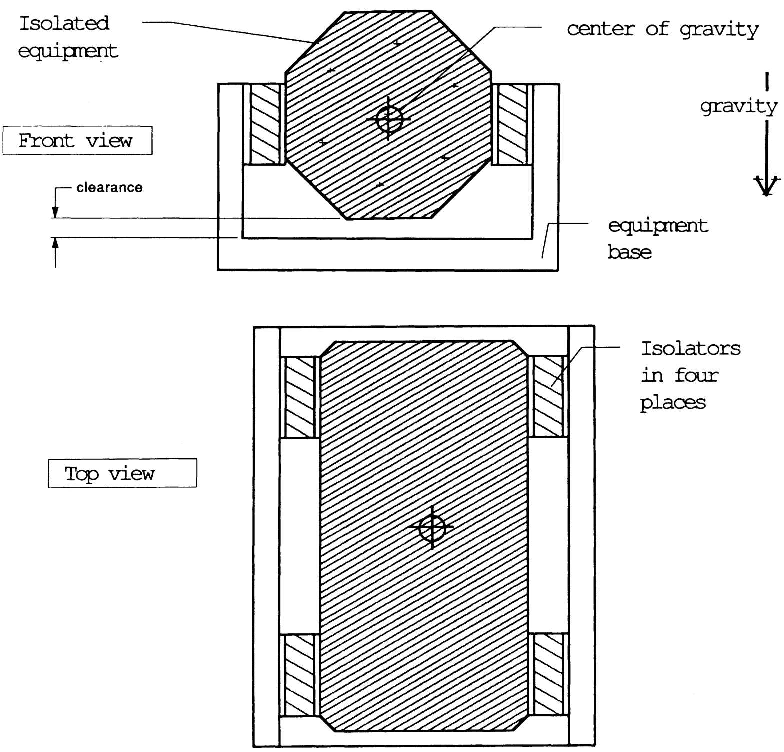

Figure 8.13 Four-mount system.

load of the equipment and applying a static stress limit, the minimum load area for the isolator can be determined using Eq. (8.3):

$$\sigma = \frac{F}{A}$$

where $\sigma = 0.069\ \text{N/mm}^2$
$F = 13.5\ \text{N}$

so

$$A_{min} = \frac{13.5}{0.069} = 196\ \text{mm}^2$$

From the specifications for the low damped elastomer selected for this isolator, we find an available range of modulus from 0.345 to 1.38 N/mm^2. From this range we select a modulus and use Eq. (8.2a) to determine the thickness of the elastomer required to get the desired dynamic spring rate:

$$K_s' = \frac{AG'}{t}$$

where $A = 196\ \text{mm}^2$, and we select $G' = 0.69\ \text{N/mm}^2$. Then

$$t = 196\left(\frac{0.69}{22.6}\right) = 6\ \text{mm}$$

This completes the first iteration of the isolator design with the following results:

Load area $A = 196\ \text{mm}^2$
Thickness $t = 6$ mm
Dynamic shear modulus $G' = 0.69\ \text{N/mm}^2$
Dynamic shear spring rate $K' = 22.6$ N/mm

The isolation system uses four isolators in shear with a total system dynamic spring rate (4)(22.6) = 90.3 N/mm. This provides a system natural frequency of 20.4 Hz for the specified weight of the isolated equipment. In turn, the equipment is isolated from 80% of the 50 Hz vertical sinusoidal disturbing vibration. Before the design is finalized, the static shear strain in the elastomer must be checked against design limits. This calls for the determination of the static spring rate for the isolator.

In general, the elastomer shear modulus is affected by frequency and strain. Therefore, the static spring rate K of an elastomeric isolator can be much softer than the dynamic spring rate K'. This difference tends to increase as the amount of damping in the elastomer increases. A dynamic-to-static ratio of 1.1 is reasonable for the low damped elastomer ($\eta = 0.02$) selected for this isolator.

Using the 1.1 factor, the static shear spring rate is given by:

$K_s = K'_s/1.1$ where $K'_s = 22.6$ N/mm

$K_s = 20.5$ N/mm

The static deflection is then given by Eq. (8.1):

$$K_s = \frac{F}{d}$$

where $K_s = 20.5$ N/mm and F (the static equipment weight) = 13.5 N. Thus,

$$d = \frac{13.5}{20.5} = 0.66 \text{ mm}$$

The static shear strain is then given by Eq. (8.4):

$$\varepsilon = \frac{d}{t}$$

where $d = 0.66$ mm and $t = 6$ mm

$$\varepsilon = \frac{0.66}{6} = 0.11 = 11\%$$

A reasonable limit for static shear strain in this case is 20%, so the isolator design meets both static stress and strain criteria. The dynamic input vibration was given as 0.4 mm SA. The transmissibility of the isolation system at the disturbing frequency is 0.2 by design; therefore the dynamic deflection transmitted to the equipment is

$$(0.2)(0.4) = 0.08 \text{ mm}$$

resulting in a dynamic strain of

$$\frac{0.08}{6} = 0.013 = 1.3\% \text{ at } 50 \text{ Hz}$$

The total clearance required to provide the desired isolation is

$$0.66 \text{ mm static} + 0.08 \text{ dynamic} = 0.74 \text{ mm}$$

In actual practice, additional allowances need to be made for temperature, fatigue, and long-term drift effects when establishing adequate clearance.

A system with a natural frequency lower than 20.4 Hz would have provided even better isolation. However, the lower natural frequency means a lower spring rate, which results in increased static deflections, requiring additional clearance in the installation. Since one of the design goals was to minimize the required clearance, the 20.4 Hz system would be considered to be the best choice.

For the final step, assume that the isolator is circular. The diameter required to provide 196 mm^2 load area is

$$\left[\frac{(196)(4)}{\pi}\right]^{1/2} = 15.8 \text{ mm}$$

The resulting isolator design is shown in Fig. 8.14.

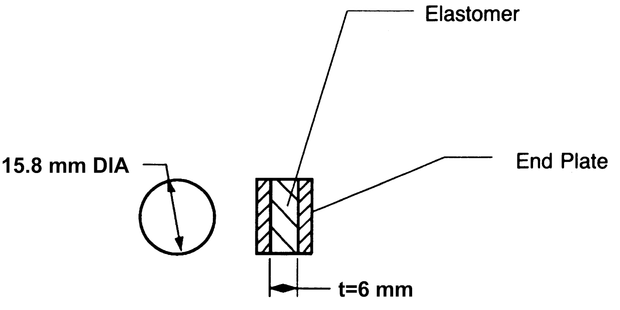

Figure 8.14 Final isolator design.

Problem 8.3.3.2

Check the design of Problem 8.3.3.1 to see whether bending will impact the performance.

Section 8.2 demonstrates that bending may affect the shear spring rate if the elastomer thickness-to-length ratio is greater than about 0.25. For the isolator of Problem 8.3.3.1, the length (= diameter) is 15.8 mm and the thickness is 6 mm. Therefore the ratio is

$6/15.8 \approx 0.38$

This means that bending may tend to reduce the actual spring rate of the isolator. From Eq. (8.5), the bending factor is calculated to be 0.96. Applying the bending correction factor has the following impact on the isolation system:

Isolator dynamic spring rate becomes (22.6)(0.96) = 21.7 N/mm
System dynamic spring rate becomes (4)(21.7) = 86.8 N/mm
Solving Eq. (8.15) for the new system natural frequency yields

$$f_n = 15.8\left(\frac{86.8}{53.9}\right)^{1/2} = 20 \text{ Hz}$$

The original design gave 20.4 Hz, so bending can be considered to have a negligible impact on performance, although some additional clearance space should be allowed.

Problem 8.3.3.3

For Problem 8.3.3.1, the only input was at 50 Hz. When the equipment and the isolation system were installed, it was found that vibrations in the 10 to 50 Hz range were also present and caused the equipment to malfunction. From additional measurements, it was determined that the input vibration disturbance was 0.25 mm SA and that the equipment could withstand a maximum vibration disturbance of 0.9 mm in the 10 to 50 Hz frequency range. What design changes are needed to maintain 80% isolation of the 50 Hz disturbance while meeting the new requirements?

The original design used a very low damped elastomer with a high transmissibility at resonance. This was chosen to provide the desired isolation with the highest possible system natural frequency to limit the necessary clearance in the installation. Note in Fig. 8.12 that as the amount of damping increases (η increasing), the isolation at any given r value decreases. In other words, additional damping tends to decrease the isolation efficiency of the isolator. The trade-off is the transmissibility at resonance T_r. At resonance $f = f_n$, and Eq. (8.16) can be reduced to

$$T_r \approx \frac{G'}{G''} \tag{8.20}$$

For the material chosen in Problem 8.3.3.1, $\eta = 0.02 = G''/G'$, therefore

$$T_r = \frac{1}{0.02} = 50$$

The resulting vibration at the 20 Hz system natural frequency would be

$$(\text{input SA})(T_r) = (0.25\text{ mm})(50) = 12.5\text{ mm}$$

which is clearly much greater than the 0.9 mm the equipment can withstand. In fact, the 12.5 mm dynamic deflection produces 209% dynamic strain, which is much greater than the isolator designed in Problem 8.3.3.1 could withstand for an extended time. Typically isolators are limited to less than 30 to 40% dynamic strain to minimize heat buildup and fatigue.

Given a 0.25 mm input and a 0.9 mm limit for transmitted vibration, the maximum allowable T_r becomes

$$T_{r\,\max} = \frac{0.9}{0.25} = 3.6$$

From a catalog of available materials, an elastomer with $\eta = 0.4$ is chosen for the redesigned isolator, because $\eta = 0.4$ is equivalent to $T_r = 2.5$.

The maximum transmitted vibration at resonance for this elastomer is then:

$$(\text{input SA})(T_r) = (0.25)(2.5) = 0.625\text{ mm}$$

Having selected an elastomer with an appropriate loss factor to meet the requirement for maximum transmitted vibration at resonance, Fig. 8.12 can be used to find the frequency ratio required to provide 80% isolation of the 50 Hz disturbing vibration. The $\eta = 0.4$ curve intersects with the $T_{ABS} = 0.2$ line at $r = 2.95$:

$$r = 2.95 = \frac{f}{f_n}$$

where $f = 50$ Hz

$$f_n = \frac{50}{2.95} = 16.9 \text{ Hz}$$

Repeating the same procedure used in Problem 8.3.3.1, the system dynamic spring rate, individual isolator spring rate, and isolator design parameters can be determined:

$$K' = \frac{(f_n)^2 W}{248.4}$$

where $f_n = 16.9$ Hz and $W = 53.9$ N

$$K' = \frac{(16.9)^2\ (53.9)}{248.4}$$

$K' = 62$ N/mm = system dynamic spring rate

$\frac{62}{4} = 15.5$ N/mm = isolator dynamic spring rate

The static stress conditions are still the same, so

$A_{min} = 196\ \text{mm}^2$ (from Problem 8.3.3.1)

Assuming a dynamic modulus G' of 0.69 N/mm^2,

$$t = \frac{(196)(0.69)}{15.5} = 8.7 \text{ mm}$$

The dynamic strain at resonance is

$$\varepsilon = \frac{(\text{input SA})(T_r)}{t}$$

$$\varepsilon = \frac{(0.25)(2.5)}{8.7} = 7\%$$

which is acceptable.

The area has not changed, so the diameter of the isolator is 15.8 and the thickness-to-length ratio now becomes

$$\frac{8.7}{15.8} = 0.55$$

This factor can be reduced by maintaining the original thickness of 6 mm to reduce any additional bending effects. In this case the elastomer modulus must be changed to obtain the desired dynamic shear spring rate:

$$K' = \frac{AG'}{t}$$

where t = 6 mm, A = 196 mm^2, and K'_s = 15.5 N/mm,

$$G' = \frac{(15.5)(6)}{196}$$

$$G' = 0.47 \text{ N/mm}^2$$

A quick check of the materials specifications for the chosen elastomer shows that this modulus is available. The static shear deflection for the system now is found as follows.

Assume:

$$\frac{K'}{K_s} \approx 1.4 \text{ for the elastomer chosen}$$

$$K_s \approx 15.5/1.4 = 11.1 \text{ N/mm}$$

$$d = \frac{F}{K_s} = \frac{13.5}{11.1} = 1.22 \text{ mm}$$

The resulting static shear strain on the elastomer is

$$\varepsilon = \frac{1.22}{6} = 0.204 = 20.3\%$$

This is marginally acceptable for this application. Finally, the total deflection for the new system is

1.22 mm static + 0.63 mm dynamic = 1.85 mm

Note that the dynamic deflection at 50 Hz (0.08 mm) is less than the dynamic deflection at resonance (0.63 mm). The larger of the two was used to determine the maximum clearance required.

In this case, the new design requirements were accommodated by changing the elastomer without changing any of the isolator geometry parameters. However, the installation had to be changed to allow for the increased clearance required by the softer system. Had the system been softened any further, changes to the thickness and area would have been required to stay within reasonable static strain limits while keeping the necessary spring rate. Many further iterations of this problem could be performed by changing the input conditions or applying new temperature and/or environmental constraints. In the problems above, the size of the isolator was determined by the static design limits. In other cases, the size may be determined by the dynamic design limits on stress and strain. This is usually the case for isolators experiencing relatively high inputs and/or transmissibilities at resonance.

8.4 Practical Design Guidelines

- Shear stress-strain performance can be assumed to be fairly linear up to 75 to 100% strain for rough sizing purposes.
- Tension and compression stress-strain performance can be assumed fairly linear up to 30% strain.
- Most product designs intentionally avoid using rubber in direct tension. Fatigue resistance and design safety requirements are usually best met by using rubber in compression and shear.
- A conservative starting point for isolator design is 0.069 N/mm^2 static stress. This minimizes potential drift for most lightly to moderately damped elastomers.
- Typical vibration isolators limit dynamic strains to 30 to 40% maximum to minimize fatigue wear and heat build-up.
- When designing within a range of available modulus for a given elastomer, it is best to stay away from both the softest and the stiffest available. Approximations in the calculations and tolerances in the elastomer manufacturing process need to be allowed for. If a design uses the softest material available, and the part then turns out to be slightly too stiff, a costly design change is required to produce a softer part. If some room were allowed for changes in the original modulus selection, a softer part could be produced simply by using a lower modulus compound.
- To complete a design project involving dynamic conditions, you need to know the dependence of modulus on frequency, strain amplitude, and temperature. In general, as damping in a material decreases, the dependence of modulus on frequency and strain amplitude also decreases.
- The relationship of dynamic to static modulus depends on the specific conditions and the specific material properties. Practically, static is assumed to mean a loading rate slower than one cycle per minute.
- For compression designs, stability generally becomes a factor when the overall elastomer thickness approaches the width or diameter of the design.

Tolerances:
In the calculations, assumptions and approximations are made about material properties, geometric factors, and loading. Examples include the effective loaded area for a tube form mounting, the effective compression modulus of the elastomer, and the assumption of infinitely rigid metal parts.

In manufacturing, tolerances in material properties and component geometry cause variations in the actual performance of a design. Also, in testing a component, many variables exist that can significantly affect test results. Loading speed, test fixture rigidity, deflection measurement and operator/machine repeatability are some of the important factors.

Given the above causes of variability, it is not unusual to have differences between actual and calculated performance. However, as long as the assumptions in the calculation are not changed, empirical relationships can be developed between calculated performance and the results obtained using standardized tests. Most manufacturers have developed their own proprietary relationships of this kind and are able to predict actual product performance very closely using closed-form solutions like those presented in this book.

8.5 Summary and Acknowledgments

This chapter presented a few specialized examples for designing elastomeric products. Resources containing additional examples and information are readily available from elastomeric product manufacturers. For example, Reference [6] contains numerous examples and additional theory for vibration and shock isolation. For examples of seal design, hose design, and so on, the reader is referred to product catalogs for the major manufacturers in those industries.

For the problems presented here, the design limits, procedures, and assumptions were based on the authors' practical design experience. The various values for elastomer modulus, assumed in the problems, were based on currently available materials.

Table 8.2 is presented to assist the reader in using both the SI and English systems of units.

Thanks to three Lord Corporation colleagues, Jesse Depriest, Paul Bachmeyer and Don Prindle for their critical review of this chapter.

Table 8.2 English to SI Conversion Factors

To convert from	To	Multiply by	Use for
Psi	kPa or kN/mm^2	6.895	Modulus
lb force	N	4.448	Force
in.	mm	25.4	Deflection
lb force/in.	N/mm	0.1751	Spring rate
	$g = 9.8$ m/s^2 = 9800 mm/s^2 = 386 in/s^2		