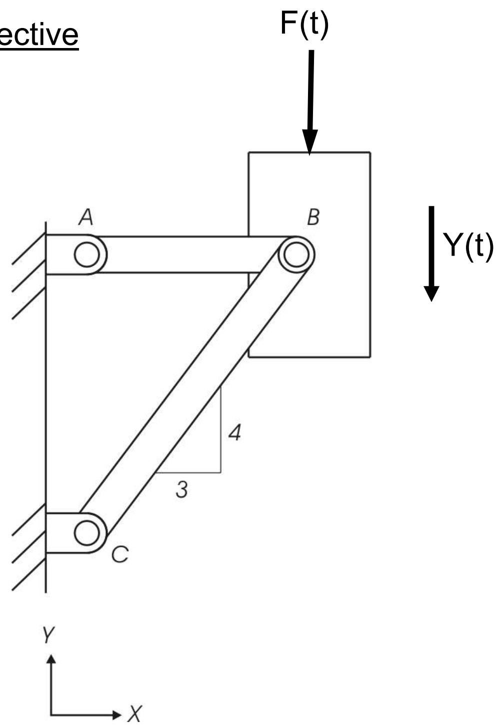
Concepts of Stress and Strain

One of our principal concerns in this course is material behavior (Strength). But strength models are often intimately related to stress. Thus, we need to be able to compute stresses. Stresses, however, cannot be directly measured, but stain is measurable and can be directly related to stress.

Stress Analysis in Perspective



Environment

↓

Forces on System

↓

Dynamic Response of System

↓

Compute Nominal Loads in Members

↓

Stress Analysis at Failure Sensitive Points

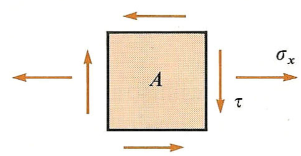
Simple Formulas

$$\sigma = \frac{F}{A}$$

My

Or **FEA** →

Stress at Point



a) Principal Stresses
b) Von Mises Stress
c) Etc.

NORMAL STRESS

A normal stress, symbolized by the Greek letter sigma σ, results when a member is subjected to an axial load applied through the centroid of the cross section. The average normal stress in the member is obtained by dividing the magnitude of the resultant internal force F by the cross sectional area A. Normal stress is:

$$\sigma_{AVG} = \frac{Force}{Area} = \frac{F}{A}$$

Consider the following free body diagram of a two-force member.

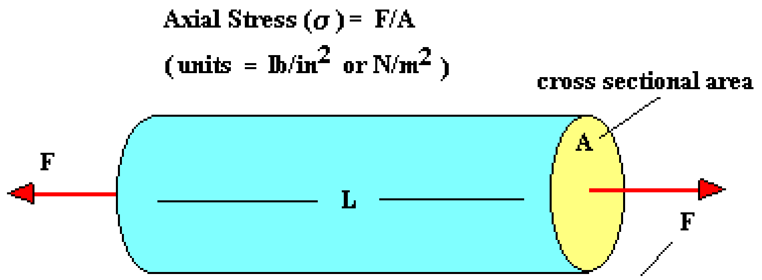


Diagram 2

Inasmuch as the stress σ acts in a direction perpendicular to the cut surface, it is referred to as a **NORMAL** stress. Thus, normal stressed may be either tensile or compressive. Our sign convention for normal stresses is:

Tensile stresses are positive (+)
Compressive stresses are negative (-)

We generally assume the normal stress distribution in an axially loaded member is uniform, except near the vicinity of the applied load know as Saint Venant's Principle. The assumption of a uniform normal stress distribution is valid if the resultant internal force is applied at the centroid of the cross section.

Saint Venant, a French mathematician, discussed the above principle for stress distributions in 1864.

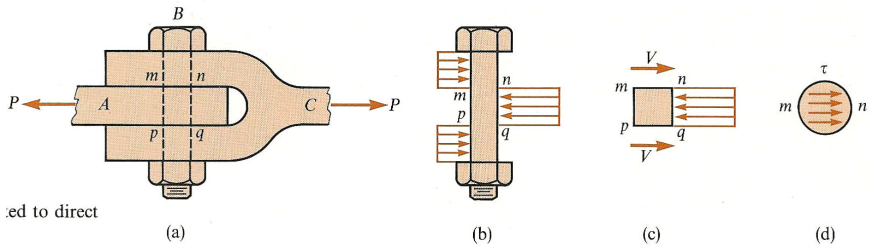
Note: A centric load is one in which the resultant force passes through the centroid of the resisting section. If the resultant passes through the centroid of all resisting sections, the loading is termed **AXIAL**.

SHEAR STRESS (Simple or Direct Shear)

A shear stress, symbolized by the Greek letter tau τ, results when a member is subjected to a force that is parallel or tangent to the surface. The average shear stress in the member is obtained by dividing the magnitude of the resultant shear force V by the cross sectional area A. Shear stress is:

$$\tau_{AVG} = \frac{ShearForce}{Area} = \frac{V}{A}$$

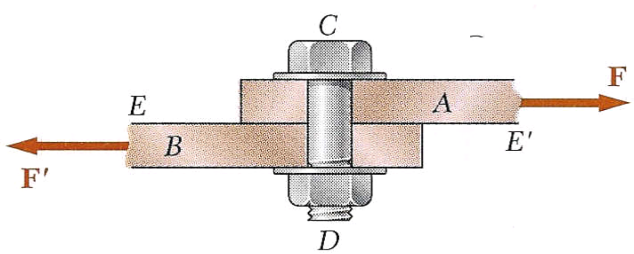
Consider the following example.



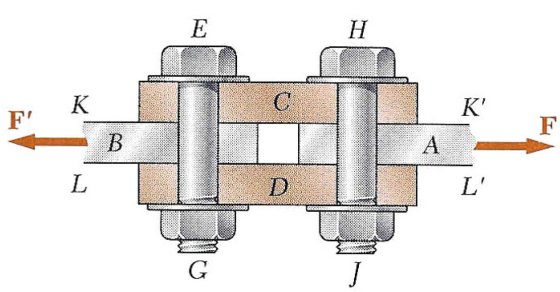
a) Typical clevis joint
b) Free body diagram of bolt
c) Free body of section mnqp
d) Shear stresses on section mn

It should be emphasized that the distributions of shear stresses is not uniform across the cross section. Shear stress will be highest near the center of the section and become zero at the edge.
Direct or simple shear arises in the design of bolts, pins, rivets, keys, welds and glued joints.

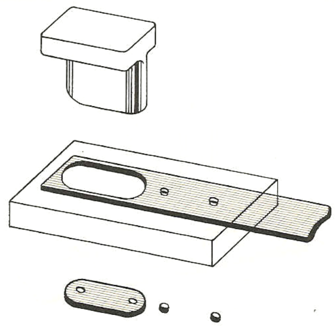
<u>Single Shear Joint</u>



<u>Double Shear Joint</u>



Punching Shear



$$\tau_{AVG} = \frac{PunchingForce}{Area} = \frac{F}{A}$$

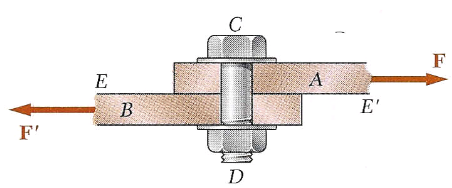
F = punching force
A = circumference x material thickness

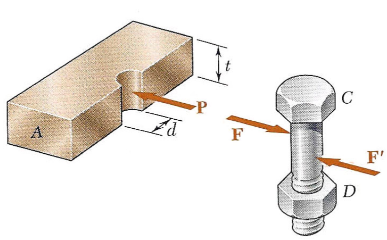
BEARING STRESS

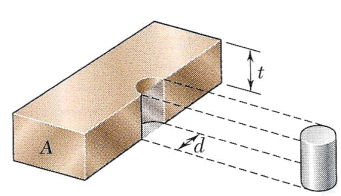
A bearing stress, symbolized by the Greek letter sigma σ_b, is a compressive normal stress that occurs on the surface of contact between two interacting members. The average normal stress in the member is obtained by dividing the magnitude of the bearing force F by the area of interest. Bearing stress is

$$\sigma_b = \frac{Force}{Area} = \frac{F}{A_b}$$

Bolts, pins and rivets create bearing stresses along the surface of contact.

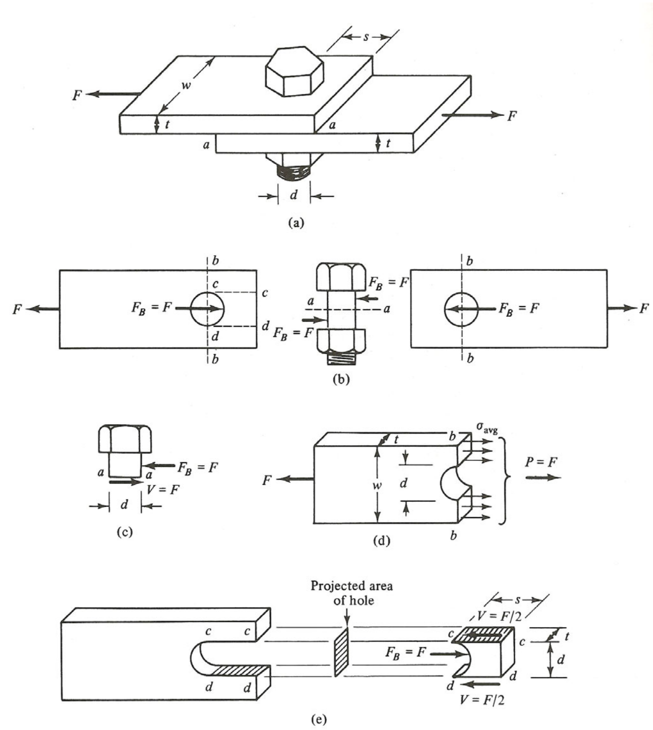






$$\sigma_b = \frac{P}{A_b} = \frac{P}{td}$$

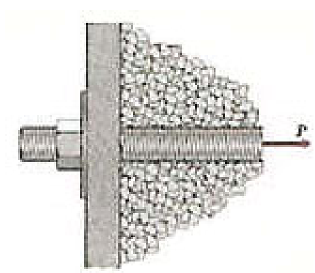
BEARING STRESS / SHEAR TEAR OUT

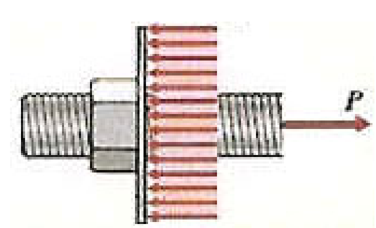


$$\tau_{avg} = \frac{V}{A_{cc}} = \frac{\frac{F}{2}}{ts}$$

BEARING STRESS - CONTINUED

Bearing stresses are also present under the heads of bolts and washers.



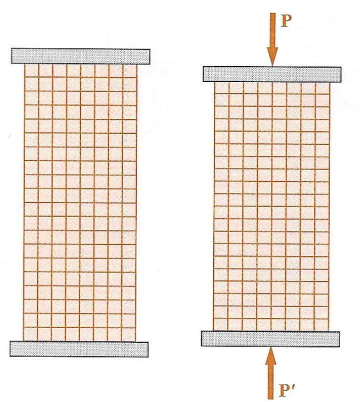


$$\sigma_b = \frac{P}{A_b} = \frac{P}{\frac{\pi}{4}(d_o^{\,2} - d_i^{\,2})}$$

STRESS DISTRIBUTION UNDER AXIAL LOADING
(Saint Venant's Principle)

As previously mentioned, we generally assume the normal stress distribution in an axially loaded member is uniform, except near the vicinity of the applied load.

Consider the following example.



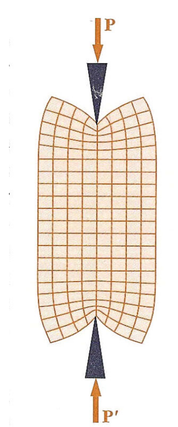
Assuming:

1. Top and bottom plates are rigid and do not rotate
2. Plates allow the member to expand laterally
3. Centroid of each load is at the center of each plate
4. Member is homogeneous and isotropic

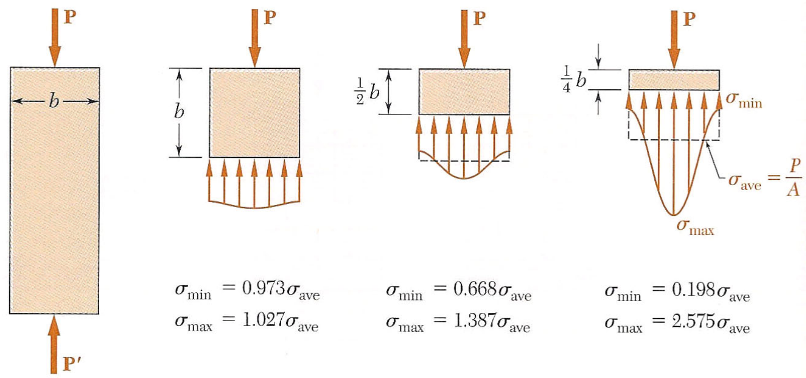
The distribution of stresses is uniform throughout the member and, at any point,

$$\sigma_{AVG} = \frac{Force}{Area} = \frac{P}{A}$$

On the other hand, if the loads are concentrated, the elements in the immediate vicinity of the points of application of the loads are subjected to very large stresses, while other elements near the ends are unaffected by the loading.

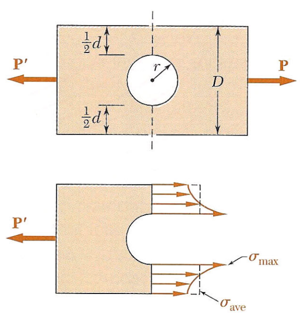


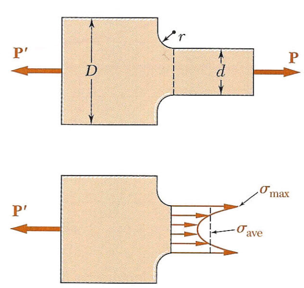
With the use of FEA or advanced mathematical methods, we can determine the distribution of stresses across various sections of a thin rectangular plate subjected to a concentrated load.



As previously shown, stresses near the point of application of concentrated loads are much higher than the average value of stress in the member (*i.e. stress concentration*). This is also true for structural members than contain a discontinuity, such as a hole or sudden change in cross section.

Lets consider two common situations, a flat bar with a circular hole and a flat bar with a reduced cross section.





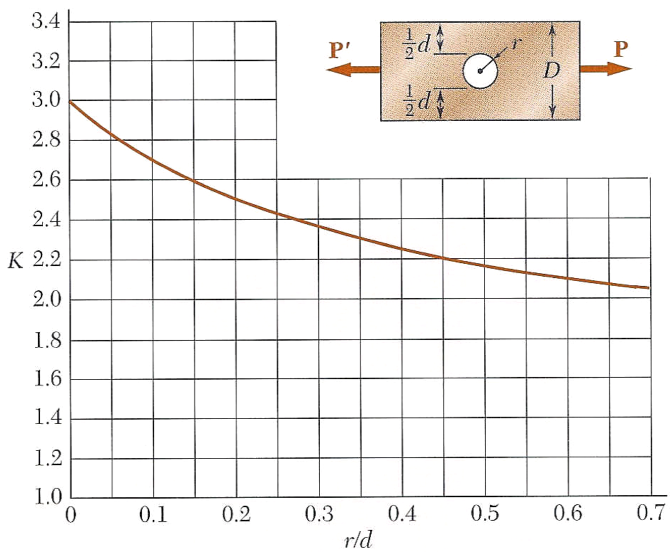
The stress concentration factor, K, is defined as follows:

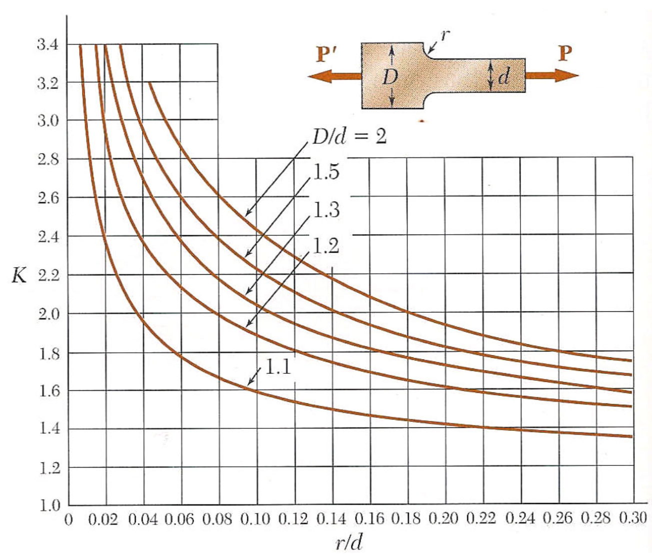
$$K = \frac{\sigma_{\max}}{\sigma_{avg}}$$

Or as typically used by engineers:

$$\sigma_{\max} = K\sigma_{avg}$$

Fortunately, stress concentration factors are available in graphs and empirical formulas from a wide variety of sources. In addition, K is dependent only on ratios of geometric parameters and is independent of member size and material.


P′
P
½d
½d
r
D
K
r/d
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7


P′
P
D
d
r
D/d = 2
1.5
1.3
1.2
1.1
K
r/d
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.02
0.04
0.06
0.08
0.10
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0.30