Beams Deflections
(Method of Superposition)

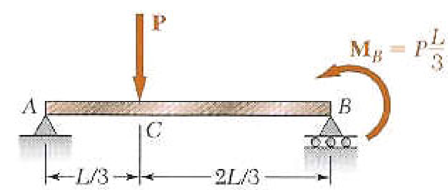
Method of Superposition:

As we previously determined, the differential equations for a deflected beam are linear differential equations, therefore the slope and deflection of a beam are linearly proportional to the applied loads. This will always be true if the deflections are small and the material is linearly elastic. Therefore, the slope and deflection of a beam due to several loads is equal to the sum of those due to the individual loads. In other words, the individual results may be superimposed to determined a combined response, hence the **Method of Superposition**.

This is a very powerful and convenient method since solutions for many support and loading conditions are readily available in various engineering handbooks. Using the principle of superposition, we may combine these solutions to obtain a solution for more complicated loading conditions.

Consider the following examples.

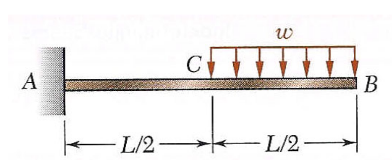
Example 1

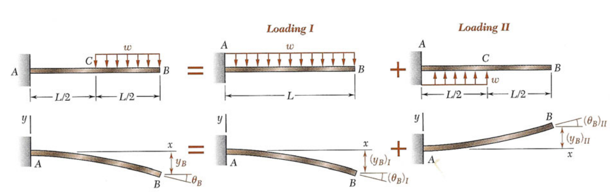


Determine:

1. The deflection at point C.
2. The slope at end A.

Example 2





Determine:

1. The slope and deflection at point B.

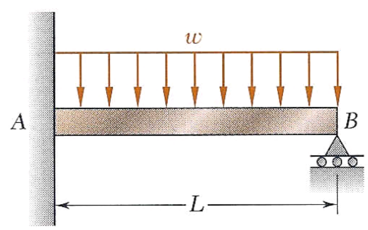
Statically Indeterminate Beams

The method of superposition is very useful for the reactions at the supports of statically indeterminate beams. As you may recall, a statically indeterminate beam is a beam with redundant supports (i.e. more supports than are required to maintain equilibrium of the beam). Therefore, the equations of equilibrium are not sufficient to determine all the reactions. By using the method of superposition, we may determine the force imposed by a redundant support and use this information to supplement the equilibrium equations.

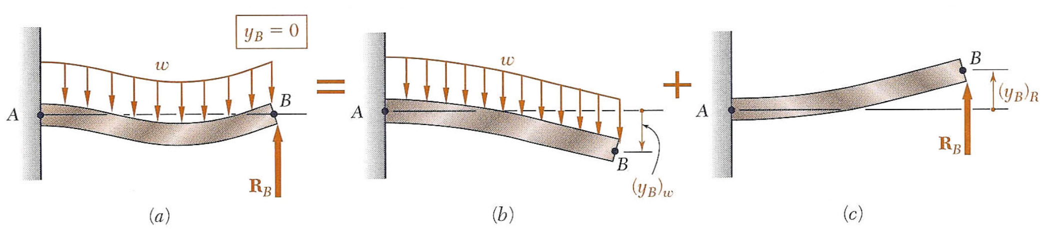
The procedure is as follows:

1. Remove enough supports to make the problem statically determinate.
2. Replace each support with the reactions they exert. The reactions are treated as part of the applied loading.
3. Solve the problem using the same procedure as a statically determinate problem to determine the reaction forces at the redundant supports. Note: the slopes and deflections due to the individual loadings must satisfy certain compatibility conditions so that the boundary conditions for the original problem are met.

Example 1



Treating the reaction at B as the redundant support, we have:



Determine:

1. The reaction force at location "B".
2. The moment at location "A".