Idealization of Supports and Distributed loads

Equilibrium: Rigid and Deformable Bodies

The following equations are the necessary conditions for equilibrium of a rigid body.

$$\Sigma F_x = 0 \qquad\qquad \Sigma F_y = 0 \qquad\qquad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \qquad\qquad \Sigma M_y = 0 \qquad\qquad \Sigma M_z = 0$$

Forces and moments that act on a body are either external or internal.

External Forces

- Interaction between bodies.
- Interaction between body and earth.
- Fluid pressure.
- Cause the body to move or ensures that it remains at rest.
- External forces are divided into applied forces and reaction forces.
    - Applied forces cause a body to move or rotate.
    - Reaction forces hold the body in equilibrium.

Internal Forces

- Forces, which hold together the particles forming a rigid body.
- Stress = the intensity of the internal forces

Free Body Diagrams

Step 1. Determine which body or combination of bodies is to be isolated. The body chosen will usually involve one or more of the desired unknown quantities.
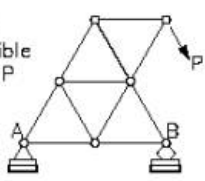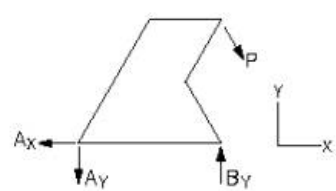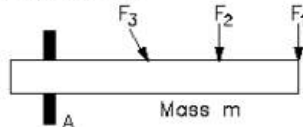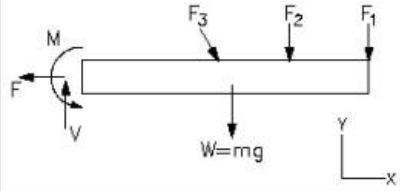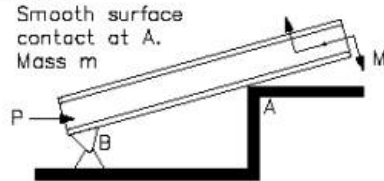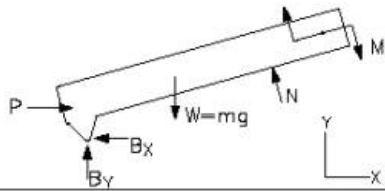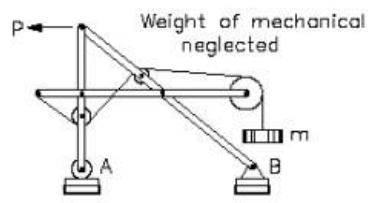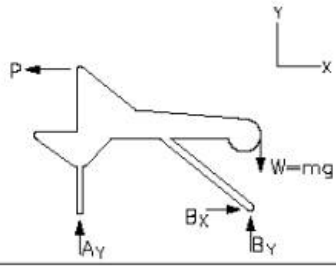
Step 2. Next, isolate the body or combination of bodies chosen with a diagram that represents its complete external boundaries.

Step 3. Represent all forces that act on the isolated body as applied by the removed contacting bodies in their proper positions in the diagram of the isolated body. Do not show the forces that the object exerts on anything else, since these forces do not affect the object itself.
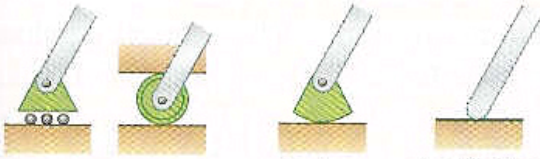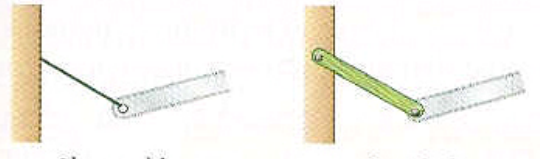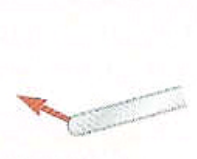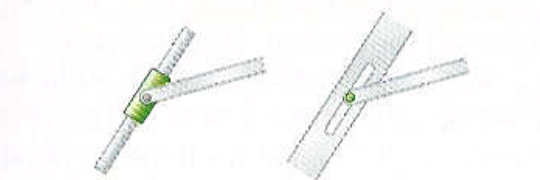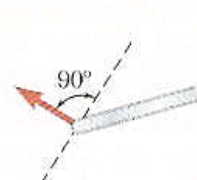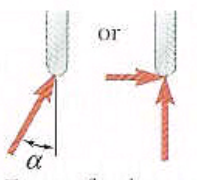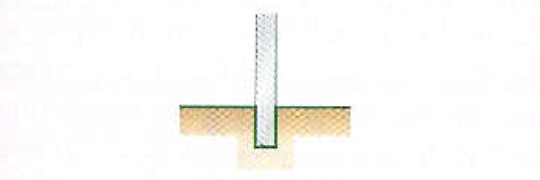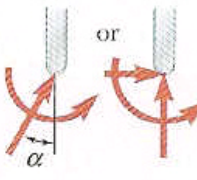
Step 4. Indicate the choice of coordinate axes directly on the diagram.   Pertinent dimensions may also be represented for convenience. Note, however, that the free-body diagram serves the purpose of focusing accurate attention on the action of the

external forces; therefore, the diagram should not be cluttered with excessive information. Force arrows should be clearly distinguished from other arrows to avoid confusion.
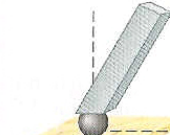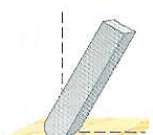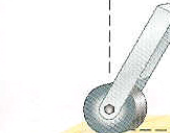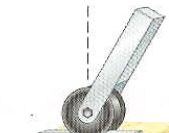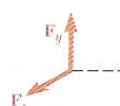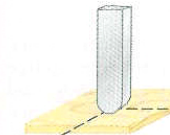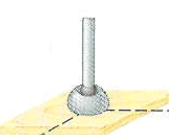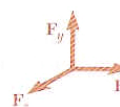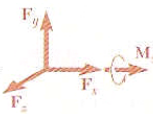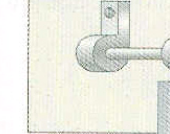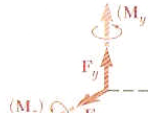
When these steps are completed a correct free-body diagram will result. Now, the appropriate equations of equilibrium may be utilized to find the proper solution.

Idealization of 2D supports

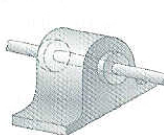| Support or Connection	Reaction	Number of Unknowns
Rollers Rocker Frictionless surface	Force with known line of action	1
Short cable Short link	Force with known line of action	1
Collar on frictionless rod Frictionless pin in slot	90° Force with known line of action	1
Frictionless pin or hinge Rough surface	or α Force of unknown direction	2
Fixed support	or α Force and couple	3

Idealization of 3D supports

Free-Body Examples

Distributed Loads on Beams

Beam – Structural member whose length is large compared to its cross sectional area which is loaded and supported in the direction transverse to its axis.
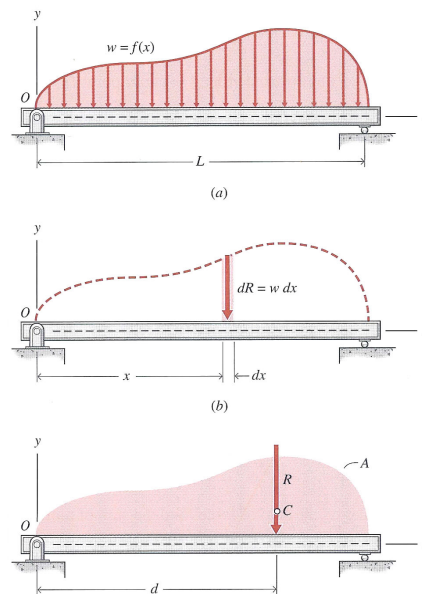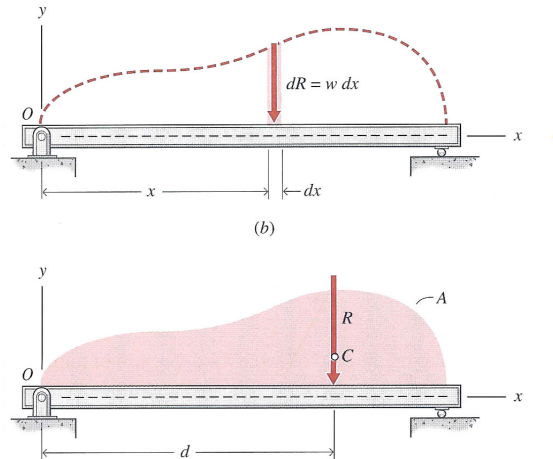

w = distributed line load (N/m or lb/ft)

$R = \int w dx$ = area under curve

(b)

$$M_o = \int x(w\,dx)$$

$$d = M_o/R$$

A distributed load on a beam can be replaced by a concentrated load; the magnitude of this single load is equal to the area under the load curve and its line of action passes through the centroid of that area.

Note:

The concentrated load is equivalent to the distributed load **only as far as external forces are concerned**. It can be used to determine reactions, but **should not** be used to determine internal forces and deflections.

Example Problem – Distributed loads
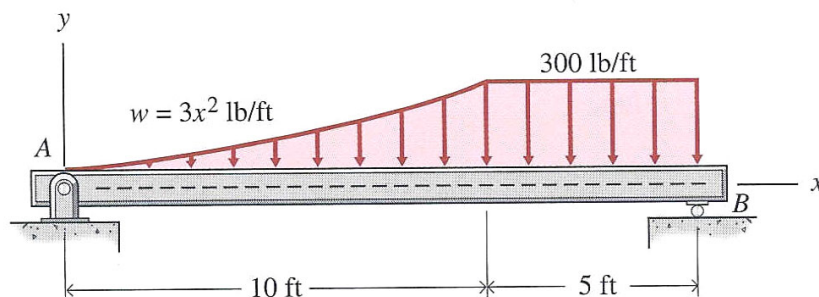
Determine the following:

1. The resultant force.
2. Its line of action with respect to A.
3. The reaction forces at A and B.