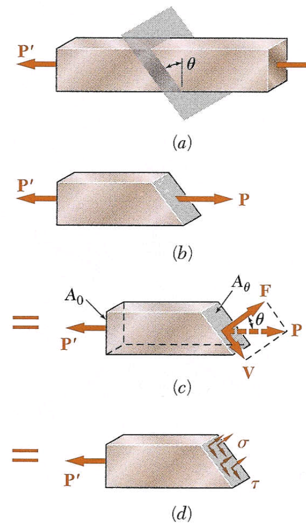
Stresses on Inclined Planes

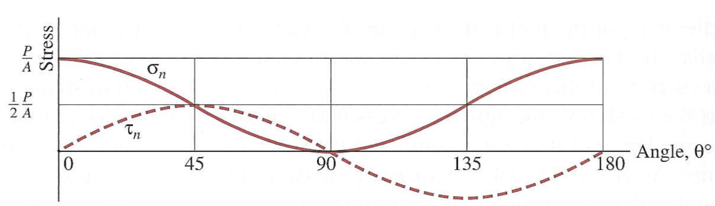
Axial forces cause both normal and shearing stresses on planes, which are not perpendicular to the axis of the member.



$$\sigma = \frac{F}{A_\theta} = \frac{P\cos\theta}{\frac{A_o}{\cos\theta}} = \frac{P}{2A_o}(1+\cos 2\theta)$$

$$\tau = \frac{V}{A_\theta} = \frac{P\sin\theta}{\frac{A_o}{\cos\theta}} = \frac{P}{2A_o}\sin 2\theta$$

The following graph shows the magnitudes of σ and τ as a function of θ.



For an axially loaded member:

$$\sigma_{\max} = \frac{P}{A_o}$$

$$\tau_{\max} = \frac{P}{2A_o}$$

Fig. 2-31 Failure along a 45° plane of a wood block loaded in compression

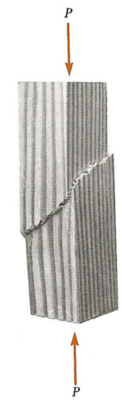


Fig. 2-32 Slip bands (or Lüders' bands) in a polished steel specimen subjected to axial tension

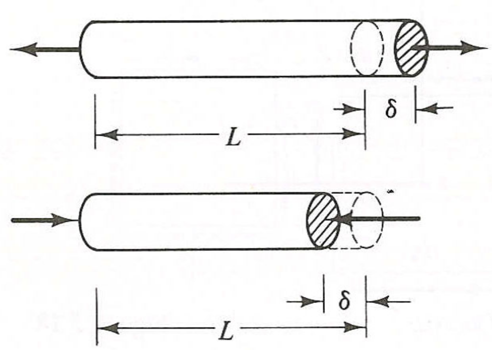
Displacement, Deformation and Strain

Displacement – Movement under load. When a system of loads is applied to a machine component, individual points of the body move (displace) from their original positions. Displacement is a vector quantity.

Deformation – A change in the geometry of a body. This may be a change is size, shape or both. Deformation, symbolized by the Greek letter **delta**, δ, is the change in dimensions associated with relative displacements. Note: deformation is not uniquely related to force or stress.

Strain – Deformation per unit length. We shall consider two classifications of strain.

Normal Strain, symbolized by the Greek letter **epsilon, ε,** measures the change in size of an arbitrary line segment of a body during deformation. In a long rod, the ***average axial strain*** and is the ratio of deformation and the original length.

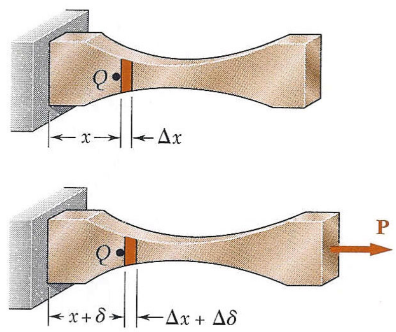


$$\varepsilon_{avg} = \frac{\delta}{L} = \frac{L_{final} - L_{initial}}{L_{initial}}$$

The above definition of strain is sometimes called engineering strain. If the member increases in length, ε is positive and called a tensile strain; if the member shortens, ε is negative and is called a compressive strain.

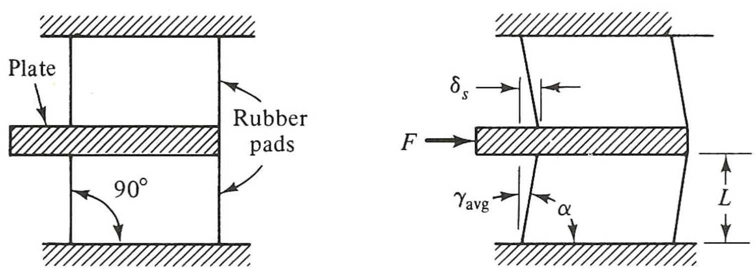
Strain at a Point

In the case of a member of variable cross-sectional area **A**, the normal stress varies along the length of the member. Therefore, it is necessary to define the strain at a given point **Q** by considering a small element of undeformed length Δ**x**. Denoting the deformation of the element as Δδ, we may define the ***normal strain at point Q*** as:



$$\varepsilon_Q = \lim_{\Delta x \to 0} \frac{\Delta \delta}{\Delta x} = \frac{d\delta}{dx}$$

Shear Strain, symbolized by the Greek letter **gamma, γ,** is a measure of the change in shape of a body during deformation. The ***average shear strain*** is the change in angle between two originally perpendicular sides of a body and is measured in radians.

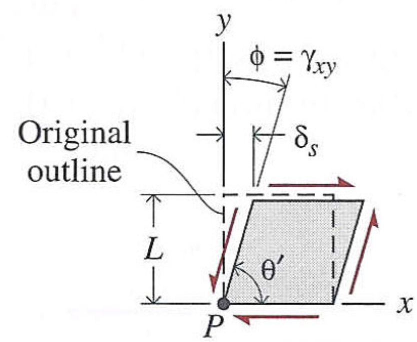


$$\gamma_{avg} = \frac{\pi}{2} - \alpha$$

For small deformations we obtain:

$$\gamma_{avg} \approx \tan\left(\gamma_{avg}\right) = \frac{\delta_s}{L}$$

Consider the following element undergoing shear strains.



Again, for cases in which the deformation is nonuniform, we may consider the shear strain at a point.

$$\gamma_{xy}(P) = \lim_{\Delta L \to 0} \frac{\Delta \delta_s}{\Delta L} = \frac{d\delta_s}{dL}$$

An equivalent expression is:

$$\gamma_{xy}(P) = \frac{\pi}{2} - \theta'$$

Both normal and shearing strains are dimensionless quantities.

Normal Strains → in./in. or μ in./in.

Shear Strains → radians or μ rad

<u>Shear Strain Example</u>

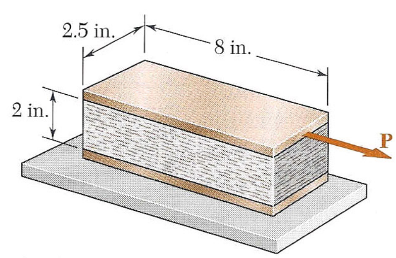


Fig. 2.51