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Thermo-elastic analysis of large optical systems

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ABSTRACT

Once the temperature distribution is known for a large optical system, there are various methods to predict its effect on the optical performance. The thermal distribution is assumed known by measurement, heat-transfer analysis, or supposition. A system consisting of reflective and refractive elements, their supporting structure, and the surrounding medium will all be affected. The reflective optical elements and structure are usually analyzed for their thermo-elastic response, while the refractive elements are subject to both elastic distortions and refractive index changes.

While it might appear almost hopeless to look for theoretical (closed form) solutions, there are some available that are both powerful and practical. Most finite difference and finite element elastic solutions can incorporate temperature effects and are used for a wide range of opto-mechanical structures. In addition to the temperature, one must also know the corresponding material parameter (for example, the coefficient of thermal expansion) These parameters are ofter themselves temperature dependent and are not constant either throughout the structure or even within a single (non-homogeneous) component.

Since temperature distributions can be irregular, variable and difficult to predict exactly, orthogonal functions can often be analyzed. Then the thermal distribution can be approximated by a sum of these functions thereby predicting the whole response.

1. INTRODUCTION

This discussion centers on large optical telescope systems. Most of the comments are just as applicable to other optical systems but the examples are expressed in terms of telescope technology. One way to start is to describe a system that would not require a thermo-elastic analysis . Such a system would be obtained if all components were made, assembled, tested and used at one single, constant temperature. Some systems are built and used in an optical laboratory where the environment is controlled. If the excursions of the actual temperature are small enough not to cause any noticeable effects, then this is a reasonable approximation to this idealization.

The special temperature in the idealization above can be called the reference temperature, Tref. This could be 22 degrees Celsius. Tref, a constant, could have been chosen as a different value BUT it is implied that you know all the necessary conditions pertaining to the system at the Tref temperature you choose. The difference of the actual temperature and Tref is the temperature distribution, T, used to analyze

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the system. Then,

$$T(x,y,z,t) = \text{actual temperature}(x,y,z,t) - Tref \quad (1)$$

where x,y,z are rectangular Cartesian coordinates (or any coordinate system you prefer) and t is time. As long as the actual temperature is equal to Tref, T will be zero, i.e., there are no thermal effects to be added to those we already know about at Tref.

Temperature, a scalar, has magnitude but no direction associated with it and its distribution is always in three dimensional space even though it can be useful at times to describe its value in only one or two directions. The units of temperature can be confusing since only differences are described relative to Tref. Only if Tref were the zero point of a known scale (e.g., Celsius) would T have the same scale values. Instead, the differences of temperature above (positive) or below (negative) the chosen reference value are used. The units are C for Centigrade degree (not degree Celsius), or F, for Fahrenheit degree, etc.

While this paper does not treat heat transfer in itself, certain ideas are assumed common knowledge; heat will transfer from a higher temperature to a lower, the heat transfer can be described as conduction, convection or radiation or some combination thereof, etc. It is assumed that the thermal distribution is known or taken by hypothesis. It is also assumed that the governing thermo-elastic equations, finite element approximations, or physical laws are known.

2. CONSTANT TEMPERATURE IN A LINEAR, HOMOGENEOUS, ISOTROPIC MATERIAL (A CLASSIC)

The body in question has a constant but non-reference value such that,

$$T(x,y,z,t) = C0 \quad (2)$$

The displacements and stresses that result from T can only be determined if the boundary conditions are known. There are two idealized conditions that can be described easily: the completely unconstrained body and the completely constrained body. The former is probably better known than the latter but we can consider them as the two extremes; intermediate cases of partial constraints would fall between them.

As long as the body is free to expand and contract (unconstrained) there is no stress induced in the body due to the temperature change. All dimensions change in a manner that is proportional to their previous values. U, v, and w are the x,y,z displacement components resulting from T:

$$u(x,y,z,t) = CTE * C0 * x$$

$$v(x,y,z,t) = CTE * C0 * y$$

$$w(x,y,z,t) = CTE * C0 * z \quad (3)$$

The linear coefficient of thermal expansion (CTE), describes the change in length per unit length per unit temperature change, or "thermal strain", for an unconstrained body. For example, take a circular concave mirror of diameter 100 cm., edge height of 10 cm. and focal length of 500 cm. Then if C0=10 C and CTE = 1.0 E-5/C, the diameter increases by 0.1 mm, the edge height increases by 0.01 mm. and the focal length increases by 0.5 mm.

For an unconstrained body, rigid body displacements (RBD's) can be added to (3) depending on the choice of the origin and any other arbitrary defining conditions. A body can be defined in space and move about as a rigid body and be unconstrained thermally as long as the temperature change does not introduce "thermal stresses". Assuming the mirror above is supported and defined in the gravity environment there are elastic stresses in the body but no change in stress due to temperature changes.

For the other idealization of the body totally constrained there are no strains and the thermal displacements are all zero but the normal stresses are

$$\text{stress} = - \text{CTE} * \text{E} * \text{C0} / (1. - 2.* \text{PR}) \qquad (4)$$

where E is the modulus of elasticity and PR is Poisson's ratio. The negative sign is conventional for compressive stresses. Again this case is never found in practice, but indicates the upper bound of thermally induced stresses which can be quite high. In the example above, if E = 10.0E+6 psi and PR is 0.25, the normal stresses are 2000 psi.

Most real systems fall somewhere between the two idealized cases and can require considerable analysis for individual situations. We also assume that when thermal stresses are induced, they are added algebraically to the other stresses. The unconstrained case is more easily achieved in practice and is usually not a problem when allowances are made for it in the system.

An example of a theoretical solution for partial constraints that might be of interest to optical engineers is the following. Assume a circular, flat optical window, constrained only at its outer edge (normal to the surface, no shear stress) with the T = C0 value. Then the normal in-plane stresses are;

$$\text{stress} = - \text{CTE} * \text{E} * \text{C0} / (1. - \text{PR}) \qquad (4a)$$

while the axial normal stress is zero. The change in the window thickness is

$$\text{change in thickness} = \text{CTE} * \text{C0} * \text{z0} *(1. + \text{PR})/(1. - \text{PR})$$

where z0 is the original thickness. The stresses are reduced while the displacement is increased.

Of even more practical interest would be the case of the window above being constrained by the structural material (of a different CTE) in which it is mounted when the window stress has a negligible effect on the structure. Now the window stresses are

$$\text{stress} = (\text{struct.CTE} - \text{CTE}) * \text{E} * \text{C0} / (1. - \text{PR}) \qquad (4b)$$

and as a further note, the window would not change thickness if

$$struct.CTE = CTE * (1. + PR)/ (2.* PR) \tag{4c}$$

even though there is still a stress in the window. If PR = .25, the struct.CTE = 2.5 * CTE.

2.1. Constant temperature changes (a contradiction in terms)

If the temperature of a solid body were truly constant (isothermal), there would be no heat transfer within the body nor any heat transfer to or from its surroundings. It is difficult to maintain constant temperature in a body (within a small error) and it is impossible to do even that while the temperature is required to change in a finite amount of time. While this seems too obvious to state, it is a request often made for thermally sensitive optical elements in a changing temperature environment. Given a value for specific heat and conductivity, an estimate of temperature gradient can be made to accomplish a temperature change in a given time frame.

3. LINEAR TEMPERATURE GRADIENTS IN A LINEAR, HOMOGENEOUS, ISOTROPIC MATERIAL

Since gradients will normally also be present in the body, it would be advantageous to understand the effect of linear gradients. We can start with the premise that

$$T(x,y,z,t) = C1 * x \tag{5}$$

For an unconstrained body, the thermal displacements (sans RBD's) are:

$$u(x,y,z,t) = .5 * CTE* C1 * (x*x -y*y -z*z)$$

$$v(x,y,z,t) = CTE * C1 * x*y$$

$$w(x,y,z,t) = CTE * C1 * x*z \tag{6}$$

Again all the thermal stresses are zero. If the gradient were in the y direction, then a similar set of displacements are obtained with the role of x above replaced by y and similarly for a gradient in the z direction. A gradient of a scalar is a vector so that a gradient in an arbitrary direction can be composed of components in the three coordinate directions. Since the equations are linear, the solutions can be superimposed. Thus,

$$T(x,y,z,t) = C0 + C1 * x + C2 * y + C3 * z \tag{7}$$

results in displacements of

$$u(x,y,z,t) = CTE *[C0*x + .5*C1*(x*x -y*y -z*z) + C2*x*y + C3*x*z]$$

$$v(x,y,z,t) = CTE *[C0*y + C1*x*y + .5*C2*(y*y -x*x -z*z) + C3*y*z]$$

$$w(x,y,z,t) = CTE *[C0*z + C1*x*z + C2*z*y + .5*C3*(z*z -y*y -x*x)] \quad (8)$$

The stresses are all zero.

Because all the stresses are zero, this result applies equally well to any homogeneous body regardless of its geometry. The same solution applies to a steel frame, a cube, or a structured, cast honey-comb mirror. While just the constant term is normally a poor approximation of T, the addition of the gradients allow one to come much closer to real situations.

We can also extend the above solution to geometries expressed in other coordinate systems. One case of very practical importance to telescope engineers is a primary mirror with a parabolic upper surface. The upper surface can be expressed most easily in cylindrical coordinates, r,θ,z, as

$$z(r) = z0 + r*r/(2. * R) \quad (9)$$

where R is the radius of curvature (twice the focal length), and z0 is the z coordinate of the mirror vertex. The relation between the two systems of coordinates is given by,

$$x = r * \cos(\theta)$$

$$y = r * \sin(\theta) \quad (10)$$

Then w(r,θ,z,t) is found by substituting the coordinate transformation relations (9,10) into the expression for w of (8) and after collecting "optically significant" terms together, one can find that the axial surface displacements for a parabolic mirror due to a constant and linear temperature gradients is (reference 1):

$$\begin{aligned} W(r,\theta) = \; & (\alpha C_3/8R^2)r^4 && \text{(spherical)} \\ & +(\alpha C_1/2R)r^3\cos(\theta) + (\alpha C_2/2R)r^3\sin(\theta) + && \text{(coma)} \\ & +(\alpha C_3 Z_0/2R - (\alpha C_3/2 + \alpha C_0/2R)r^2 && \text{(focus)} \\ & +(\alpha C_1 Z_0 - C_4)\, r\cos(\theta) + (\alpha C_2 Z_0 - Cr_5)\, r\sin(\theta) && \text{(tilt)} \\ & +(\alpha C_3 Z_0^2/2 + \alpha C_0 Z_0 - C_6) && \text{(piston)} \quad (11) \end{aligned}$$

where

W = mirror surface deformation caused by temperature

r = radius position on surface ($r = \sqrt{x^2+y^2}$)

θ = angular position on surface ($\theta = \arctan(y/x)$)

Here, C4, C5, C6 include arbitrary tilts and piston that depend on the mirror defining system for completeness. As noted in the reference, we used this theoretical solution to verify the finite element solution of a structured mirror with this same temperature distribution. The same procedure can be used to rewrite (8) for other special geometries as well.

It must again be stressed that the homogeneous, unconstrained conditions must obtain. In addition, a mirror of non-uniform thickness with a linear gradient implies that while a flat back could have a uniform T, the curved front could not. The temperature difference from front to back is not the same everywhere. Steady-state heat conduction requires linear temperature gradients. With the assurance that no thermal stresses are introduced and that the displacements are known, it may be possible to change the temperature of elements with fewer optical "side effects".

3.1. Some comments about CTE

For most materials, CTE is a positive, non-zero number; for positive T there is expansion. The term linear can be misleading. For a small range of T, the CTE can be approximated as a constant but for large T (e.g., cryogenic temperature-room temperature) it is not normally a good approximation. If CTE(T) is known, the effect can be integrated, but only if the body remains unconstrained through the entire range of T. It can happen that boundary conditions change and an additional non-linearity is suddenly introduced in mid-range. This has been used to "lock" optical elements at one temperature while allowing them freedom at another. When it is an unplanned event, it can change the elastic properties of the system.

The term homogeneous is also an idealization. A material can have variations of CTE from one point to another. Then the variations of CTE at different temperatures exhibit strain/stress patterns similar to those due to T alone. As an example, we have determined that a pyrex mirror with CTE = 3.0 E-6/C can have only .02 C temperature variation to meet our requirements. If we then change its temperature 20 C the variation of CTE tolerable is then 3.0E-9/C (i.e., one part in 1000) to satisfy the same requirement. As another example, the Hubble space telescope primary mirror is kept at the temperature of the polishing shop where it was made to avoid unknown effects at the lower space temperature.

One can also speculate on a possible solution to at another problem facing telescope design. Boules of glass can be fused together to make large mirrors but the individual pieces often have variations of CTE (reference 2). Since the role of CTE variation is essentially the same as T, it might be worthwhile to arrange the pieces so that approximately linear variations of CTE are present along the coordinate directions. To a first approximation, little thermal stress would be introduced and the thermal distortion effect could be calculated as an equivalent temperature gradient to see its effect on the error budget.

3.2. Extension to systems of multiple components

The solution above also applies to an assembly of components as long as the individual members are all of the same, homogeneous material. If a telescope structure is made entirely of steel components (sans optical elements) then any of its dimensions also change according to (8) as long as this does not introduce constraints.

4. ORTHOGONAL T FUNCTIONS

One can devise additional T functions and a common approach is to use power series. It can also be shown that past the linear terms there are always thermally induced stresses even for unconstrained bodies. If theoretical solutions could be found, they would be geometry dependent and would lack the universality of those above. However, modern FEA programs have temperature inputs that give good approximations to thermo-elastic problems. However, many runs would be required to solve all the possible temperature distributions possible in a practical situation.

Orthogonal functions have the advantage that individual terms are independent of each other and thus can be included, or not, in a sum of component terms. If any arbitrary T can be decomposed into a finite series of orthogonal functions, and each of the individual terms is run as a unit T case, then the stress-displacement result is the sum of unit cases weighted by the component terms of the thermal distribution of T. An example of this technique is presented in reference 1. The functions used were two dimensional Zernike polynomials. The two dimensional (x,y) functions could be used in this case since the nodes of the structured mirror were located in z=constant planes. Zernike terms are also used for optical surfaces but there is no direct relation between the thermo-elastic results and the optical surface terms.

4.1. *Non-homogeneous and anisotropic materials.*

At some small error level, all materials exhibit both characteristics. This can cause errors in optical elements and should be accounted for. Some processes, such as powdered, sintered beryllium, improve the isotropy of the material. Composite materials are designed to be anisotropic to enhance certain directional properties. This increases the analysis difficulty, but can result in light-weight, stiff, thermally stable structures. When we can afford them, an added advantage is the internal damping afforded for higher frequency regimes now required of telescopes.

4.2. Finite element models.

Most problems will require the use of finite element analysis (FEA). In fact, there are FEA for heat transfer problems and the thermal results can be used as the input for the thermo-elastic solution. There are even software programs that will do both solutions, sequentially, for the same model. There is then a tendency to use the same model for both. This saves time but few, if any, heat transfer problems are the same as elastic problems from the point of view of modeling. The choice of nodes, type of elements, input conditions, etc., have to be good representations of the problem to be solved. The only single-model-solution is to combine the complexities of both such that you have "over-modeled" both regimes. This requires a sensitivity to both models, time-consuming model building, and longer solution runs.

4.3. The time factor, t

Any idealization that T is not changing with time is suspect for the same reasons that "isothermal" and "steady state" are. It makes the analyst's life a little easier but real systems never like to cooperate. FEA heat transfer analysis will solve for time dependent conditions, T(x,y,z,t), and elastic FEAs also find dynamic and time response solutions. HOWEVER, not only is there the modeling problem, but a mathematical and physical distinction between the two time dependent solutions. For the time intervals, ti, i=1,n, each resulting T(x,y,z,ti) can be considered a "snapshot" of thermal input for thermo-elastic FEA. This is a laborious process for most software.

5. SUMMARY

1. Theoretical solutions can be easily misapplied to the wrong conditions but they also can produce good results quickly in many cases.
2. FEAs can be used for both the estimation of T as well as the thermo- elastic stress/displacement relations but dissimilar models probably are required and time dependent cases are truly "time consuming".

6. REFERENCES

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