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Bonding stresses between piezoelectric actuators and elastic beams

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ABSTRACT

A beam with piezoceramic elements bonded to the beam's top and bottom surfaces is investigated. Simple pin-force models allow to predict the deformation of the beam very well for the static case. As the pin-force models are simple they are well suited for active control algorithms of such structures. In experiments, however a debonding of the ceramics can occur due to high stresses in the bonding layer. The simple pin-force models are not able to predict these stresses. Therefore, a Finite Element (FEM) calculation is used to estimate the shear and normal stresses. As could be expected a stress concentration occurs at the ends of the ceramics. The maximum stress obtained by the FEM calculations strongly depends on the discretization. The analysis shows that these stresses are moderate in the static case but may be very high if the structure is excited in resonance. In the future a more sophisticated beam model will be developed to get analytic approximations of the interface stresses due to the interaction between the ceramics and the beam.

Keywords: Pin-Force models, Actuators, smart materials, piezoceramics, FEM of smart structures

1 INTRODUCTION

During the last decades piezo-actuated structures have become important in many industrial applications. They are used for example for the active damping of slender beams and space structures, excitation of the bending waves of ultrasonic motors and for adaptive wings to name only a few. Figure 1 shows the stator of a travelling wave linear motor with piezoceramics bonded to the surfaces of the beams. Also the principle of an actively damped beam can be seen. Two piezoceramic elements are bonded to a beam to control the damping mechanism. One of the elements is used for sensing, the other for actuation. The sensor signal is fed into a controller which

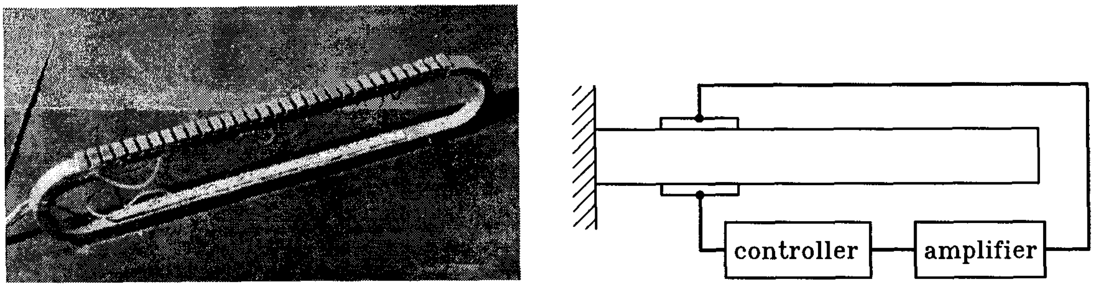

Figure 1: Two applications of piezoceramics: Stator of a travelling wave motor and an actively damped beam

excites the actuator via an amplifier such that an optimal damping of the beam may be achieved.

In many cases piezoceramic elements are bonded to the surfaces of beams or plates or in the case of composite materials they may be embedded into the structures. The piezoceramic elements may then be used for actuation as well as for sensing. The advantage of piezoceramic actuators is that they are light-weighted and easy to handle. Many actuators are used in parallel to form a so-called smart or intelligent structure.

Especially for the case of active damping, many articles have been published recently and it would be beyond the framework of this paper to cite all of them. These investigations focus on the modelling of the actuators and sensors such that they may be incorporated into simple control algorithms.[1–4] This leads to the pin-force models for bending and longitudinal actuation. For these models constant strain in the piezoceramic elements or Bernoulli-Euler hypothesis is used, assuming that the electric field in the piezoceramic is given.[5–17] Most of the authors introduce contact forces between the piezoceramics and the beam and incorporate the piezoelectric effect as an additional strain. To our opinion it is more straightforward to use for example Hamilton's principle for electromechanical systems to derive the equations of motion. Therefore, a brief summary of the pin-force models is given with respect to a consistent derivation of the equations and with respect to the results.

The kinematics of the deformation introduced in the pin-force models lead to a singular shear stress distribution in the bonding layer. As debonding was observed in experiments done in the field of ultrasonic motors, it is important to gain knowledge of the realistic shear stress distribution in the interface layer is important. Furthermore, it could be expected that there are not only high shear stresses but also a high normal stress.

In this paper we describe first three pin-force models for a static deformation. With a derivation of the equations of motion by Hamilton's principle the dynamic case may be considered as well. In a second step the results for the transverse deflection of the different pin-force models are compared with results of a Finite Element Analysis (FEA). The FEA also allows to estimate the stresses in the bonding layer.

The system under consideration is a beam with two piezoceramic elements bonded to the upper and lower surfaces. The beam of length ℓ may be divided into three sections each having a length ℓ_i and described by a local coordinate x_i (Fig. 2). The thickness of the beam is $2h$, the thickness of the piezoceramic elements h_P. For the dynamic case it is assumed that the beam is simply supported at the ends $x_1 = 0$ and $x_3 = \ell_3$. The FEA has the disadvantage in comparison with simple beam models that all geometrical and material parameters have to be given with numerical values instead of analytical expressions.

2 PIN-FORCE-MODELS

In this section a survey is given of the simple pin-force-models which may be found in the literature. They are applied for bending actuation of the beam shown in Figure 2. The polarizations of the piezoceramic elements are in direction of the z-axis and the electric field has only a component E_3 also in direction of the z-axis (upper ceramic) or in opposite direction (lower ceramic). For all these models ideal bonding of the piezoceramic elements to the beam is assumed. Therefore, the displacements of the beam and of the ceramic at corresponding points of

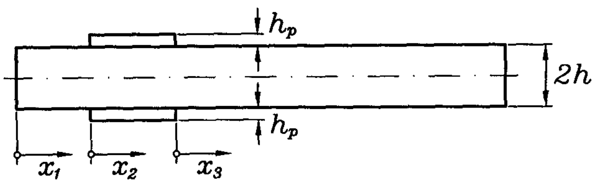

Figure 2: Geometry of the beam

the bonding surface are equal. In reality there is a finite thickness of the bonding layer which limits the maximum stress to a certain amount. The same assumption holds for the model investigated in section 3 by the Finite Element method.

In the piezoceramic elements the electric field and the electric displacement are coupled with the mechanical stress and strain. In the following the abbreviated form of the constitutive equations is applied which is described for example in the IEEE-standard.[18] Instead of the strain and stress tensors the corresponding matrices containing the six independent quantities of the tensors or multiples thereof are used

$$\begin{aligned} T &= (T_1, T_2, T_3, T_4, T_5, T_6)^T = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{xy})^T & (1)\\ S &= (S_1, S_2, S_3, S_4, S_5, S_6)^T = \left(\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial w}{\partial z}, \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right). & (2) \end{aligned}$$

In equation (2) u, v, w denote the displacements in direction of the x, y and z axes, respectively. The dielectric displacement D and the electric field are vectors with components D_i and E_i. In reality the relations between D, E, S and T are nonlinear, especially if large strains or large electric fields are applied. Also hysteretic effects may occur. For the present analysis, however, it is assumed that the constitutive equations may be linearized. For the pin-force models it seems to be convenient to write the constitutive equations in the form

$$\begin{aligned} D_i &= \varepsilon_{ij}^S E_j + e_{iJ} S_J, & (3)\\ T_I &= -e_{iI}^T E_i + c_{IJ}^E S_J. & (4) \end{aligned}$$

In these equations ε_{ij}^S denotes the permittivity matrix for constant strain, c_{IJ}^E the stiffness matrix for constant electric field and e_{iJ} the piezoelectric matrix which couples the mechanical and electrical fields. The indices i, j range from 1 to 3 and I, J from 1 to 6.

The matrices c_{IJ}^E and e_{iJ} for piezoceramic materials with a polarization in direction of the 3-axis have the form

$$c_{IJ}^E = \begin{pmatrix} c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & 0 \\ c_{12}^E & c_{22}^E & c_{13}^E & 0 & 0 & 0 \\ c_{13}^E & c_{13}^E & c_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(c_{11}^E - c_{12}^E) \end{pmatrix}, \qquad (5)$$

$$e_{kJ} = \begin{pmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{pmatrix}. \qquad (6)$$

In most cases it is assumed that the electric field in the piezoceramic elements is given. In practice not the electric field but the electric potential is prescribed which is applied at surface of the piezoceramic. Therefore, an exact analysis requires that within the ceramic the divergence of the dielectric displacement vanishes

$$\nabla \cdot D = 0. \qquad (7)$$

This equation is not taken into account in the pin-force models which may be found in the literature. This means that normally the influence of the deformation (strain) on the electric field is neglected.

The constitutive equations (3) and (4) describe the 3-dimensional case. In beam theories normally it is assumed that the stresses T_2 and T_3 vanish. This leads to plane stress or, for a beam theory, which takes into account only the bending deformation, it leads to a one-dimensional stress. According to plane or one-dimensional stress we have

$$T_1 \neq 0, \qquad T_2 = 0, \qquad T_3 = 0 \tag{8}$$

so that the constitutive equation (4) together with equations (5) and (6) imply

$$\begin{pmatrix} S_2 \\ S_3 \end{pmatrix} = -\begin{pmatrix} c_{22}^E & c_{13}^E \\ c_{13}^E & c_{33}^E \end{pmatrix}^{-1} \begin{pmatrix} c_{12}^E \\ c_{13}^E \end{pmatrix} S_1 + \begin{pmatrix} c_{22}^E & c_{13}^E \\ c_{13}^E & c_{33}^E \end{pmatrix}^{-1} \begin{pmatrix} e_{31} \\ e_{33} \end{pmatrix} E_3 \tag{9}$$

or

$$T_1 = \bar{c}_{11}^E S_1 - \bar{e}_{31} E_3, \tag{10}$$

$$D_3 = \bar{\varepsilon}_{33}^S E_3 + \bar{e}_{31} S_1 \tag{11}$$

with

$$\bar{c}_{11}^E = c_{11}^E - \begin{pmatrix} c_{12}^E & c_{13}^E \end{pmatrix} \begin{pmatrix} c_{22}^E & c_{13}^E \\ c_{13}^E & c_{33}^E \end{pmatrix}^{-1} \begin{pmatrix} c_{12}^E \\ c_{13}^E \end{pmatrix}, \tag{12}$$

$$\bar{e}_{31} = e_{31} - \begin{pmatrix} c_{12}^E & c_{13}^E \end{pmatrix} \begin{pmatrix} c_{22}^E & c_{13}^E \\ c_{13}^E & c_{33}^E \end{pmatrix}^{-1} \begin{pmatrix} e_{31} \\ e_{33} \end{pmatrix}, \tag{13}$$

$$\bar{\varepsilon}_{33}^S = \varepsilon_{33}^S + \begin{pmatrix} e_{31} & e_{33} \end{pmatrix} \begin{pmatrix} c_{22}^E & c_{13}^E \\ c_{13}^E & c_{33}^E \end{pmatrix}^{-1} \begin{pmatrix} e_{31} \\ e_{33} \end{pmatrix}. \tag{14}$$

These constants are important if we want to compare the results of beam theories with a Finite Element calculation. In the sequel these modified constants are used in the beam theories. For simplicity however the overbars are dropped.

Equation (7) then leads to

$$D_{3,z} = \varepsilon_{33}^S E_{3,z} + e_{31} S_{1,z} = 0 \tag{15}$$

which clearly shows that the electric field in the piezoceramic cannot be constant if the strain is a function of z.

For the derivation of the equations of motion Hamilton's principle for electromechanical systems is used

$$\delta \int_{t_0}^{t_1} (T - H) dt = 0 \tag{16}$$

with the kinetic energy T and the electric enthalpy

$$H = \int_V (\frac{1}{2}(T_I S_I - D_i E_i) dV = \int_V (\frac{1}{2} c_{IJ}^E S_I S_J - e_{iJ} E_i S_J - \frac{1}{2} \varepsilon_{ij}^S E_i E_j) dV. \tag{17}$$

For the static case investigated in many papers the kinetic energy vanishes and Hamilton's principle reduces to

$$\delta \int_V (\frac{1}{2} c_{IJ}^E S_I S_J - e_{iJ} E_i S_J - \frac{1}{2} \varepsilon_{ij}^S E_i E_j) dV = 0. \tag{18}$$

If the polarization or the electric field are oriented such that a bending deformation of the beam results, then mainly two models were investgated in the past. For the first model it is assumed that the thickness of the piezoceramic elements is small so that the strain in axial direction does not depend on the z-coordinate. Therefore, it is denoted 'constant strain model'. The strain in the beam is given by

$$S_1 = -w''(x)z \tag{19}$$

and in the piezoceramic by

$$S_1 = \begin{cases} -w''(x)h & \text{upper ceramic} \\ w''(x)h & \text{lower ceramic.} \end{cases} \tag{20}$$

All other strains vanish identically. As the strain is constant over the cross-section of the ceramic the electric field is also constant.

The second pin-force-model assumes a linearly varying strain not only in the beam but also in the piezoceramic elements. This model is refered to as the Bernoulli-Euler model. The strain in the beam and in the piezoceramics is therefore

$$S_1 = -w''(x)z. \tag{21}$$

Though the strain is a linear function of the thickness coordinate in the ceramic the electric field is assumed to be constant.

Introducing the strain and electric field into Hamilton's principle for the static case (18) leads to

$$\begin{aligned} &\int_0^{\ell_2} \left(c_{11B}\frac{2}{3}bh^3w'' + 2bh_Pc_{11P}^Eh^2w'' - 2e_{31}E_3bh_Ph\right)\delta w''dx_2 \\ &\quad + \int_0^{\ell_1} c_{11B}\frac{2}{3}bh^3w''\delta w''dx_1 + \int_0^{\ell_3} c_{11B}\frac{2}{3}bh^3w''\delta w''dx_3 = 0 \end{aligned} \tag{22}$$

for the first model and to

$$\begin{aligned} &\int_0^{\ell_2} \left(b\frac{2}{3}h^3c_{11B}w'' + 2b\left(h^2h_P + hh_P^2 + \frac{h_P^3}{3}\right)c_{11P}^Ew'' - 2e_{31}E_3b(hh_P + \frac{h_P^2}{2})\right)\delta w''dx_2 \\ &\quad + \int_0^{\ell_1} c_{11B}\frac{2}{3}bh^3w''\delta w''dx_1 + \int_0^{\ell_3} c_{11B}\frac{2}{3}bh^3w''\delta w''dx_3 = 0 \end{aligned} \tag{23}$$

for the second. For the static case these variational equations allow to determine the curvature due to the electric excitation of the piezoceramics. For model 1 the curvature in the region with the attached piezoceramics is given by

$$w'' = \frac{2e_{31}E_3bh_Ph}{c_{11B}\frac{2}{3}bh^3 + 2bh_Pc_{11P}^Eh^2} \tag{24}$$

and for the Bernoulli-Euler model by

$$w'' = \frac{2e_{31}E_3b(hh_P + \frac{h_P^2}{2})}{b\frac{2}{3}h^3c_{11B} + 2b\left(h^2h_P + hh_P^2 + \frac{h_P^3}{3}\right)c_{11P}^E}. \tag{25}$$

Especially for an excitation near resonance the change of the electric field in the piezoceramics due to the strain may be important. To incorporate this, a third model is investigated. Beginning with (7) an expression for the electric fields for the upper and lower ceramics in the form

$$E_{3u} = \frac{e_{31}}{\varepsilon_{33}^S}w''z - \frac{e_{31}}{\varepsilon_{33}^S}(h + \frac{h_P}{2}) - \frac{\phi}{h_P}, \tag{26}$$

$$E_{3l} = \frac{e_{31}}{\varepsilon_{33}^S}w''z + \frac{e_{31}}{\varepsilon_{33}^S}(h + \frac{h_P}{2}) + \frac{\phi}{h_P} \tag{27}$$

can be obtained. In these equations ϕ denotes the electric potential applied at the surface of the ceramics. Introducing this into Hamilton's principle yields (static case)

$$\begin{aligned}\delta H \quad = \quad & \delta\Big(\int_0^{\ell_1}\int_{-h}^{h}\frac{1}{2}bc_{11B}w''^2z^2dzdx_1+\int_0^{\ell_2}\int_{-h}^{h}\frac{1}{2}bc_{11B}w''^2z^2dzdx_2+\int_0^{\ell_3}\int_{-h}^{h}\frac{1}{2}bc_{11B}w''^2z^2dzdx_3 \\ & +\int_0^{\ell_2}\int_{-h-h_P}^{-h}\frac{1}{2}bc^E_{11P}w''^2z^2-\frac{1}{2}\varepsilon^S_{33}b(E_{3a}zw''-E_{3b}w''-E_{30})^2+e_{31}(E_{3a}z^2w''-E_{3b}zw''-E_{30}z)bw''dzdx_2 \\ & +\int_0^{\ell_2}\int_{h}^{h+h_P}\frac{1}{2}bc^E_{11P}w''^2z^2-\frac{1}{2}\varepsilon^S_{33}b(E_{3a}zw''+E_{3b}w''+E_{30})^2+e_{31}(E_{3a}z^2w''+E_{3b}zw''+E_{30}z)bw''dzdx_2\Big)\end{aligned} \tag{28}$$

with

$$E_{3a}=e_{31}/\varepsilon^S_{33} \qquad E_{3b}=-e_{31}(hh_P+h_P^2/2)/(\varepsilon^S_{33}h_P), \qquad E_{30}=-\frac{\phi}{h_P}. \tag{29}$$

The integration with respect to z yields

$$\begin{aligned}\delta H \quad = \quad & \delta\Big(\int_0^{\ell_1}\frac{1}{2}b\frac{2}{3}h^3c_{11B}w''^2dx_1+\int_0^{\ell_2}\frac{1}{2}b\frac{2}{3}h^3c_{11B}w''^2dx_2+\int_0^{\ell_3}\frac{1}{2}b\frac{2}{3}h^3c_{11B}w''^2dx_3 \\ & +\int_0^{\ell_2}b\left(h^2h_P+hh_P^2+\frac{h_P^3}{3}\right)c^E_{11P}w''^2-\varepsilon^S_{33}b[E^2_{3a}\frac{3h^2h_P+3hh_P^2+h_P^3}{3}w''^2 \\ & \qquad +E_{3a}E_{3b}(2hh_P+h_P^2)w''^2+E_{30}E_{3a}(2hh_P+h_P^2)w''+(E_{3b}w''+E_{30})^2h_P] \\ & \qquad -e_{31}b[E_{3b}(2hh_P+h_P^2)w''^2+E_{30}(2hh_P+h_P^2)w''+2E_{3a}\frac{3h^2h_P+3hh_P^2+h_P^3}{3}w''^2]dx_2\Big)\end{aligned} \tag{30}$$

so that after some integration by parts the equations of motion and the boundary conditions may be obtained. In the static case once again the curvature change due to the piezoelectric actuation is

$$w''=\frac{-2e_{31}E_3(hh_P+\frac{h_P^2}{2})}{\frac{2}{3}h^3c_{11B}+2\left(h^2h_P+hh_P^2+\frac{h_P^3}{3}\right)c^E_{11P}+\frac{e^2_{31}}{6\varepsilon^S_{33}}h_P^3}. \tag{31}$$

It can be seen that the influence of the nonconstant electric field leads to an additional term in the denominator of the curvature in comparison to the curvature determined by Bernoulli-Euler theory and constant electric field, equation (25).

The equations of motion for excited vibration or for free vibration may be derived also by the equations given above if the kinetic energy is taken into account. They are beyond the scope of this paper. Especially the influence of the changing electric field in thickness direction due to the strain in the ceramics will be shown in another paper.

The results of the different models will be compared later with results of the Finite Element Analysis.

3 FINITE ELEMENT CALCULATIONS

In the previous section the different models for the actuation of beams by piezoceramic elements have been presented. The advantage of these models is that they may be incorporated in control algorithms, for example. In our laboratory, however, a debonding of the elements could be observed in experiments done in the field of ultrasonic motors. The simple models for the actuation are not capable of giving any prediction of the stress distribution in the bonding layer. In a previous paper[22] a piezoceramic element was investigated which is bonded

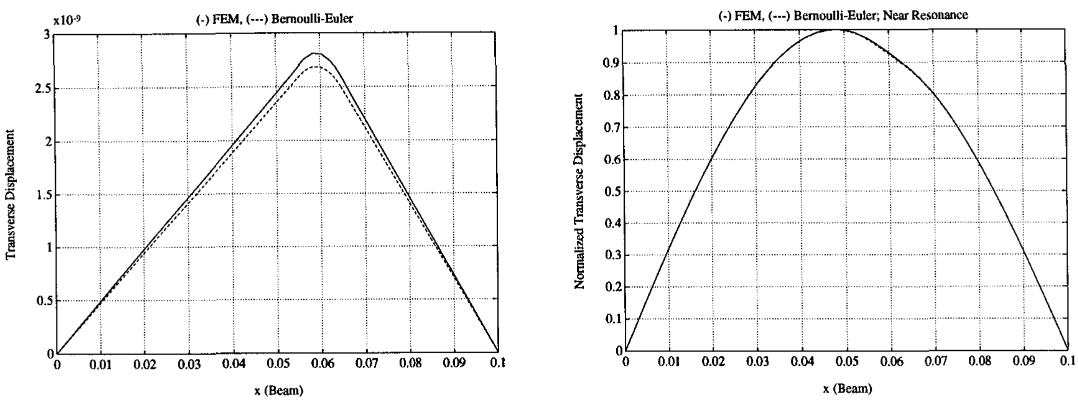

Figure 3: Comparison between the Bernoulli-Euler model (— — —) and FEM (-.-) for the transverse deflection for the static case (left) and near resonance (right).

ideally to a half-space. For a very thin piezoceramic element and an elastic half-space stress singularities at the ends of the ceramic were observed. For a rigid foundation and a piezoceramic which deforms both in thickness and longitudinal direction an approximation of the stress distribution in the bonding layer was obtained which is qualitatively the same as that obtained for a Finite Element model.

A first approximation for the bonding stresses for a beam-piezoceramic structure is determined by the Finite Element method (FEM). In the literature some papers can be found which deal with piezo-actuated structures.[19–21] In our calculation the Finite Element code ANSYS was used which allows to take into consideration the piezoceramic effect also in a two-dimensional model. The problem with the currant version of the FEM code is that for the beam structure the plane stress model may be incorporated which gives good results for slender beams whereas the piezoceramic may be modelled as a twodimensional structure only for plane strain.

This leads to difficulties if the results of the beam models and FEM are compared. Another difficulty is that for FEM numerical values for the geometrical and material parameters have to be given. Therefore, a comparison can only be done for special sets of parameters. For the actual investigations the following values were chosen: $\ell_1 = 55$ mm, $\ell_2 = 10$ mm, $\ell_3 = 35$ mm, $h = 4$ mm, $h_P = 1$ mm. This corresponds to a nonslender beam for which the results of the pin-force models will be questionable. Material parameters were taken for SONOX P1.

In a first step the displacements in transverse direction of the Bernoulli-Euler model and of the FEM are compared. On the left side of Figure 3 the transverse displacement for the static case is shown. The results of the two models agree quite well and it has to be taken into account that for the FE model plane stress was used for the beam and plane strain for the ceramics. Modelling the structure with solid elements leads to high computation times as the grid especially near the edges of the ceramics has to be very fine. On the right side of Figure 3 the transverse displacement is plotted for an excitation near resonance. As no damping is introduced in the models the displacement is normalized by the maximum displacement to account for the fact that the resonance frequencies may be slightly different for the two models. Though the mode shape is not sensitive to changes in frequency near resonance, the amplification factor is strongly affected by such a difference which make it necessary to normalize the mode shapes by the amplification factor.

Figure 4 shows the contour lines for equal shear and normal stresses in the beam and the piezoceramics for the static case. Clearly the stress concentration at the edges of the piezoceramics can be seen. In Figure 5 the contour lines for equal shear and normal stresses is plotted for an excitation near resonance.

For the same model also for the static case the shear stress distribution in the bonding layer is shown on the left side of Figure 6. Clearly the stress concentration at the ends of the ceramics can be seen. The corresponding distribution of the normal stress in transverse direction which is neglected in practically all beam theories is depicted on the right side of Figure 6. The corresponding curves are given in Figure 7. In order to see the

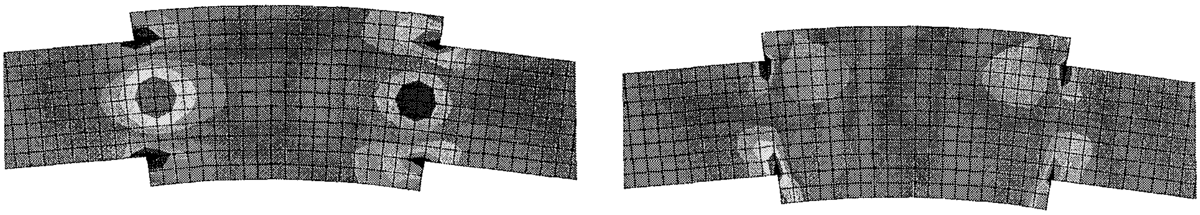

Figure 4: Contour lines for the distribution of the shear stress and transverse normal stress in the beam and piezoceramics (static case, detailed view).

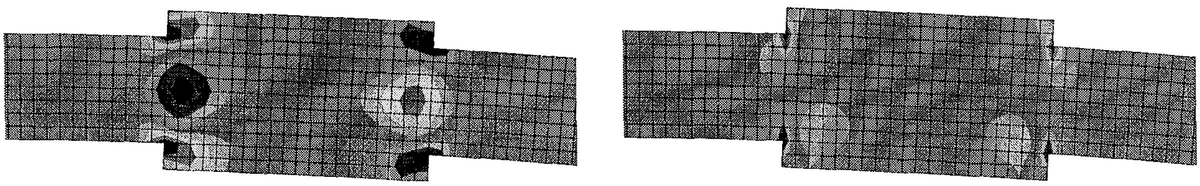

Figure 5: Contour lines for the distribution of the shear stress and transverse normal stress in the beam and piezoceramics (near resonance, detailed view).

influence of the discretization the shear and normal stresses in the bonding layer are plotted in the two figures for three different element sizes of the finite element mesh. Curves are plotted for the discretization shown above and in addition for double and half resolution. It can be seen that the distribution at the edges of the ceramics depends much on the discretization of the FE mesh. To obtain precise results the mesh has to be very fine. At this point it has to be taken into account that the corner leads to a singularity which will not be observed in experiments due to the finite stiffness of the bonding layer and due to plastic deformations.

The last figure shows the difference between results obtained by a plane FE model and a system modeled by 3D elements. The 3D model was assumed to be very thin and it can be seen in Figure 8 that the differences for corresponding FE mesh sizes are small.

Though the results depend much on the discretization it may be concluded from the figures that the stresses occuring for the static case are an order of magnitude smaller than the stresses which occur near resonance.

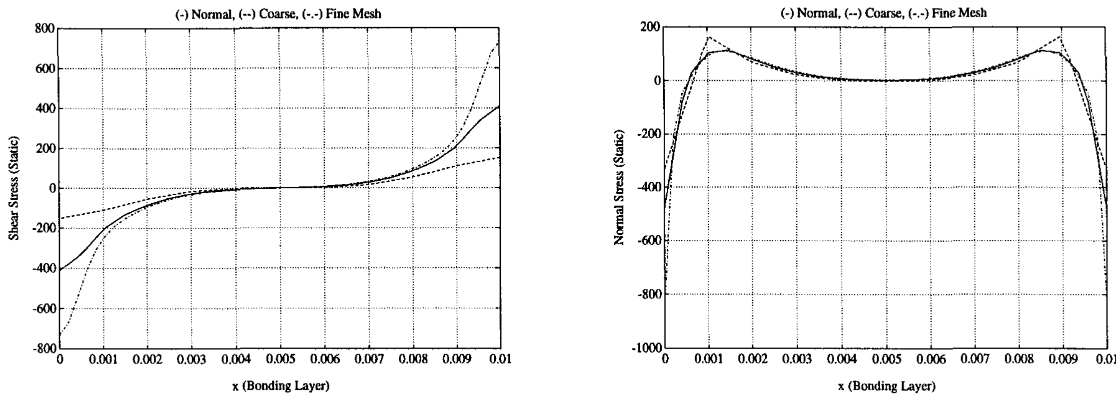

Figure 6: Shear stress and normal stress in the bonding layer for different meshes (static case).

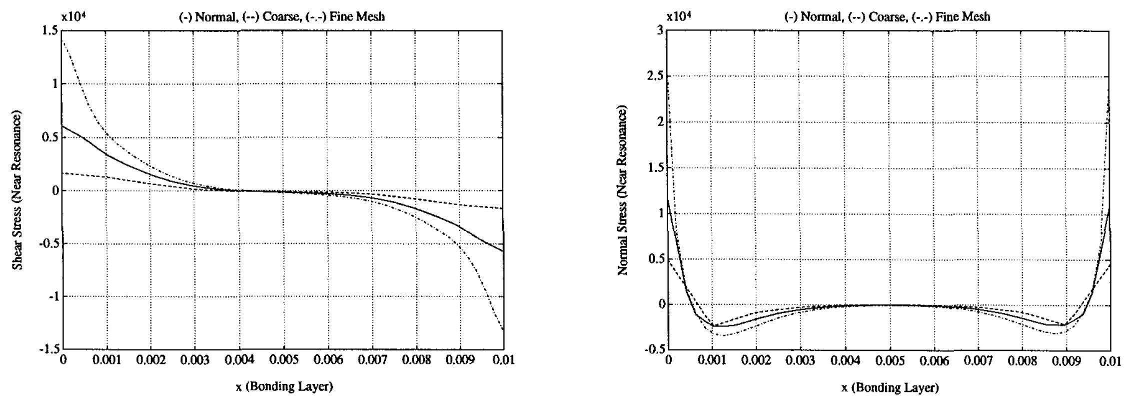

Figure 7: Shear stress and normal stress in the bonding layer for different meshes (near resonance).

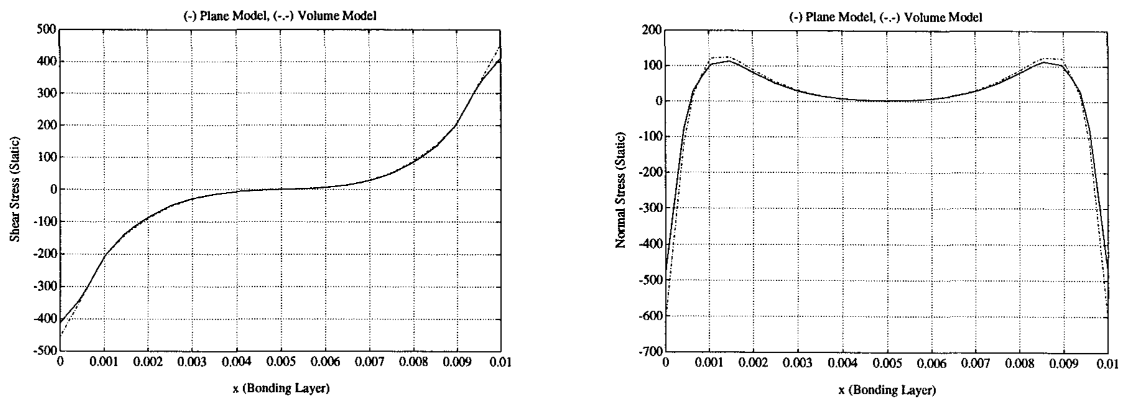

Figure 8: Comparison of results for the static case obtained by plane finite elements (—) and volume elements (– – –).

4 CONCLUSIONS

In this paper the different models for the actuation of beams by piezoceramic elements were presented. These models give simple results which may be incorporated in control algorithms. But they are not suited for determining the stresses which occur in the bonding layer. The stresses in the bonding layer are important, because delamination effect were observed in experiments. The Finite Element calculation presented in this paper show that high stresses occur only for an excitation in resonance. On the other hand an actuation with low frequencies only leads to small displacements.

Future investigations will include developing more refined beam theories to allow the estimation of the bonding stresses without FEM. Other investigations will be done for piezo beam structures which are excited not in a bending mode but in longitudinal direction.

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