

Design of a kinematic coupling for precision applications

C. H. Schouten, P. C. J. N. Rosielle, and P. H. J. Schellekens

Department of Mechanical Engineering, Precision Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands

To machine a complex precision product, several tools are needed. These tools are placed on a tool turret. A tool must return several times to its original position. To attain a very high repeatability between the upper part and the base of the tool turret mounted on a precision lathe, it is preferable that the parts of the tool turret are statically determined in their contacts. This is attained by using a kinematic coupling. To attain the required stiffness this coupling is provided with a preload of $1.5 \cdot 10^3$ N. The machining forces are typically less than 1 Newton. A special kinematic coupling, consisting of grooves and balls, was designed, made, and tested. By providing the grooves with self-adjusting surfaces, hysteresis is reduced to less than one-tenth of a micrometer. Maximum stiffness is aimed at by using cemented carbide, a material with a high admissible stress, at the contact points. Experiments show that this kinematic coupling, under a preload of $1.5 \cdot 10^3$ N, has a static stiffness of more than $1 \cdot 10^8$ N/m in every direction and a repeatability better than one-tenth of a micrometer. © Elsevier Science Inc., 1997

Keywords: kinematic coupling; repeatability; static stiffness; hysteresis; self-adjusting surfaces; elastic hinges

Introduction

The position of a *rigid* body is fully defined if all six degrees of freedom are constrained; then the body is said to be "statically determined." Constraining a degree of freedom more than once requires elasticity (so the body cannot be *rigid* any more), which introduces difficulties in the design. It is, therefore, preferable to design statically determined, especially for precision machines.^{1,2} With such a statically determined body, it is possible to replace *that* body to the original position, with respect to the rest of the machine, with high repeatability. There are no redundant elements that constrain the same degree of freedom. Repeatability of such a machine can be much better than the manufacturing accuracy.

In the literature, such statically determined bodies are known as "kinematic couplings"³⁻⁵ or

"Kelvin clamps."⁶ A kinematic coupling consists of two parts. The number of contact points between these two parts equals the number of degrees of freedom that must be constrained. One of the possibilities is a kinematic coupling consisting of three radial V-grooves in one part and three balls (or segments of balls) on the other part, as shown in *Figure 1*. Every ball has two contact points with a V-groove. This design consists of similar parts, and it shows predictable thermal behavior, with the thermal center located in the middle of the coupling. A high stiffness is obtained if the coupling is provided with a preload. The value of the maximum static stiffness that will be attained depends upon such parameters as the admissible stress of the material and the dimensions of the balls and grooves. If these parameters are known, the value of the static stiffness is highly predictable.⁷ Because of the symmetry of the coupling, a central preload in the vertical direction causes a similar deflection of every contact.

Calculations are often based on the assump-

Address reprint requests to Ir. C. H. Schouten, Tinelstraat 166, 5654 LX, Eindhoven, The Netherlands.

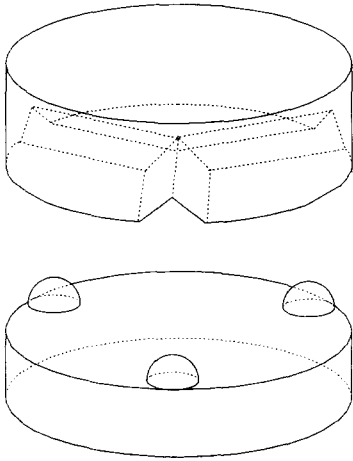


Figure 1 A kinematic coupling with three V-grooves and three balls

tion of ideal Hertzian contacts^{3,5} between the bodies. In this context, “ideal” means no friction between the bodies in contact and no surface roughness. In this frictionless situation, contact stresses do not change if there is a relative movement between surfaces. However, in real contacts, every relative movement between surfaces introduces a friction force F_f in the direction opposite to the movement. This friction force is tangential to the surface and causes a tangential deflection δ_t . Expressions for the tangential displacement have been found by Mindlin for the case of no-slip and by Deresiewicz for partial slip (reported in Johnson⁸). In the first instance there is no slip at the interfaces. The tangential deflection δ_t is a function of the tangential friction force F_f , the radius of the contact area a , the shear modulus G , and the Poisson ratio ν of the contact material.⁸ The tangential stiffness is:

$$c_t \equiv \frac{F_f}{\delta_t} = \frac{4aG}{2-\nu} \quad \text{with} \quad G = \frac{E}{2(1+\nu)} \quad (1)$$

With a “local” normal stiffness $c_n \equiv dF_n/d\delta_n$ of $E_e \cdot a$ (Hertzian theory) and an equivalent elastic Young’s modulus $E_e = E/(1-\nu^2)$ Equation (1) becomes:

$$\frac{c_t}{c_n} = \frac{2(1-\nu)}{2-\nu} \quad (2)$$

Contact between a ball and a conventional V-groove is shown schematically in *Figure 2*. Variation of the preload causes a variation in (elastic) deformation (symbolized by an elastic spring) in the contact. It is also possible that there is a movement between the surfaces (symbolized by friction). When the preload varies so much that there is a relative movement between ball and V-groove,

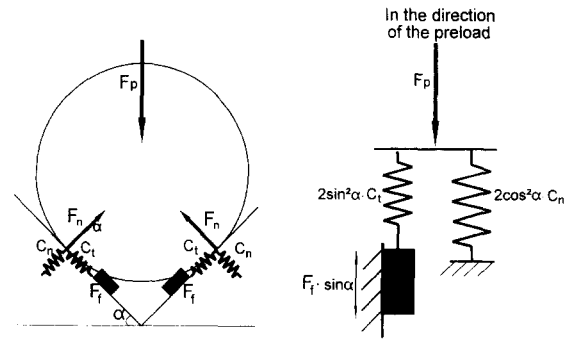


Figure 2 Schematic diagram of a ball in a conventional V-groove

it is almost certain that the original position will not be regained once this variation no longer exists. This uncertainty of position is termed hysteresis s_v ¹ and can be described as:

$$s_v = \frac{2 \cdot |F_f|}{c} \quad (3)$$

In this equation, F_f is the friction force and c the stiffness in the same direction. If the preload on the ball in the V-groove varies, the maximum friction force that will appear is $F_f = f \cdot F_n$ (with coefficient of friction f and reactive force F_n in the contacts normal to the surfaces of the V-groove). By using this relationship for F_f in Equation (3), maximum hysteresis in the direction of the preload is given by:

$$s_v = \frac{2 \cdot |F_{f,p}|}{c_{n,p}} = \frac{2 \cdot |2 \cdot f \cdot F_n \cdot \sin\alpha|}{2 \cdot \cos^2\alpha \cdot c_n} \quad (4)$$

In this equation, $F_{f,p}$ is the friction force in the direction of the preload and $c_{n,p}$ the normal stiffness in the same direction, *Figure 2*. With $F_n = F_p/2\cos\alpha$, Equation (4) becomes:

$$s_v = \frac{f \cdot F_p \cdot \tan\alpha}{\cos^2\alpha \cdot c_n} \quad (5)$$

For our application, repeatability needs to be better than one-tenth of a micrometer. Hysteresis is very undesirable when such a high repeatability is required. Equation (5) shows that hysteresis is small if the angle of inclination α is small and/or the coefficient of friction f is small and/or the normal stiffness c_n is large for a certain preload F_p . Hysteresis is theoretically zero if there is no relative movement between the ball and the surfaces of the V-groove.

Minimizing hysteresis

Repeatability improves when hysteresis decreases, because then the indefiniteness of position is less after a force or temperature variation. According

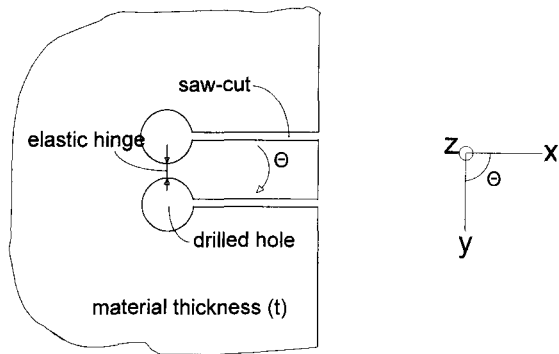


Figure 3 Elastic hinge principle

to Equation (5), this can be realized by, for example, the choice of a suitable material for the contacts: (1) with a low coefficient of friction f ; and (2) with a high admissible (Hertzian) stress (for a high feasible stiffness c_n).

A high value of the normal static stiffness leads not only to minimum hysteresis, but also to a high total stiffness of the coupling. That is why cemented carbide, a material with a very high admissible stress ($\approx 5 \cdot 10^3 \text{ N/mm}^2$), is used at the contacts. Because this cemented carbide is used, instead of steel, the maximum contact stiffness that will be attained with the same contact geometry and preload F_p is approximately twice as high.⁷ Unfortunately, the coefficient of friction of cemented carbide (≈ 0.2) is similar to the coefficient of friction of steel. So hysteresis decreases, but if the preload varies so much that there is a relative movement between ball and surface of the groove, there is still the same friction force. Friction between the balls and grooves is theoretically zero when the surfaces of the V-grooves follow the movement of the balls, when the preload increases or decreases. Then there is *no* relative movement between balls and surfaces of the grooves. This can be achieved by providing the V-grooves with self-adjusting surfaces, by means of elastic hinges in the material beneath the surfaces of the V-grooves. Such elastic hinges are created by drilling two holes next to each other in the material and making saw-cuts from the surface to these holes, as shown in *Figure 3*.

In *Figure 4* the V-groove–ball situation is depicted, showing the elastic hinges in the material beneath the surfaces of the V-groove. The situation is simplified by assuming that the *left* elastic hinge has a very low rotational stiffness k_θ (θ means a rotation around the z -axis) and the *right* half of the V-groove has stiffnesses c_n and c_t in the ball–groove-contact and stiffnesses of the elastic hinge; c_x normal to the surface and c_y tangential to the surface. Because of the very low rotational

stiffness of the *left* elastic hinge, the tangential deflection δ_t in is also very low; whereas, the tangential stiffness $c_t (= F_t/\delta_t)$ in the left contact is very high. If α is approximately 45° , the movement u_θ of the *left* surface of the V-groove by means of pure rotation of the elastic hinge almost equals the normal deflection $\delta_{n,x}$ in the contact of the ball with the *right* half of the V-groove. Of course c_t is not infinite, but because of the movement of the surface with the movement of the ball, the tangential force F_t is very small and so is the deflection δ_t .

For a certain ball diameter in contact with flat surfaces of the V-groove, the maximum contact stiffness, normal to a surface of the V-groove $c_{n,max}$ is calculated with the help of the Hertzian theory. So the stress in the contacts equals the admissible stress of cemented carbide.⁷ Stiffness of the elastic hinge normal to the surface c_x is chosen to be ten times as large as $c_{n,max}$. The total normal stiffness $c_{n,x}$ (this is a series connection of stiffness c_n and c_x) is only 9% less than the normal stiffness without the elastic hinge. With the help of a theory reported by Johnson,⁸ a relationship between the tangential and the normal stiffness in the contact is derived (Equation (2)). When the maximum normal stiffness $c_{n,max}$ is given, it is now possible to calculate the maximum tangential stiffness $c_{t,max}$ in the contact. The stiffnesses of the elastic hinge; c_x (normal to the surface), c_y (tangential to the surface) and k_θ (rotational stiffness) are calculated with equations given in a university textbook.¹ When the distance between hinge and contact is a , the tangential stiffness in the contact, caused by the rotational stiffness of the hinge, is $c_\theta \approx k_\theta/a^2$. The total tangential stiffness in contact $c_{t,y,\theta}$ is a series connection of the stiffnesses c_p , c_y , and c_θ . The total normal stiffness almost equals the normal stiffness without the elastic hinge. However, the rotational stiffness has to be low to achieve the required mobility of the surfaces, so the tangential stiffness (with the elastic hinge) has to be much less than the tangential stiffness without elastic

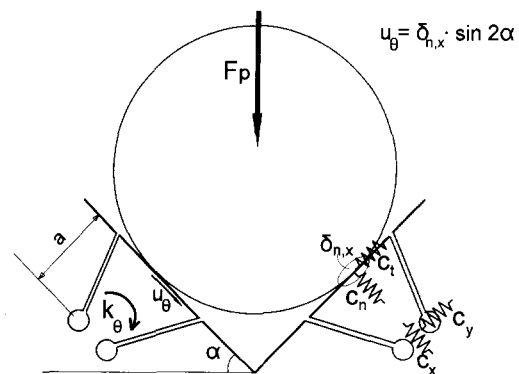


Figure 4 Ball in a V-groove with elastic hinges

hinge. So, unfortunately, a disadvantage of a coupling with elastic hinges is that the total stiffness is lower than the total stiffness of a coupling with conventional V-grooves.

The total axial stiffness $c_{a,\text{total}}$ of the coupling with elastic self-adjusting V-grooves can be described with⁷:

$$c_{a,\text{total}} = 6 \cdot \cos^2 \alpha \cdot c_{n,x} + 6 \cdot \sin^2 \alpha \cdot c_{t,y,\theta} \quad (6)$$

The total radial stiffness $c_{r,\text{total}}$ of this coupling can be described with⁷:

$$c_{r,\text{total}} = 3 \cdot \sin^2 \alpha (c_{n,x} - c_{t,y,\theta}) + 3 \cdot (c_{t,y,\theta} + c_t) \quad (7)$$

The total axial and radial stiffness of a coupling with *conventional* V-grooves can be described with the same Equations (6) and (7), when c_n and c_t are substituted for $c_{n,x}$ respectively $c_{t,y,\theta}$.

Because $c_t < c_n$ (with $V \approx 0,25$ equation [2] becomes: $c_t \approx 0,86 \cdot c_n$) and $c_{t,y,\theta} \ll c_{n,x}$ ($c_{t,y,\theta} \approx 0,01 \cdot c_{n,x}$), the axial stiffness decreases, and the radial stiffness increases as the angle of inclination α increases. So, the best choice for the angle of inclination is if $c_{r,\text{total}} = c_{a,\text{total}}$. For the kinematic coupling with elastic self-adjusting V-grooves, this is 37° ⁷. For the conventional coupling it is 55° ⁷ but this value is less critical, because the tangential stiffness c_t has the same order of magnitude as the normal stiffness c_n .

Also important is the stiffness of the total coupling under eccentric load (in tangential direction). The equation for the so-called total tangential stiffness $c_{t,\text{total}}$ of the coupling with elastic self-adjusting V-grooves can be described with⁷:

$$c_{t,\text{total}} = \frac{c_{r,\text{total}} \cdot 3 \cdot (c_{n,x} \cdot \sin^2 \alpha + c_{t,y,\theta} \cdot \cos^2 \alpha)}{c_{r,\text{total}} + 3 \cdot (c_{n,x} \cdot \sin^2 \alpha + c_{t,y,\theta} \cdot \cos^2 \alpha)} \quad (8)$$

The angle of inclination α has to be of such a value that the coupling does not self-lock during positioning. Positioning of the coupling with elastic self-adjusting surfaces does not give that problem, because of the mobility of the V-groove surfaces. In case of a conventional coupling, the change that self-locking does occur is the smallest for an angle of inclination of 43° ⁷. This angle of inclination is not critical. For an angle of 29 – 56° , the admissible coefficient of friction is more than 0.2 (the coefficient of friction of cemented carbide).

Design of a kinematic coupling and test results

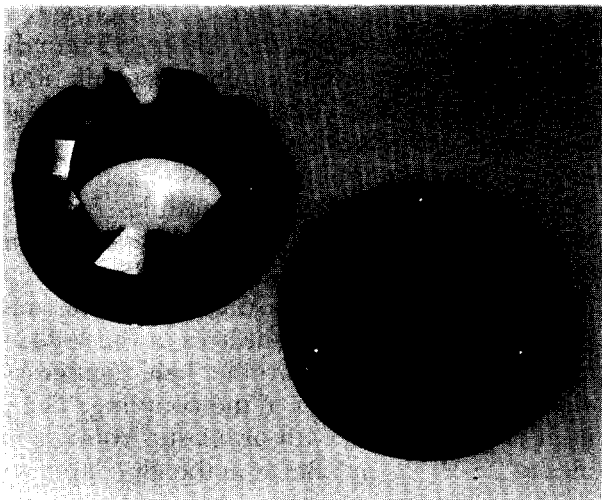
A special kinematic coupling was designed. One part of this coupling has six V-grooves: three conventional and three self-adjusting, all with an an-

gle of inclination of 40° . On the counterpart of the coupling, consisting of a disc, three (15-mm diameter) ball segments are mounted equally divided on the same radius. The coupling is made of steel; cemented carbide is used only at the points of contact. The material properties of cemented carbide are: an elastic Young's modulus E of $6 \cdot 10^5$ N/mm², an admissible stress of $5 \cdot 10^3$ N/mm², Poisson ratio ν of 0.25, and a coefficient of friction f of 0.2. Cemented carbide tiles, with a thickness of 2 mm, are glued on the surfaces of the V-grooves, and three cemented carbide ball segments are glued on the other part of the coupling. The glue used is a thermoset with an elastic Young's modulus of $2 \cdot 10^9$ N/m². Because the glue layer is very thin (only a few micrometers) and the contact area fairly large ($1,8 \cdot 10^{-4}$ m²), this layer is very stiff. This design enables comparison between a conventional coupling and a coupling with self-adjusting surfaces (by means of elastic hinges). For this purpose, two holes are drilled next to each other in the material beneath the two surfaces of three V-grooves. The middle of the dam between the holes coincides with the normal to the surface of the V-groove through the contact (of the ball with the V-groove). Cuts make the surfaces relatively flexible in the tangential direction with respect to the rest of the coupling. *Figure 5* is a photograph of the kinematic coupling.

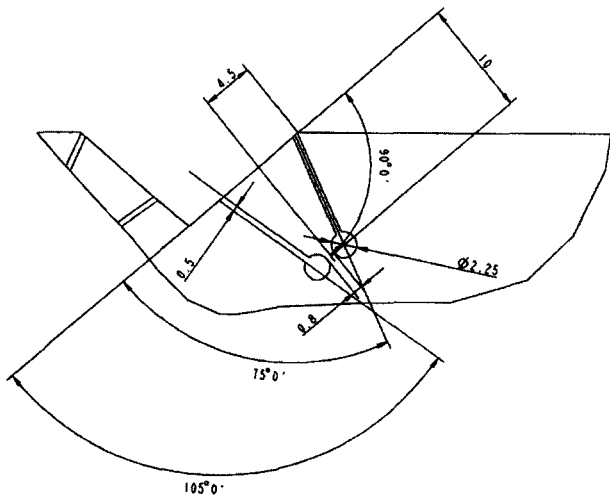
The coupling was tested with the help of a testing system, as shown in *Figure 6*. By moving a mass M , the kinematic coupling is provided with a varying preload in vertical direction, by way of a beam, two rods, a second beam, and finally a ball on the middle of the coupling. The relative movement of the disc with the three ball segments with regard to the part with the V-grooves, attributable to the compression in the vertical direction (by elastic deflection of the contact surfaces), is measured with a displacement transducer fixed to the upper disc of the coupling. The preload varies progressively and is measured with a dynamometer, fixed to the bottom of the lowest beam. The dynamometer measures the pulling force on the beam. Therefore, the measurements have to be corrected with the static mass of the disc, the beams, the rods and the ball (a total weight of 3.5 kg).

Elastic deflection and hysteresis

The coupling should be provided with a load to attain the required stiffness. This load is called a preload and is much larger than the external loads in use, which are typically one Newton. Preload versus displacement is recorded. The vertical dis-



a



b

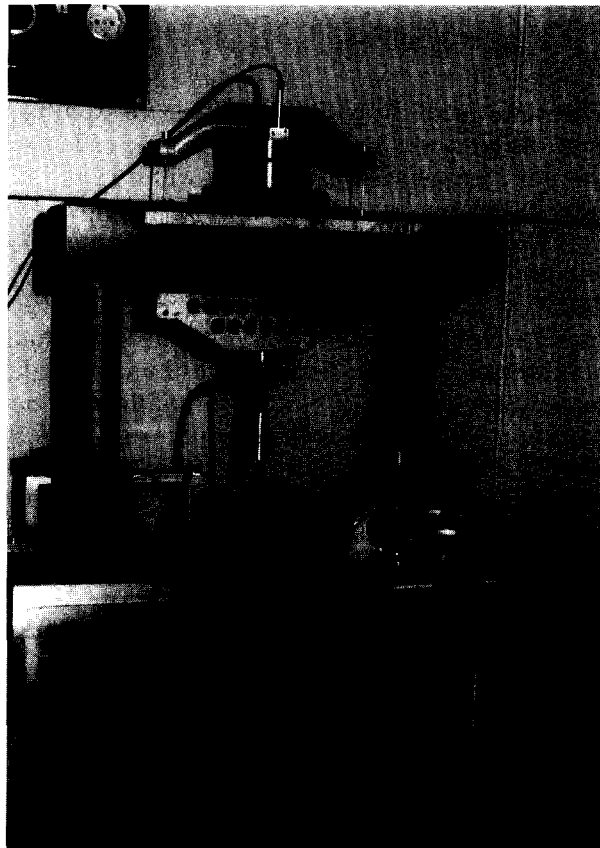
Figure 5 (a) The kinematic coupling parts; (b) detail drawing: an elastic hinge

placement is the relative movement between the two halves of the coupling, which is the result of the elastic deflection of the contact surfaces, is plotted on the x -axis. The preload is plotted on the y -axis. In *Figure 7*, the preload–compression graphs for both couplings are shown next to each other.

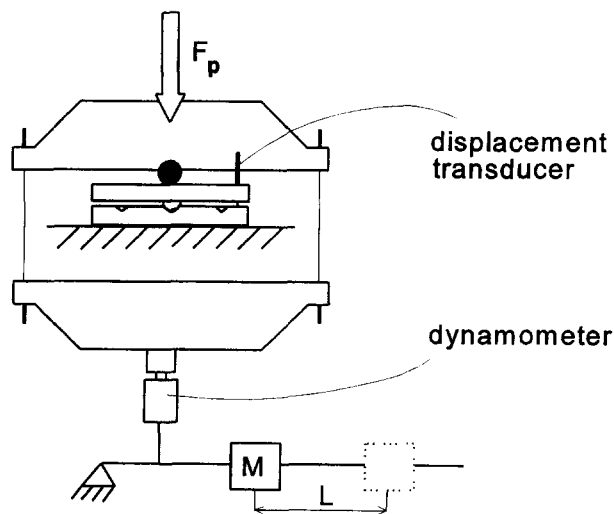
Hysteresis is the difference between the compression when the preload increases and the compression when the preload decreases. As explained earlier, this hysteresis is the result of friction between balls and V-grooves. When the preload decreases, the friction force is in the opposite direction to the friction force with an increasing preload. There is no relative movement of the bodies in contact until the friction has changed direction. *Figure 7* shows that hysteresis decreases by a factor of 10 if the surfaces of the V-grooves are provided with these elastic hinges.

Total axial stiffness of the coupling

The tangent (for a specific preload) on the preload–compression graphs of *Figure 7* gives the axial stiffness of the coupling. If the preload is $1.5 \cdot 10^3$ N, the axial stiffness of the conventional coupling is about $6.1 \cdot 10^2$ N/ μ m and for the coupling with self-adjusting surfaces about $3.6 \cdot 10^2$ N/ μ m.



a



b

Figure 6 (a) The testing system; (b) diagram of the testing system

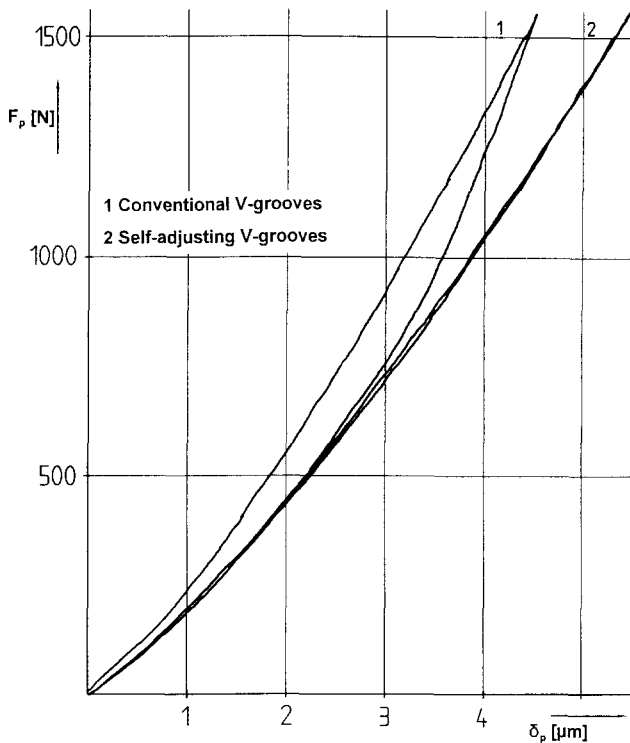


Figure 7 Measurements of preload F_p versus elastic deflection δ_p ; results: hysteresis and axial stiffness

Total radial stiffness of the coupling

The radial stiffness of the coupling is measured with the same setup. Again, the kinematic coupling is provided with a central preload of $1.5 \cdot 10^3$ N in the vertical direction, as shown in *Figure 6*. Using a spring-type force gauge, an additional force in the radial direction is introduced to the bottom of the upper disc (at the height of the contacts). The radial deviation of the disc with respect to the base, which is the result of the radial force, is measured with a displacement transducer. An approximation of the radial stiffness is ob-

tained if the force in the radial direction is divided by the radial deviation. The radial stiffness of the conventional coupling is about $6.0 \cdot 10^2$ N/ μm and about $3.7 \cdot 10^2$ N/ μm for the coupling with self-adjusting surfaces.

Total tangential stiffness of the coupling

In the same way as for the radial stiffness, the tangential stiffness of the coupling (stiffness under eccentric load) is measured with the set up. Now the additional force is in the tangential direction and the deviation is measured in the direction of this force.

In *Table 1*, the results of the measurements and the calculated values are shown. All measurements are taken with a preload F_p of $1.5 \cdot 10^3$ N, and calculations are made for the same value of the preload. The uncertainty of the experiments was estimated to be $\pm 5 \cdot 10^1$ N/ μm .

Calculated values and measured values of the axial and radial stiffness of the coupling agree well (Equations [6] and [7]). These values are highly predictable. It is more difficult to make a good estimate of the tangential stiffness of the coupling (Equation [8]).

For the conventional coupling, the maximum value of hysteresis is calculated with Equation (5) at a total preload of $7.5 \cdot 10^2$ N (after a variation from 0 to $1.5 \cdot 10^3$ N). The experimental value of hysteresis of the conventional coupling is less than this calculated value, but not less than one-tenth of a micrometer. By providing the grooves with self-adjusting surfaces, hysteresis is reduced considerably. Because the maximum value of the friction force $F_{w,\max}$ ($= f \cdot F_n \approx 66$ N) is much larger than the real tangential force F_t (≈ 3.3 N) hysteresis caused by macro slip is theoretically zero. Results of the experiments show that there still is some micro slip in the contacts.

Table 1 Comparison of theory with experiments for a preload F_v of $1.5 \cdot 10^3$ N

	Conventional V-grooves		Self-adjusting V-grooves	
	Calculations	Experiments	Calculations	Experiments
Axial stiffness, $c_{a,\text{total}}$, N/ μm	$6.5 \cdot 10^2$	$6.1 \cdot 10^2$	$3.7 \cdot 10^2$	$3.6 \cdot 10^2$
Radial stiffness, $c_{r,\text{total}}$, N/ μm	$6.1 \cdot 10^2$	$6.0 \cdot 10^2$	$4.3 \cdot 10^2$	$3.7 \cdot 10^2$
Tangential stiffness $c_{t,\text{total}}$, N/ μm	$3.1 \cdot 10^2$	$1.4 \cdot 10^2$	$1.6 \cdot 10^2$	$1.1 \cdot 10^2$
Hysteresis, μm	max 0.79	0.42	None	0.03

Conclusions

Hysteresis is very undesirable when repeatability between the upper part and the base of a tool turret mounted on a precision lathe (to return a tool several times to the original position) must be better than one-tenth of a micrometer. After an external force or temperature variation, hysteresis can occur. By providing the grooves of a kinematic coupling with self-adjusting surfaces, hysteresis is reduced by 95% and is less than one-tenth of a micrometer for a preload of $1.5 \cdot 10^3$ N. This preload is necessary to attain the required stiffness in all directions. If it is possible to reproduce the preload with high accuracy, repeatability only depends upon the stability of temperature and expansion of the design. The static stiffness of the coupling is highly predictable, with the equations given in this article, for the conventional coupling as well as for the coupling with self-adjusting surfaces. If the preload is as large as possible, which depends upon the admissible load of the material, the stiffness of both kinematic couplings are sufficient for most purposes.

Acknowledgments

The experiments were carried out in the mechanical department of the Philips Research Laboratories Eindhoven. We thank all contributors, especially F. Jaartsveld and J. J. Baalbergen, for their advice, help, and skill, which were essential to this project.

Notation

a	Radius of a circular contact area, mm
a	Arm, distance contact-hinge, mm
c	Stiffness, N/ μ m
D	Diameter, mm
E	Elastic Young's modules, N/mm ²
f	Coefficient of friction, [–]
F	Force, N
G	Shear modules, N/mm ²
h	Dam, m, mm
k	Rotational stiffness, Nm/rad
M	Moment, Nm
R	Radii of curvature, mm
s_v	Virtual play (hysteresis), μ m
t	Thickness, mm

T	Temperature, °C, K
u	Movement, m, mm
x	Axis of coordinates, m, mm
y	Axis of coordinates, m, mm
z	Axis of coordinates, m, mm

Greek

α	Angle of inclination, [°], rad
δ	Compression (elastic deflection), μ m
θ	Spin parameter, rad
ν	Poisson ratio, [–]
φ	Spin parameter, rad
Ψ	Spin parameter, rad

Subscripts

a	Axial
e	Equivalent
f	Friction
F	of the force
max	Maximum
n	Normal
p	Pre-
r	Radial
t	Tangential
total	Total
x	in direction of x
y	in direction of y
z	in direction of z
θ	Rotation around z
φ	Rotation around x
Ψ	Rotation around y

References

- 1 *Design Principles*. Eindhoven, The Netherlands: Eindhoven University of Technology (no. 4007), 1996
- 2 Slocum A. H. *Precision Machine Design*. Englewood Cliffs, NJ: Prentice-Hall, 1992
- 3 Slocum A. H. Kinematic couplings for precision fixturing—Part 1—Formulation of design parameters. *Prec Eng*, 1988, **10**, 85–91
- 4 Slocum A. H., Donmez A. Kinematic couplings for precision fixturing—Part 2—Experimental determination of repeatability and stiffness. *Prec Eng*, 1988, **10**, 115–121
- 5 Slocum A. H. Design of three-groove kinematic couplings. *Prec Eng*, 1992, **14**, 67–73
- 6 Sherrington I., Smith E. H. Design of a Kelvin clamp for use in relocation analysis of surface topography. *Prec Eng*, 1993, **15**, 77–85
- 7 Schouten C. H. "Het ontwerp van een precisiegereedschap-wisselaar" "Design of a precision tool-changer", Master's thesis (in Dutch), Eindhoven University of Technology, Eindhoven, The Netherlands, 1994
- 8 Johnson K. L. *Contact Mechanics*. Cambridge, UK: Cambridge University Press, 1992