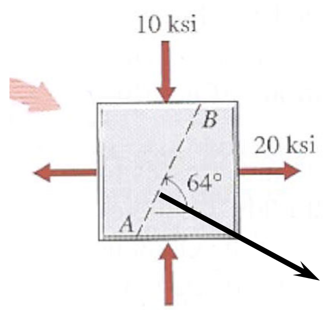
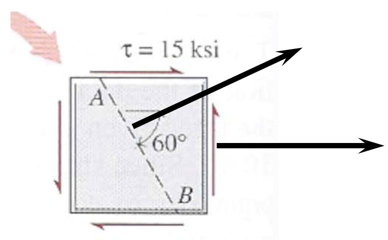
Mohr's Circle for Plane Stress (*Additional Topics*)

Homework Examples:

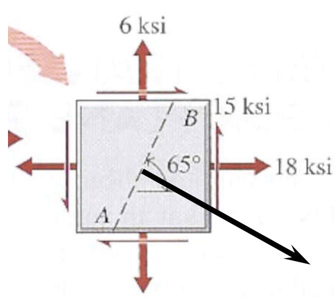
$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$



θ = -26°



θ = 30°



θ = -25°

Theta, θ, is the rotation of the positive vertical face using the right hand rule. In other words, theta is the angle of the normal of any arbitrary plane with respect to the X axis.

Mohr's Circle for Plane Stress:

In the previous lecture, we examined the construction of Mohr's circle for plane stress. Some important geometric observations are:

Center of the circle is the average stress.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Radius of the circle is the maximum shear stress.

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

Principal stresses equal the center of the circle +/- the radius.

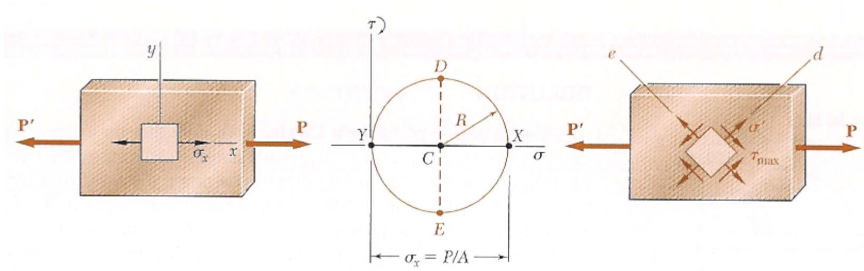
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

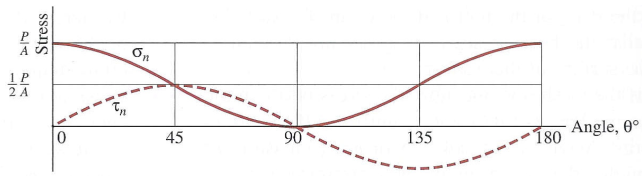
Note: The coordinates of each point on Mohr's circle represent σ_n and τ_{nt} for one particular plane through the stressed point. When interpreting shear stresses one must consider the sign. Recall when plotting Mohr's circle, a clockwise shear is plotted as positive and a counter-clockwise shear is plotted as negative. Use this sign convention when sketching the stressed element.

Mohr's Circle Examples:

Centric Axial Loading



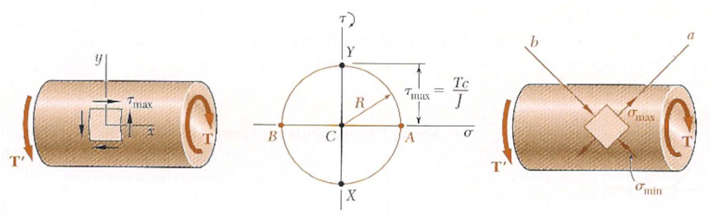
In lecture 3, we found that axial forces cause both normal and shearing stresses on planes, which are not perpendicular to the axis of the member.



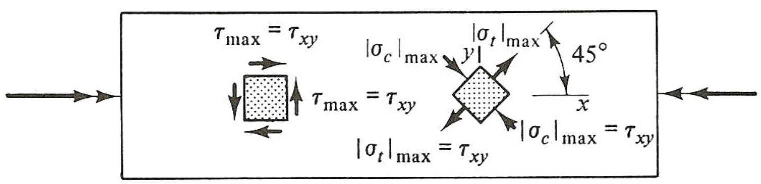
$$\sigma_{\max} = \frac{P}{A_o}$$

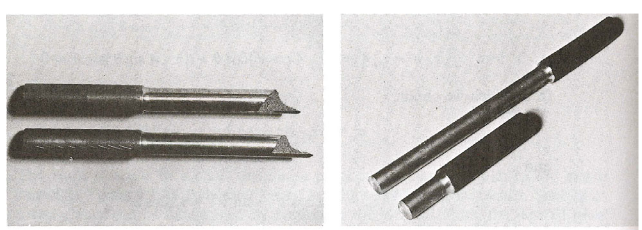
$$\tau_{\max} = \frac{P}{2A_o}$$

Torsional Loading



In lecture 15, we found that in a circular member subjected to pure shear, the normal stresses are a maximum on a 45° plane.





(a) Cast Iron (b) Mild Steel

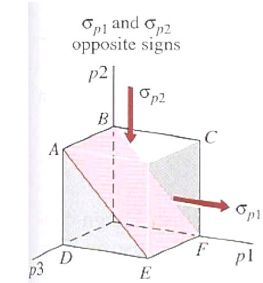
In previous lectures, we found that the maximum in-plane shear stress may be determined as follows.

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

or

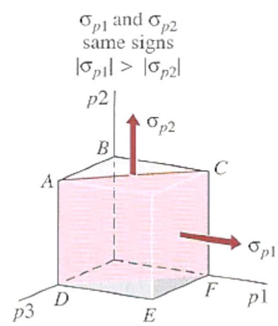
$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

However, when a state of plane stress exists, one of the principal stresses is zero. If the values of σ_1 and σ_2 have the same sign, Mohr's circle is shifted to the right or left of the vertical, τ axis and the minimum stress is zero. Therefore:



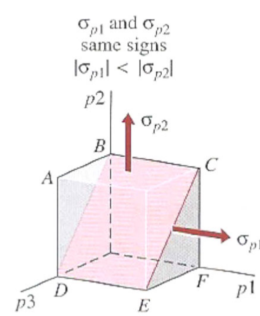
Maximum in-plane shear stress occurs with rotation about the p_3 axis and:

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$



Maximum in-plane shear stress occurs with rotation about the p_2 axis and:

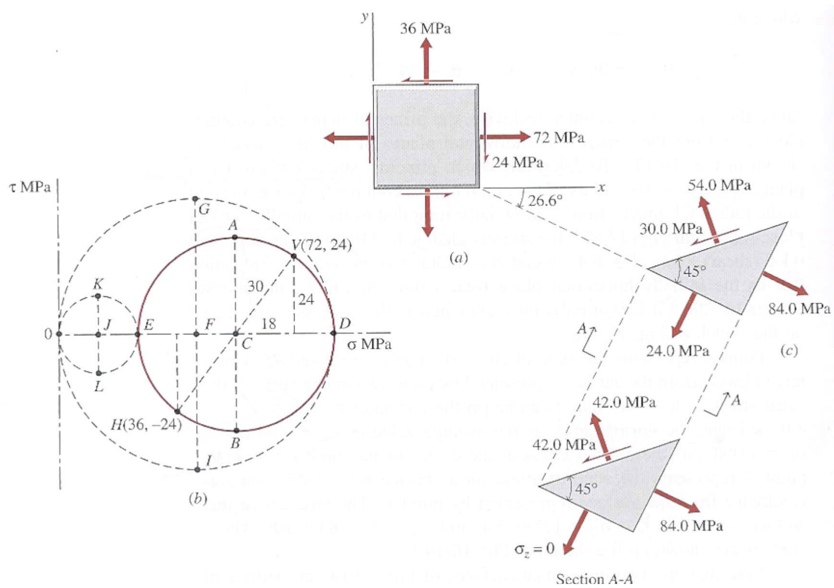
$$\tau_{\max} = \frac{\sigma_1 - 0}{2}$$



Maximum in-plane shear stress occurs with rotation about the p_1 axis and:

$$\tau_{\max} = \frac{0 - \sigma_2}{2}$$

This is illustrated with the following example of Mohr's circle.


36 MPa
72 MPa
24 MPa
26.6°
(a)
τ MPa
σ MPa
G
A
V(72, 24)
30
24
18
K
J
E
F
C
D
0
L
H(36, −24)
B
I
(b)
54.0 MPa
30.0 MPa
45°
84.0 MPa
24.0 MPa
(c)
42.0 MPa
42.0 MPa
45°
84.0 MPa
$\sigma_z = 0$
Section A-A