## Beams Deflections

Deflection of Beams:

In many optical systems structural deformation may be the controlling design criteria. Opto-mechanical engineers are generally concerned with deflections. Deflection controls the shape of optical surfaces, system alignment and bore sight.

As we have seen, when a straight beam is loaded the neutral axis deforms into a curve. As shown below, the deflection v is measured from the original position of the neutral axis to the deflected position of the beam's neutral axis at any location x. Likewise, the slope of the deformed beam at any location x is dv/dx and is typically denoted as θ(x).
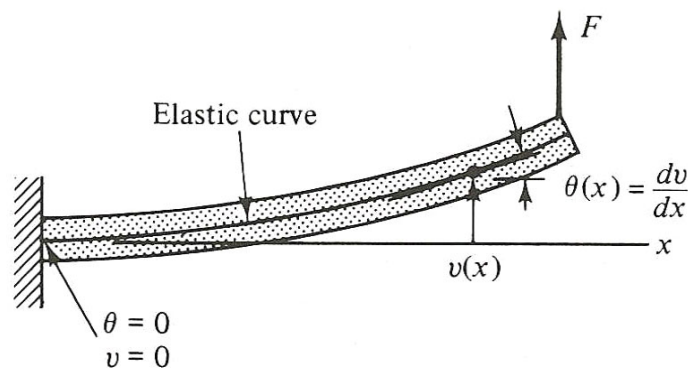
In lecture 9, we saw that a beam subjected to pure bending is bent into an arc of a circle and that the moment-curvature relationship can be expressed as follows:

$$M = \frac{EI}{\rho} \qquad \Rightarrow \qquad \frac{1}{\rho} = \frac{M}{EI}$$

Where:

M = bending moment
EI = flexural rigidity
ρ = radius of curvature

From calculus and analytic geometry we find:

$$\frac{1}{\rho} = \frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

Where:

y = deflection at any position x

However, small deflections also imply that the slope dy/dx will be small, in which case the term $(dy/dx)^2$ is negligible compared to unity. Therefore the above equation reduces to:

$$\frac{1}{\rho} \cong \frac{d^2y}{dx^2}$$

Combining with the moment curvature relationship we have:

$$M = EI\frac{d^2y}{dx^2}$$

For the illustration on page 57 where the deflection of the beam is defined as $v(x)$

$$M = EI\frac{d^2\upsilon}{dx^2}$$

Defining deflection as $y(x)$ we obtain:

$$M = EIy"$$

which is a second-order linear differential equation and is the governing differential equation for the elastic curve.

In addition to the pure bending assumptions (*lecture 9*) the following assumptions were used in the development of the above differential equation.

- The material is linear elastic and that the beam geometry and load conditions are such that the moment curvature relationship applies.
- That deflections are small and that the square of the slope of the beam is negligible compared to unity.
- That the contribution of shear force to the deflection is negligible (*reasonable unless the beam is very short*).

In lecture 8 we examined shear forces and bending moments in beams and derived the following relationships:

$$\frac{dM}{dx} = V \qquad\qquad \frac{dV}{dx} = -q$$

Using successive derivatives and prime notation we have:

|	
Deflection	= y
Slope	= dy/dx
Moment	= EIy"
Shear	= EIy"'
Load	= -EIy""

Deflections by Integration

Solve the differential equation as follows:

1. Select the interval(s) of the beam to be evaluated and place a coordinate system with the origin at one end of an interval. Beams with concentrated loads or beam with abrupt changes in uniform load must be evaluated over multiple intervals.
2. List all available and matching boundary conditions. One boundary condition can be used to determine one and only one constant of integration.
3. Determine the bending moment as a function of x for each interval and equate it to EIy''.
4. Solve the equation(s) and evaluate the constants of integration. Calculate deflection as required.

Example Problems
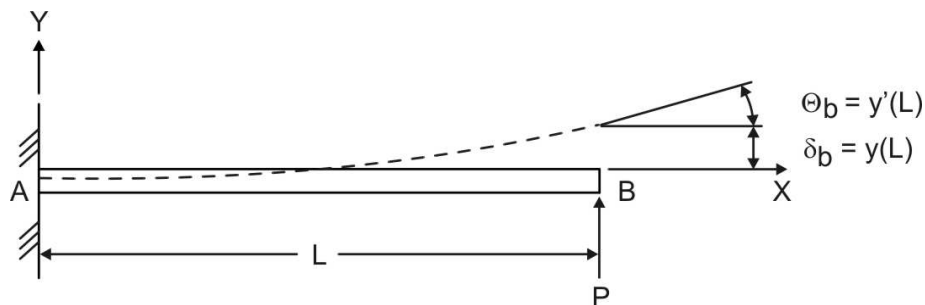
a) Using the method of integration, derive the **slope** and **deflection** equations as a function of "x".

b) Using the results of part "a", determine the equation for the maximum deflection at the tip (x = L).

c) Using the results of part "a", determine the equation for the maximum rotation (i.e. slope) at the tip (x = L).

Example Problems (cont.)
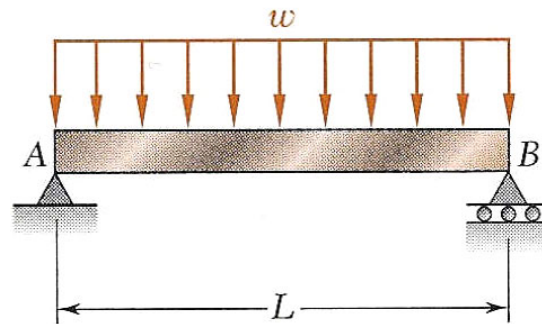
a) Using the method of integration, derive the **slope** and **deflection** equations as a function of "x".

b) Using the results of part "a", determine the equation for the maximum deflection at the midpoint (x = L/2).

c) Using the results of part "a", determine the equation for the maximum rotation (i.e. slope) at the endpoint (x = 0).