Stresses on Inclined Planes

Axial forces cause both normal and shearing stresses on planes, which are not perpendicular to the axis of the member.

\[ \sigma = \frac{F}{A_\theta} = \frac{P \cos \theta}{A_o} = \frac{P}{2A_o} (1 + \cos 2\theta) \]

\[ \tau = \frac{V}{A_\theta} = \frac{P \sin \theta}{A_o} = \frac{P}{2A_o} \sin 2\theta \]

The following graph shows the magnitudes of \( \sigma \) and \( \tau \) as a function of \( \theta \).

For an axially loaded member:

\[ \sigma_{\text{max}} = \frac{P}{A_o} \]

\[ \tau_{\text{max}} = \frac{P}{2A_o} \]
**Displacement, Deformation and Strain**

**Displacement** – Movement under load. When a system of loads is applied to a machine component, individual points of the body move (displace) from their original positions. Displacement is a vector quantity.

**Deformation** – A change in the geometry of a body. This may be a change in size, shape or both. Deformation, symbolized by the Greek letter delta, $\delta$, is the change in dimensions associated with relative displacements. **Note:** deformation is not uniquely related to force or stress.

**Strain** – Deformation per unit length. We shall consider two classifications of strain.

**Normal Strain**, symbolized by the Greek letter epsilon, $\varepsilon$, measures the change in size of an arbitrary line segment of a body during deformation. In a long rod, the average **axial strain** and is the ratio of deformation and the original length.

\[
\varepsilon_{\text{avg}} = \frac{\delta}{L} = \frac{L_{\text{final}} - L_{\text{initial}}}{L_{\text{initial}}}
\]

The above definition of strain is sometimes called engineering strain. If the member increases in length, $\varepsilon$ is positive and called a tensile strain; if the member shortens, $\varepsilon$ is negative and is called a compressive strain.
Strain at a Point

In the case of a member of variable cross-sectional area $A$, the normal stress varies along the length of the member. Therefore, it is necessary to define the strain at a given point $Q$ by considering a small element of undeformed length $\Delta x$. Denoting the deformation of the element as $\Delta \delta$, we may define the normal strain at point $Q$ as:

$$\varepsilon_Q = \lim_{\Delta x \to 0} \frac{\Delta \delta}{\Delta x} = \frac{d \delta}{dx}$$

Shear Strain, symbolized by the Greek letter gamma, $\gamma$, is a measure of the change in shape of a body during deformation. The average shear strain is the change in angle between two originally perpendicular sides of a body and is measured in radians.

$$\gamma_{avg} = \frac{\pi}{2} - \alpha$$

For small deformations we obtain:

$$\gamma_{avg} \approx \tan\left(\gamma_{avg}\right) = \frac{\delta_s}{L}$$
Consider the following element undergoing shear strains.

Again, for cases in which the deformation is nonuniform, we may consider the shear strain at a point.

\[
\gamma_{xy}(P) = \lim_{\Delta L \to 0} \frac{\Delta \delta_s}{\Delta L} = \frac{d\delta_s}{dL}
\]

An equivalent expression is:

\[
\gamma_{xy}(P) = \frac{\pi}{2} - \theta'
\]

Both normal and shearing strains are dimensionless quantities.

Normal Strains → in./in. or µ in./in.
Shear Strains → radians or µ rad

Shear Strain Example