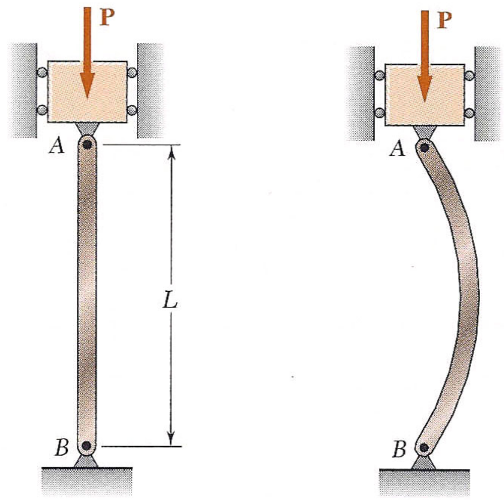
Buckling - Other End Conditions

Buckling (Pin Ended Columns):



For a column with pinned ends, we have the following conditions:

1. Force P that is applied through the centroid of the cross section and aligned with the longitudinal axis of the column.
2. Force P is guided such that P is always aligned with the pin joints
3. Both ends of the beam are free to rotate about one or more axes.

Assuming the material is made of a linear elastic material that follows Hooke's law, we found that the critical load is:

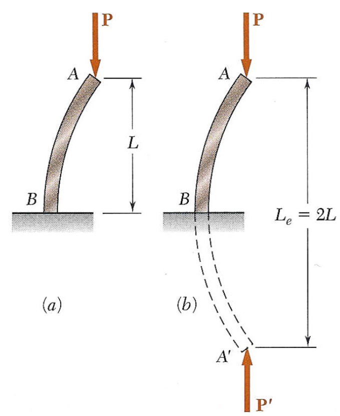
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

and that the critical stress is:

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

Buckling (Columns With Other End Conditions):

However, in many engineering problems we are faced with columns with other end conditions. The first condition we would like to consider is a column with one fixed end and one free (unguided) end.



By observation we see that this is identical to a pinned end column with a length of 2L. Likewise, the solution of this problem is identical to the solution of a pinned end an ***effective length*** of 2L. Using the concept of effective length, Euler's equation becomes:

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

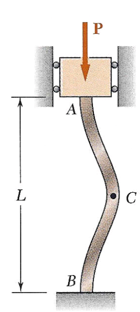
Using the same concept, we may also rewrite our expression for critical stress.

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$

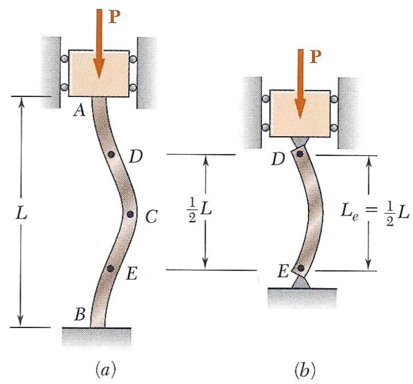
Therefore for a column with one free end and one fixed end, we use an effective length of:

$$L_e = 2L$$

Now lets consider a column with two fixed ends. The fixed end is guided such that the force P acts through the centroid of the cross section at each end.



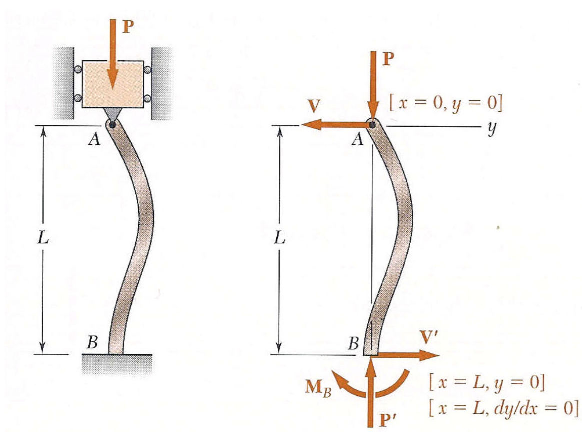
The symmetry of the supports and of the loading about a horizontal axis through the midpoint C requires that the shear at C and the horizontal components at the reactions at A and B be zero. The upper half of the column AC is identical to the lower half of the column therefore portion AC must be symmetric about its midpoint D. In addition, point D must be a point of inflection, where the bending moment is zero.



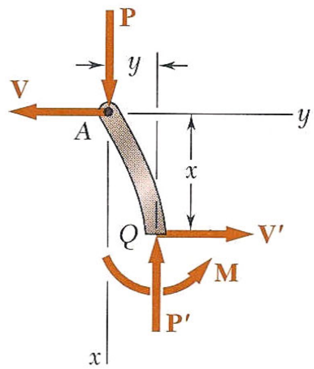
Therefore for a column with two fixed ends, we use an effective length of:

$$L_e = \tfrac{1}{2} L$$

Finally lets consider a column with one fixed end and one pinned end. Again, the pinned end is guided such that the force P acts through the centroid of the cross section at each end.



In this case, we must write and solve the differential equation of the elastic curve to determine the effective length of the column. Considering a free body diagram of a section of the beam and summing moments about Q we have:



$$M = -Py - Vx$$

Substituting this value into our differential equation and setting $k^2 = P/EI$ we obtain:

$$\frac{d^2y}{dx^2} + k^2 y = -\frac{V}{EI}x$$

This equation is a linear, nonhomogeneous differential equation of the second order with constant coefficients. The particular solution for this equation is:

$$y_p = -\frac{V}{k^2 EI}x = -\frac{V}{P}x$$

Adding the particular solution to the general solution we found in lecture 16, we obtain the following:

$$y = C_1 \sin kx + C_2 \cos kx - \frac{V}{P}x$$

In this case C_1, C_2 and V are unknown and must be determined from the boundary conditions. Solving we determine that:

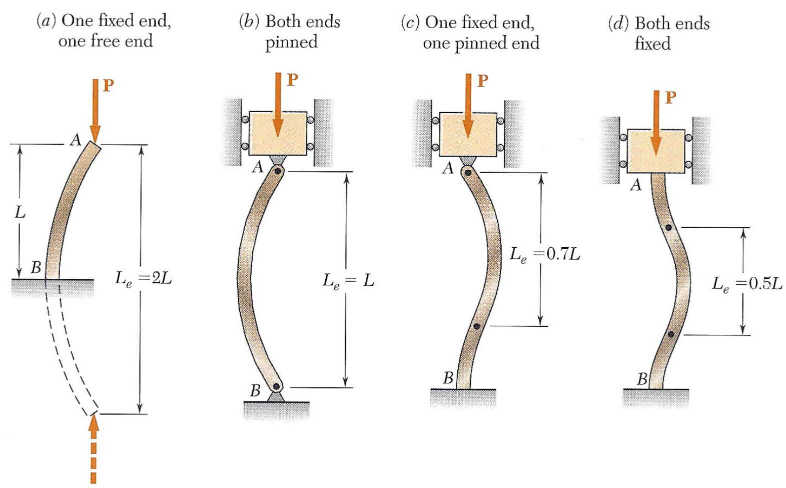
$$P_{cr} = \frac{20.19EI}{L^2}$$

Equating the above equation to Euler's equation we have:

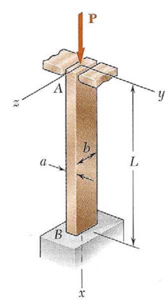
$$\frac{\pi^2 EI}{L_e^{\,2}} = \frac{20.19EI}{L^2}$$

and $L_e = 0.699L \approx 0.7L$.

In summary we have:



Buckling Example 5:



Specifications:

1. P = 5000 lbs.
2. Material is aluminum (E = 10.1 x 10^6 psi).
3. L = 20 in.
4. FS = 2.5.

Determine:

1. The ratio of a/b corresponding to the most efficient design.
2. The size of the column given the specifications above.