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Method for the determination of complex shear modulus of viscoelastic adhesives

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ABSTRACT

Viscoelastic adhesives are commonly used as vibration damping treatments for mechanical systems. When using these adhesives, knowledge of the shear modulus and loss factor at a given temperature is essential. The standard method for determining shear modulus and loss factor yields values at only a few frequency points for any given temperature. The method presented in this report offers a continuous curve for the two material properties over a broad frequency range.

Keywords: viscoelastic adhesives, damping treatments, shear modulus, loss factor

1. INTRODUCTION

Viscoelastic adhesives have two primary mechanical properties. The first property describes the energy storage of the adhesive and is called the shear modulus and has the symbol G. The second property describes the energy dissipation of the adhesive and is characterized by the loss factor. The loss factor is denoted by η. These properties are derived from the complex shear modulus. The complex shear modulus has the following form:

$$ShearModulus = G + \hat{\imath}\eta G. \tag{1}$$

The real part represents the "stiffness" of the adhesive. The imaginary part divided by the real part represents the loss factor and it indicates how well the material dissipates energy. Both the real and imaginary parts of the complex shear modulus are dependent on the frequency of the motion that induces shear and the temperature of the adhesive itself. Since viscoelastic adhesives are very sensitive to both frequency and temperature, it is convenient to account for both parameters through the use of a reduced frequency function which is a function of temperature. G and η are then plotted verses reduced frequency at the reference temperature. To obtain the data for another temperature one divides the frequency scale by the associated shift factor[1].

1.1. Current methods

Most available data for viscoelastic adhesives have a great deal of scatter. This is due to the method used to determine the material properties. The standard method involves using the resonant response of a cantilevered laminated beam [1,2]. Measured response data can be analyzed to determine the modal loss factor of selected resonant modes. Each modal loss factor can then be used in the Ross-Kerwin-Ungar equations for laminated beams to solve for the material shear modulus and loss factor. One is typically able to obtain data for the material at only four resonant frequencies for each given temperature. The resulting data are often very irregular and curve fitting techniques are employed to get a continuous representation of the material properties. While this standard method is reliable and widely used, it is desirable to employ additional approaches to check the results. A common alternative method is to directly measure the stiffness of a sample of the material using a force gage and an accelerometer when it is deformed dynamically. Depending on the type of excitation signal used, one can process these measured data to estimate the complex shear modulus over a range of frequencies. This approach is referred to as an impedance or direct stiffness method[3].

Some difficulties with the impedance method are that resonances in the fixture can limit the useful frequency range and any misalignment can lead to erroneous force measurements. In the present study we have used a similar approach which relies on random base excitation which provides a system which tends to be insensitive to resonances in the test fixture (and the exciter) and in which the effective excitation force (imposed motion) is accurately measured with an accelerometer. A similar technique is used in reference 4 to measure the bulk modulus of a viscoelastic material.

2. PROPOSED METHOD

The device used in this measurement system consists of an aluminum block suspended between two aluminum brackets by the viscoelastic adhesive we wish to examine. The brackets are mounted to a shaker head that is aligned with the vertical direction. This configuration is similar to a mass at the end of a spring. The shaker will provide white noise base excitation that will induce shear in the adhesive and relative motion to occur between the brackets and the block. A block diagram of the equipment setup is shown in figure 1.

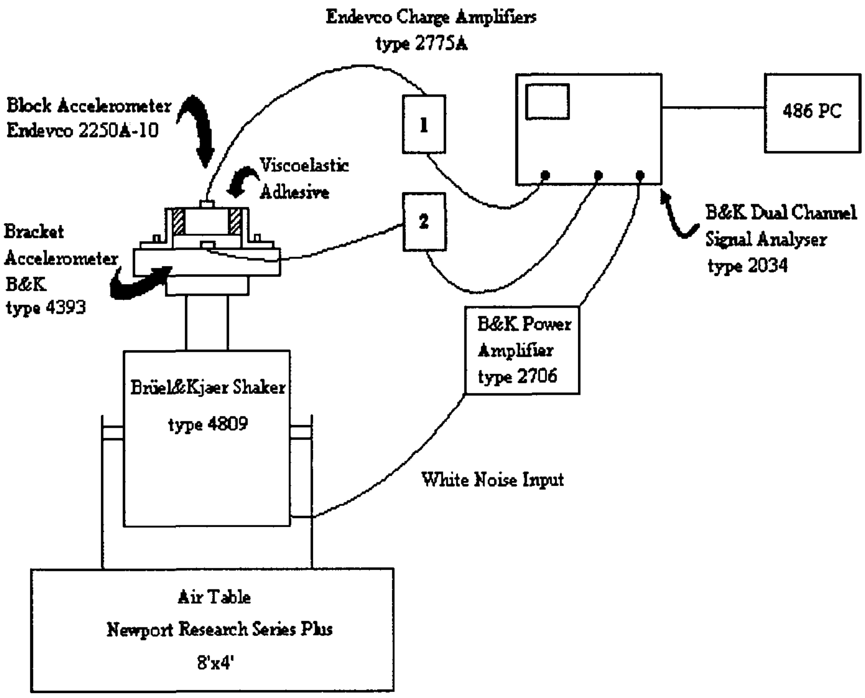

Figure 1. Block diagram showing all components used in acquiring data with exception of the space heater used for changing the temperature of the adhesive.

In the following we will derive an equation relating the complex modulus of the adhesive to measurements of the random motion of the brackets and the attached mass. Let $k(t)$ be the relaxation function which accounts for the properties of the viscoelastic material. The differential equation of motion for this system is

$$m\ddot{x} + \int_0^t k(t-\tau)\frac{d}{d\tau}(x(\tau) - y(\tau))d\tau = 0, \tag{2}$$

where x is the displacement of the block, y is the imposed random displacement of the bracket, and m is the mass of the block[5]. The complex stiffness, $K(\omega)$ is

$$K(\omega) = \hat{\imath}\omega \int_{-\infty}^{\infty} e^{-\hat{\imath}\omega t}k(t)dt, \tag{3}$$

where $\hat{\imath} = \sqrt{-1}$. By measuring $x(t)$ and $y(t)$, one can determine $K(\omega)$ as shown in the following. Our result will depend on the transfer function between $x(t)$ and $y(t)$.

It is convenient to rewrite equation (2) as

$$m\ddot{x} + \int_0^t k(t-\tau)\frac{d}{d\tau}x(\tau)d\tau = \int_0^t k(t-\tau)\frac{d}{d\tau}y(\tau)d\tau. \tag{4}$$

Multiplying equation (4) by $\dot{y}(t+\tau')$, taking the expected value and rearranging give

$$mE[\ddot{x}(t)\dot{y}(t+\tau')] + \int_0^t k(t-\tau)E[\frac{d}{d\tau}x(\tau)\dot{y}(t+\tau')]d\tau = \int_0^t k(t-\tau)E[\frac{d}{d\tau}y(\tau)\dot{y}(t+\tau')]d\tau. \quad (5)$$

If $x(t)$ and $y(t)$ are stationary random processes the expected value operations, E[·], are equivalent to the following correlation functions

$$\begin{aligned} R_{\ddot{x}\dot{y}}(\tau') &= E[\ddot{x}(t)\dot{y}(t+\tau')] \\ R_{\dot{x}\dot{y}}(t+\tau'-\tau) &= E[\dot{x}(\tau)\dot{y}(t+\tau')] \\ R_{\dot{y}\dot{y}}(t+\tau'-\tau) &= E[\dot{y}(\tau)\dot{y}(t+\tau')]. \end{aligned} \quad (6)$$

The correlation functions may be expressed in terms of the inverse Fourier transforms of the corresponding cross spectra,

$$\begin{aligned} R_{\ddot{x}\dot{y}}(\tau') &= \int_{-\infty}^{\infty} e^{\hat{\imath}\omega\tau'} S_{\ddot{x}\dot{y}}(\omega)d\omega \\ R_{\dot{x}\dot{y}}(t+\tau'-\tau) &= \int_{-\infty}^{\infty} e^{\hat{\imath}\omega(t+\tau'-\tau)} S_{\dot{x}\dot{y}}(\omega)d\omega \\ R_{\dot{y}\dot{y}}(t+\tau'-\tau) &= \int_{-\infty}^{\infty} e^{\hat{\imath}\omega(t+\tau'-\tau)} S_{\dot{y}\dot{y}}(\omega)d\omega, \end{aligned} \quad (7)$$

where $S_{ab}(\omega)$ is the cross power spectral density of $a(t)$ and $b(t)$.

Substituting equations (6) and (7) into equation (5) and rearranging give

$$\begin{aligned} &m\int_{-\infty}^{\infty} e^{\hat{\imath}\omega\tau'} S_{\ddot{x}\dot{y}}(\omega)d\omega + \int_0^t k(t-\tau)\int_{-\infty}^{\infty} e^{\hat{\imath}\omega(t+\tau'-\tau)} S_{\dot{x}\dot{y}}(\omega)d\omega d\tau \\ &= \int_0^t k(t-\tau)\int_{-\infty}^{\infty} e^{\hat{\imath}\omega(t+\tau'-\tau)} S_{\dot{y}\dot{y}}(\omega)d\omega d\tau. \end{aligned} \quad (8)$$

Since we are interested in the response long after the initial transients have died away, the lower limit in the convolution integrals may be changed from $t = 0$ to $t = -\infty$. Also, since the relaxation function $k(t)$ is defined to be zero for $t < 0$, the upper limit in the convolution integrals may be changed to ∞. Further rearranging allows equation (8) to be written as

$$\begin{aligned} &m\int_{-\infty}^{\infty} e^{\hat{\imath}\omega\tau'} S_{\ddot{x}\dot{y}}(\omega)d\omega + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k(t-\tau)e^{\hat{\imath}\omega(t-\tau)}e^{\hat{\imath}\omega\tau'} S_{\dot{x}\dot{y}}(\omega)d\omega d\tau \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k(t-\tau)e^{\hat{\imath}\omega(t-\tau)}e^{\hat{\imath}\omega\tau'} S_{\dot{y}\dot{y}}(\omega)d\omega d\tau. \end{aligned} \quad (9)$$

If we change the integration variable from τ to τ_1 according to $\tau = \tau_1 - t$, equation (9) becomes

$$\begin{aligned} &m\int_{-\infty}^{\infty} e^{\hat{\imath}\omega\tau'} S_{\ddot{x}\dot{y}}(\omega)d\omega + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k(\tau_1)e^{-\hat{\imath}\omega\tau_1}e^{\hat{\imath}\omega\tau'} S_{\dot{x}\dot{y}}(\omega)d\omega d\tau_1 \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k(\tau_1)e^{-\hat{\imath}\omega\tau_1}e^{\hat{\imath}\omega\tau'} S_{\dot{y}\dot{y}}(\omega)d\omega d\tau_1. \end{aligned} \quad (10)$$

The integration over τ_1 gives the complex stiffness defined in equation (3). Grouping the three terms of equation (10) and using equation (3) give

$$\int_{-\infty}^{\infty} e^{\hat{\imath}\omega\tau'}\{mS_{\ddot{x}\dot{y}}(\omega) + (K(\omega)/(\hat{\imath}\omega))S_{\dot{x}\dot{y}}(\omega) - (K(\omega)/(\hat{\imath}\omega))S_{\dot{y}\dot{y}}(\omega)\}d\omega = 0. \quad (11)$$

It may be shown that $S_{\ddot{x}\dot{y}}(\omega) = \hat{\imath}\omega S_{\dot{x}\dot{y}}(\omega)$, $S_{\dot{y}\dot{y}}(\omega) = \omega^2 S_{yy}(\omega)$, and $S_{\dot{x}\dot{y}}(\omega) = \omega^2 S_{xy}(\omega)$. Using these results and setting the integrand of equation (11) to zero give

$$K(\omega) = \omega^2 m(1 - S_{yy}(\omega)/S_{xy}(\omega)). \quad (12)$$

The cross power spectral density $S_{xy}(\omega)$ and the autospectrum $S_{yy}(\omega)$ may be measured by readily available methods. The ratio $S_{xy}(\omega)/S_{yy}(\omega) = H_{xy}(\omega)$ is the transfer function between $x(t)$ and $y(t)$.

Equation (12) gives the effective complex stiffness of the damping material used in the experimental setup of figure 1. The adhesive of thickness h is attached to each side of the block. The attachment surface has width b and length l. The relationship between the complex shear modulus $G(\omega)$ and the complex stiffness $K(\omega)$ may be shown to be

$$G(\omega) = \frac{K(\omega)h}{2bl}, \tag{13}$$

where the factor of 2 accounts for the fact that there is adhesive on two sides of the block.

3. RESULTS

The apparatus shown in figure 1 was used to obtain complex shear moduli at three different temperatures 22.0^oC, 36.5^oC and 43.1^oC. The temperature of the system was controlled by an ordinary space heater set at various distances from the apparatus. A Simpson Digital Thermometer was used to monitor the temperature. Data were taken from the apparatus only when the temperature reached a steady state. The actual dimensions of the block were 2.12"x1.0"x1.0" with a mass of 93.58 grams. The viscoelastic adhesive 3M ISD112 was used in the experiment and the dimensions of each strip were $h = .108$", $l = 1.77$" and $b = .5$". The raw data that is obtained from the B&K spectrum analyzer contains the real and imaginary parts of the transfer function between the block and the base. This data was then imported to an ASCI file in a 486 personal computer. The scientific spread sheet program ORIGIN was used to calculate $G(\omega)$ using equations (12) and (13). An example of how our raw data compares with experimental and curve fit data given in reference 1 is shown in figure 2. The present method produces smooth data up to a frequency of 2kHz where resonances in the fixture influence the results. Note that the raw data of the present method is fairly close to that of reference 1.

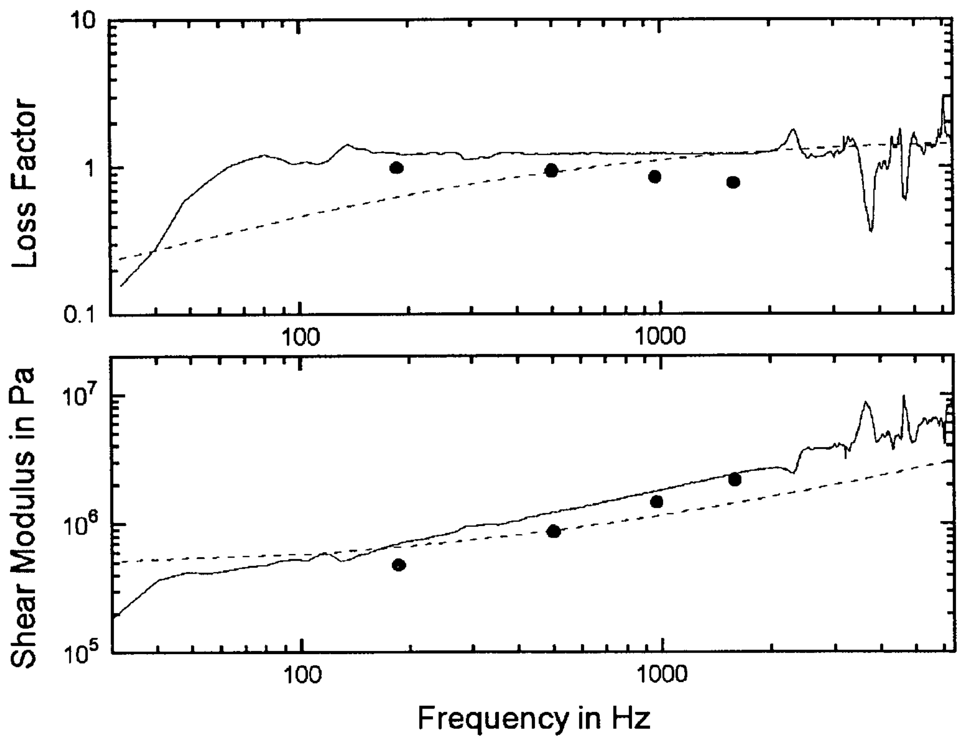

Figure 2. Our data is the solid line, Damping Design Guide's experimental data are black circles and their curve fit data is a dashed line[1]. This example is for 36.5^oC.

Since data was taken with our apparatus at three different temperatures, a reduced frequency nomogram could be prepared for shear modulus and loss factor. The reference temperature for our data was 22.0^oC. For each run of our experiment, the high and low ends of the data were unreliable. The low frequency problem was due to bad coherence and the very high frequency region was affected by resonances in the system. Only the best segments of our data at each temperature were retained for each nomogram constructed. The resulting nomograms are shown in figure 3. Our data is the continuous line and the curve fit from the Damping Design Guide[1] is shown as a dashed line.

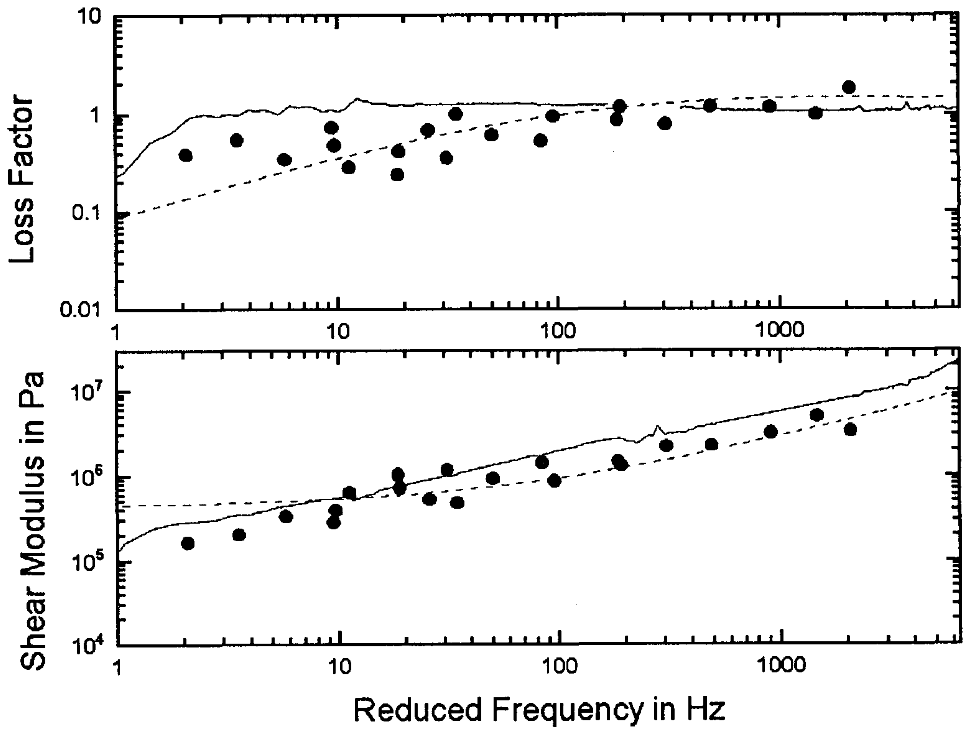

Figure 3. Loss Factor and Shear Modulus for 3M ISD112 versus reduced frequency. Solid line is from present method based on measurements at three temperatures 22.0^oC, 36.5^oC and 43.1^oC with shift factors of $\alpha_t = 1, .09, .045$ respectively. The dashed line (curve fit) and the points are from the damping design guide[1].

4. CONCLUSIONS

The proposed method provides another useful way of determining the complex shear modulus of viscoelastic adhesives. The method relies on relatively simple data processing of a transfer function between two accelerometers. Results obtained for 3M ISD112 compare favorably with previously published data.

5. REFERENCES

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