

Torsion (Additional Topics)

Torsion:

As we learned in the previous lecture, torsion refers to the twisting of a structural member that is loaded by couples (torque). For a circular shaft manufactured out of a homogeneous and isotropic material, we obtained the following:

$$\text{Shear Strain} \quad \gamma = \frac{\rho\theta}{L} \quad (1)$$

$$\text{Shear Stress} \quad \tau = \frac{T\rho}{J} \quad (2)$$

$$\text{Angle of Twist} \quad \theta = \frac{TL}{GJ} \quad (3)$$

Where:

θ = Angle of twist (radians)

T = Applied torque (moment)

L = length of member

G = Shear modulus of material

J = Polar moment of inertia

ρ = Radius from center of shaft

The angle of twist formula can only be used if the shaft is homogeneous, has a uniform cross section and is loaded by couples at its ends. If the shaft has torques applied at other or multiple locations or the shaft has multiple sections with different diameters, we must divide the shaft into component parts that individually satisfy Equation 3.

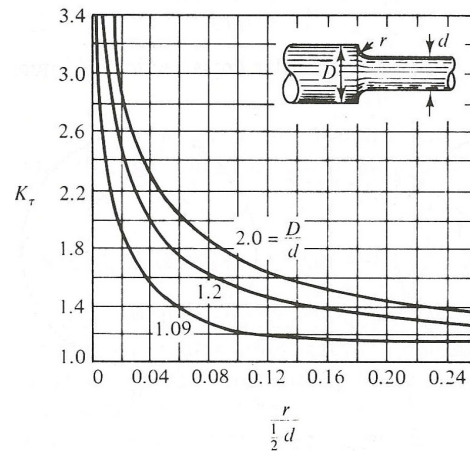
In this case, Equation 1 takes the form:

$$\theta = \sum_{i=1}^n \frac{T_i L_i}{J_i G_i}$$

Stress Concentrations:

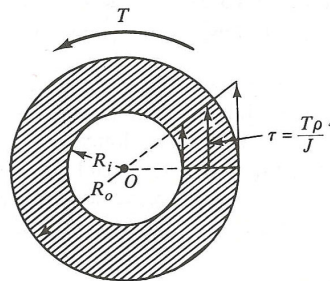
In many cases, shafts contain keyways, fillets and other geometric discontinuities, which can be accounted for by multiplying the shear stress obtained from Eq. 2 by a stress concentration factor K_τ . Thus,

$$\tau_{\max} = K_\tau \frac{TR}{J}$$



Hollow Shafts (Thick Tubes):

As shown in the previous lecture, shear stresses are smallest near the center of the shaft and the moment arms of the forces due to these stresses are also small. Consequently, the material near the center contributes little to the torsional capacity of the shaft. That is why hollow shafts (*i.e. thick wall round tubes*) are commonly used for power transmission.



The polar moment of inertia for a thick wall tube is:

$$J = \frac{\pi}{2}(R_o^4 - R_i^4) = \frac{\pi}{32}(D_o^4 - D_i^4)$$

Thin Walled Circular Tubes:

When the wall thickness is thin, the shear strain is nearly uniform through the wall thickness. Consequently, the stress distribution is also nearly uniform. Recall the polar moment of inertia is:

$$J = \int \rho^2 dA$$

For a round tubular section we have:

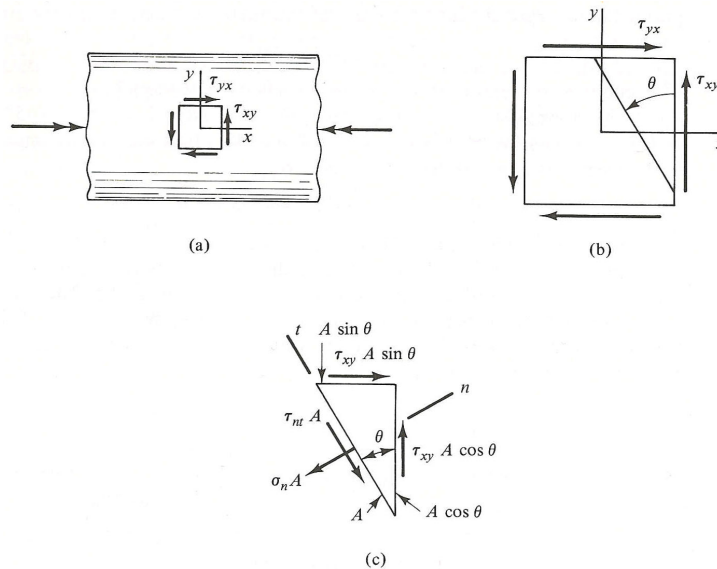
$$J = \int_{R_i}^{R_o} \rho^2 (2\pi\rho d\rho)$$

For a thin tube we can approximate J as follows:

$$J \approx 2\pi r^3 t$$

Stresses on Oblique Planes:

Shown below are the torsional shear stresses both perpendicular and parallel to the axis of the shaft ($\tau_{xy} = \tau_{yx}$). Stresses on any other plane can be determined by sectioning the element and considering the equations of equilibrium.

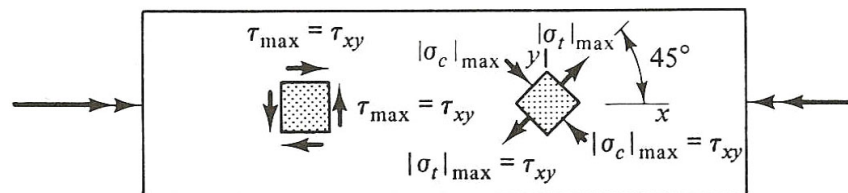


Considering equilibrium in the n and t directions we obtain:

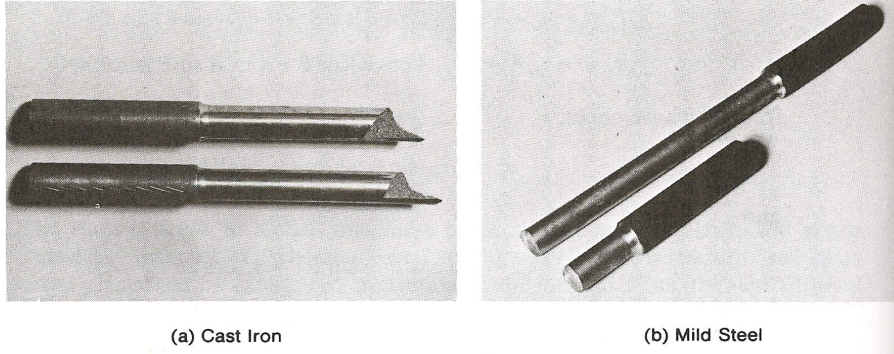
$$\sigma_n = 2\tau_{xy} \sin\theta\cos\theta = \tau_{xy} \sin 2\theta$$

$$\tau_{nt} = \tau_{xy}(\cos^2\theta - \sin^2\theta) = \tau_{xy} \cos 2\theta$$

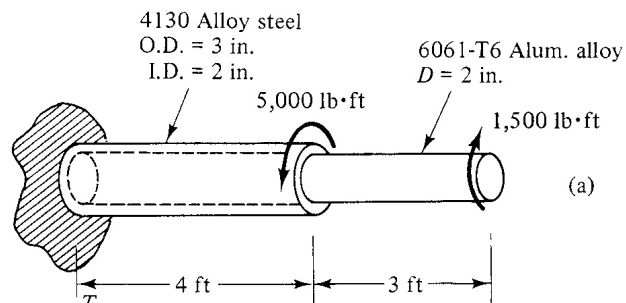
As shown below, normal stresses are a maximum on a 45° plane.



The maximum tensile stress, which is numerically equal to the maximum shear stress, is the significant stress in shafts made from brittle materials. As shown below, the fracture surface of a cast iron shaft is oriented at 45° with respect to the axis. While in a mild steel shaft, the failure plane is perpendicular to the axis of the shaft



Torsion - Problem 1



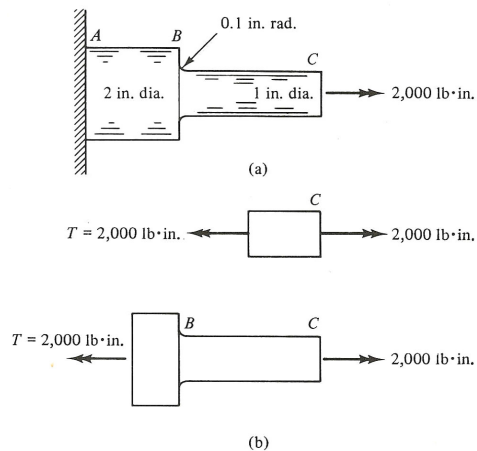
Specifications:

Steel $\Rightarrow G = 12 \times 10^6$, $\tau_{YS} = 60$ ksi.

Aluminum $\Rightarrow G = 3.8 \times 10^6$, $\tau_{YS} = 21$ ksi.

Determine:

- a) The angle of twist of the free end with respect to the wall.

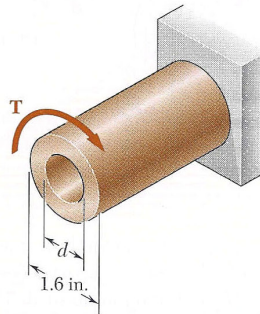
Torsion - Problem 2

Specifications:

Stepped shaft of 2024-T3 aluminum

Determine:

- The maximum shear stress in each section of the shaft and at the fillet.

Torsion - Problem 3

Specifications:

Inner diameter, d , = 1.52 inches

Applied torque, T = 2238 in-lbs

Material = 1026 DOM steel tubing.

Determine:

- The maximum shear stress, τ using the tubular equation for J .
- The maximum shear stress, τ using the thin wall approximation for J .
- The percent error.