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# Analysis of Elastomer Lens Mountings

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## ABSTRACT

The equation for determining the decentering of lenses mounted in a circumferential flexible elastomer is derived. A closed form analytical solution was derived for a circular lens mounted in an elastomer to describe this deflection. The theoretical expression was used to verify finite element models which then may be used for more complex mounts. Proper modeling such as element type and aspect ratios are addressed as well as applications of the method for various mounting applications.

## 1. ANALYTICAL DERIVATION

Consider the model of the tangent bar support system shown in Figure 1. This model comprises six springs, three of equal stiffness, $k_r$, which act radially and three of equal stiffness, $k_t$, which act tangentially. For typical tangent bar designs the tangent spring constant is several orders of magnitude greater than the radial spring constant. Moreover, since the mirror or lens may usually be treated as a rigid body, the stiffness of the tangent bar support system in the plane normal to the optical axis is given by the following equation.

$$K = \frac{3}{2}(k_r + k_t) \tag{1}$$

where K is invarient, i.e., equal in all directions, in the plane of the support system.

If these springs are considered to have differential stiffness per unit length of $\overline{k}_r$ and $\overline{k}_t$ acting over a differential length of $R d\theta$ on the optical element boundary as shown in Figure 2, the differential stiffness of this support system is

$$dK = \frac{3}{2}(\overline{k}_r + \overline{k}_t)\, R\, d\theta \tag{2}$$

where R is the radius of the optical element.

For a continuous elastic support system then

$$K = \int_0^{\frac{2\pi}{3}} \frac{3}{2}(\overline{k}_r + \overline{k}_t)\, R\, d\theta \tag{3}$$

$$K = \pi R\, (\overline{k}_r + \overline{k}_t) \tag{4}$$

where for an elastomer:

$\bar{k}_r$ = extensional stiffness per unit length
$\bar{k}_t$ = shear stiffness per unit length

When the elastomer thickness (radial) is approximately equal to the lens thickness, the elastomer may be modelled as a plane strain rectangular (quadrilateral) element. For a unit length,

$$\bar{k}_r = \left[\frac{E}{1 - \nu^2}\right]\frac{d}{t} \tag{5}$$

$$\bar{k}_t = G\,\frac{d}{t} \tag{6}$$

Where:

E = Young's Modulus
G = Shear modulus
$\nu$ = Poisson's ratio
d = optical element thickness
t = elastomer thickness (radial)

The stiffness of the annulus of the elastomer then is

$$K = \pi R\,\frac{d}{t}\left[\frac{E}{1 - \nu^2} + G\right] \tag{7}$$

Decentering, $\Delta$, of the optical element due to self weight acting normal to the optical axis of the mirror may then be computed as follows:

$$\Delta = \frac{W}{K} \tag{8}$$

Substituting for K:

$$\Delta = \frac{W}{\pi R\,\frac{d}{t}\left[\frac{E}{1 - \nu^2} + G\right]} \tag{9}$$

where W = weight of the optical element.

However, when the elastomer thickness (radial) is small compared to the lens thickness, the term $\frac{E}{1-\nu^2}$ in Equations 5, 7, and 9 should be replaced by $\frac{E}{1+\nu}\left[1 + \frac{\nu}{1-2\nu}\right]$ to account for both tangential and radial elastomer confinement.

## 2. VERIFICATION OF FINITE ELEMENT MODELING

To verify finite element solutions, models were constructed using the GIFTS program. Shown in Figure 3 is a four inch radius lens with material properties of KZFSN9 glass. The first study used a lens of thickness 0.50 inches whereas the second study used a lens of 1.00 inch. In both cases the radial thickness of the elastomer was 0.20 inches. For the first case the ring of quadrilateral membrane elements were used as plane stress elements since the depth to thickness aspect ratio is 2.5. For the second case these elements were used as plane strain elements since the depth to thickness ratio was 5.0. The results of this study are presented in Figure 5 where the finite element solutions which use a low depth to thickness aspect ratio formulation for the thin lens and a high aspect ratio formulation for the thick lens are given. These analyses demonstrate the dominating influence of the mechanical properties of the elastomer, especially Poisson's ratio. Manufacturers and suppliers of elastomers are often unable or reluctant to specify the specific mechanical properties (E, $\nu$, and G) of the elastomer due to the variabilities which result from temperature, humidity, curing environment, etc. For this reason it is often advisable to perform parametric studies to assess the effects of these variables.

## 3. SUMMARY

A simple analytical formula has been developed to determine the deflection normal to the optical axis of a circular lens when mounted in an annulus of elastomer. Two finite element models were then verified using the analytical solution to demonstrate that the finite element method may be used with confidence for more complex lens geometries.

## 4. ACKNOWLEDGEMENTS

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## 5. REFERENCES

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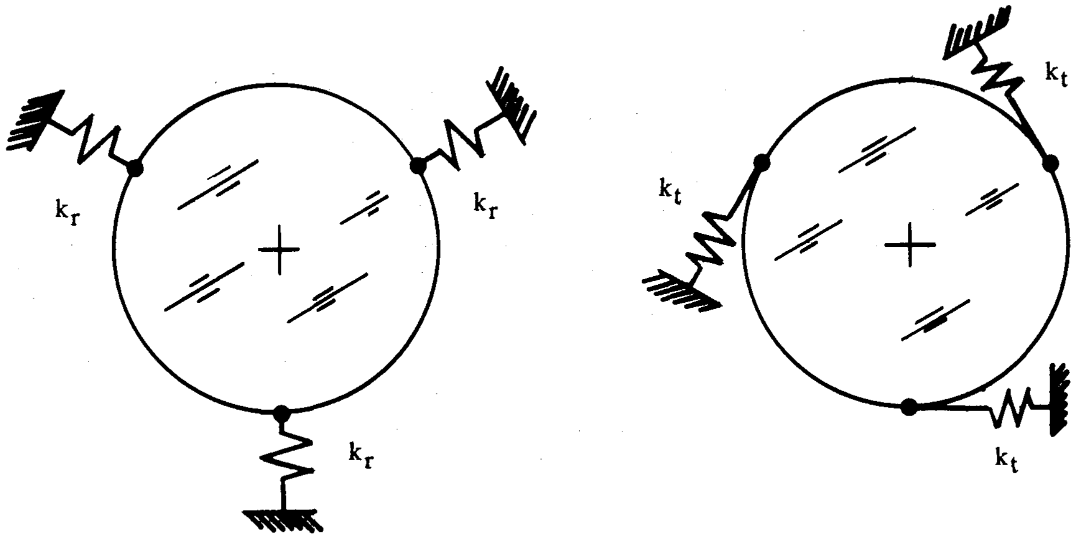


$K = \frac{3}{2} k_r$ $\qquad$ $K = \frac{3}{2} (k_r + k_t)$ $\qquad$ $K = \frac{3}{2} k_t$

Figure 1. Combined tangent bar support system

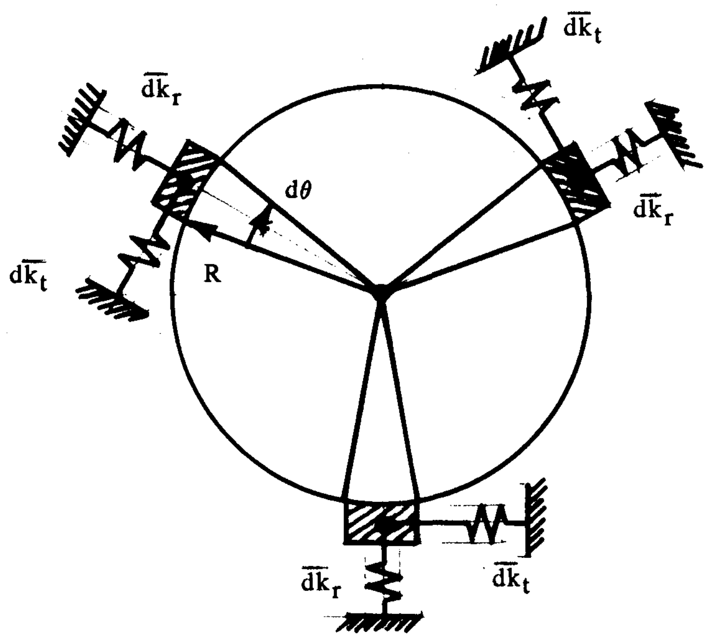


Figure 2. Notation for analytical derivation

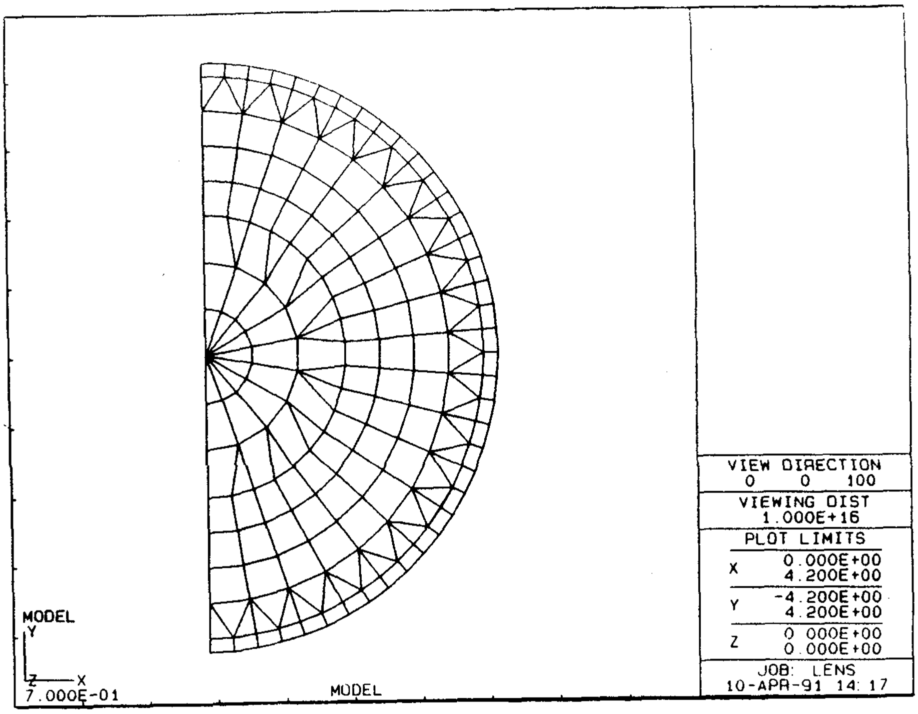


Figure 3. Gifts finite element model of a lens mounted in an elastomer

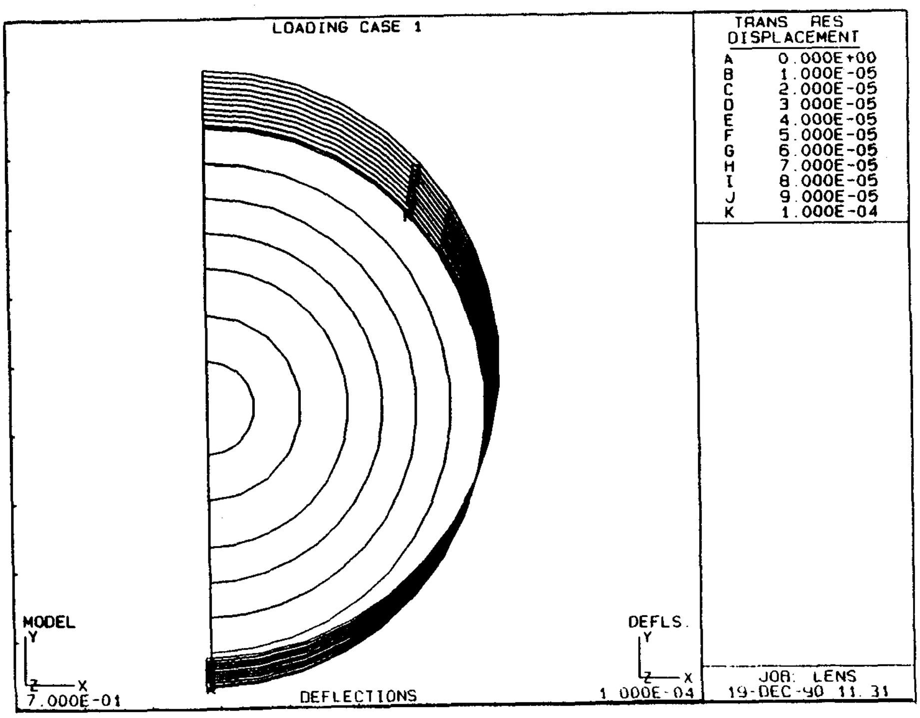


Figure 4. Deflection of the lens model in the elastomer subjected to gravity loading

**GIVEN:**

LENS MATERIAL = KZFSN9

$E_{lens}$ = 9.573E6 psi

$\nu_{lens}$ = 0.271

LENS RADIUS = 4 in.

$E_{elastomer}$ = 500 psi

$\nu_{elastomer}$ = 0.40

ELASTOMER THICKNESS = 0.20 in.

| **CASE 1** | **CASE 2** |
|---|---|
| LENS THICKNESS = 0.5 in. | LENS THICKNESS = 1.0 in. |
| LENS WEIGHT = 2.714 lb | LENS WEIGHT = 5.4286 lb |

| **METHOD** | **Δ (IN) CASE 1** | **Δ (IN) CASE 2** |
|---|---|---|
| ANALYTICAL | 11.2 E -5 | 8.06 E -5 |
| GIFTS (FEM) | 11.4 E -5 | 7.10 E -5 |

Figure 5. Comparison of deflections for GIFTS and analytical solutions