

Mohr's Circle for Plane Stress

Analysis of Stress and Strain:

As we learned in the previous two lectures, when a structural element is subjected to several types of loads acting simultaneously, say bending and torsion, principal stresses occur. These stresses act on principal planes where the shear stresses are zero. In addition, many engineering problems, such as axial bars, beams in bending and circular members in torsion, are examples of a state of stress called **plane stress** ($\sigma_z = \tau_{zx} = \tau_{zy} = 0$). Our procedure for determining principal stresses for a state of plane stress is as follows:

1. Determine the point on the body in which the principal stresses are to be determined.
2. Treating the load cases independently and calculated the stresses for the point chosen. When applicable combine the stresses to determine the state of stress at the point.
3. Choose a set of x-y reference axes and draw a square element centered on the axes.
4. Identify the stresses σ_x , σ_y , and $\tau_{xy} = \tau_{yx}$ and list them with the proper direction.
5. Calculate the principal stresses, the maximum shear stress and the principal plane if required.

Principal Stresses (Shear Stress = 0):

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum Shear Stress:

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

Principal Planes (Planes on which Principal Stresses Act):

$$2\theta_p = \tan^{-1} \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Plane of Maximum Shear Stress:

$$2\theta_s = \tan^{-1} - \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Average Stress (Shear Stress is Maximum):

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Important Observations:

1. Principal stresses occur on mutually perpendicular planes.
2. Shear stresses are zero on principal planes.
3. Planes of maximum shear stress occur at 45° to the principal planes.
4. The maximum shear stress is equal to one half the difference of the principal stresses.

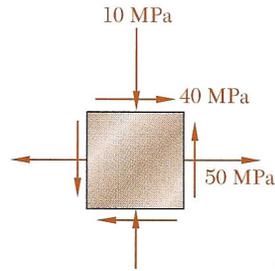
It should be noted that the equation for principal planes, $2\theta_p$, yields two angles between 0° and 360°. Which one corresponds to σ_1 and which to σ_2 ? This can be determined by substituting one of the values into the stress transformation equation for normal stress.

Normal Stress Transformation

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

Shear Stress Transformation

$$\tau_{nt} = -(\sigma_x - \sigma_y) \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta)$$

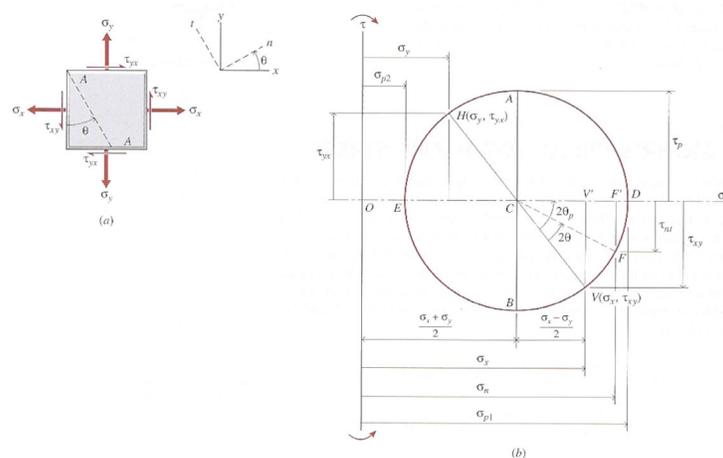
Example Problem:

Determine:

- The principal planes.
- The principal stresses.
- The maximum shear stress and the corresponding normal stress.

Mohr's Circle for Plane Stress:

The transformation equations for plane stress can be represented in a graphical format known as Mohr's circle. This representation is useful in visualizing the relationships between normal and shear stresses acting on various inclined planes at a point in a stressed body.



Before we discuss the procedure for constructing Mohr's circle there are several rules that apply.

Stress Component

Normal Stresses
(*Horizontal Axis*)

Shear Stresses
(*Vertical Axis*)

Plot

+ For Tension

- For Compression

+ For Clockwise Shear

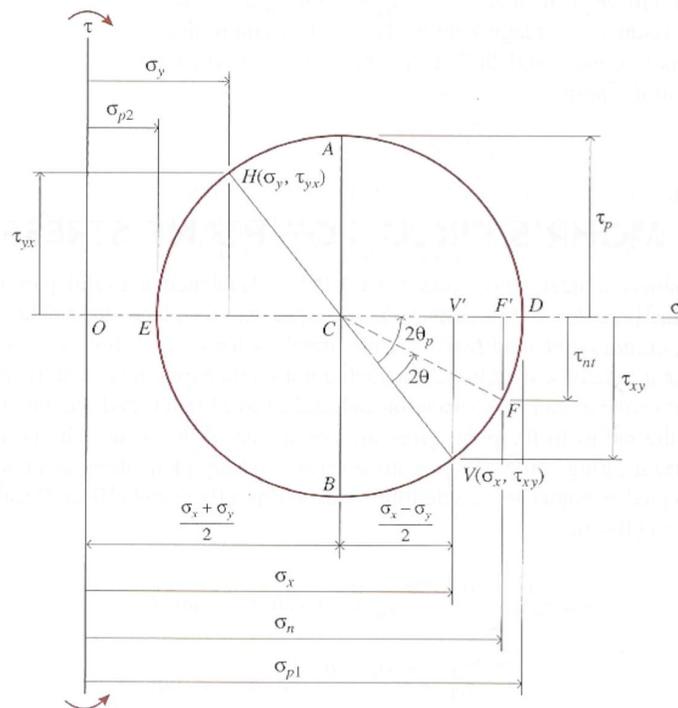
- For Counterclockwise Shear

Construct Mohr's circle as follows:

1. Determine the point on the body in which the principal stresses are to be determined.
2. Treating the load cases independently and calculated the stresses for the point chosen.
3. Choose a set of x-y reference axes and draw a square element centered on the axes.
4. Identify the stresses σ_x , σ_y , and $\tau_{xy} = \tau_{yx}$ and list them with the proper sign.
5. Draw a set of $\sigma - \tau$ coordinate axes with σ being positive to the right and τ being positive in the upward direction. Choose an appropriate scale for the each axis.
6. Using the rules on the previous page, plot the stresses on the x face of the element in this coordinate system (*point V*). Repeat the process for the y face (*point H*).
7. Draw a line between the two point V and H. The point where this line crosses the σ axis establishes the center of the circle.
8. Draw the complete circle.
9. The line from the center of the circle to point V identifies the x axis or reference axis for angle measurements (*i.e.* $\theta = 0$).

Note: The angle between the reference axis and the σ axis is equal to $2\theta_p$.

Mohr's Circle:



Consider the previous example.

