Optical distortion coefficients of high-power laser windows

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Abstract. This paper concerns the problem of describing and evaluating thermal lensing phenomena that occur as a result of the absorption of laser light in solid windows. The aberration function expansion method is applied for deriving the two optical distortion coefficients χ_+ and χ_- that characterize the degradation in light intensity at the Gaussian focus of an initially diffraction-limited laser beam passing through a weakly absorbing stress-birefringent window. In a pulsed mode of operation, the concept of an effective optical distortion coefficient χ_{eff}, which properly combines the coefficients χ_+ and χ_- in terms of potential impact on focal irradiances, then leads to the definition of a figure of merit for distortion. The theory and calculations presented in this and earlier papers provide simple analytical tools for predicting the optical performance of a window-material candidate in a specific system's environment.

Subject terms: optical distortion; laser windows; thermal stress; thermal lensing; aberration function; figure of merit.

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CONTENTS

1. INTRODUCTION

Wavefront distortion caused by "thermal lensing" of optical components can have a major impact on the operation of high-power laser systems.[1] Windows, in particular, can distort the incident beam in a complex manner because thermally induced phase shifts reflect changes in the optical path length arising from position-dependent variations in window thickness as well as from position- and polarization-dependent variations in refractive index.[2,3] This problem is now well understood in the sense that a suitable theory has been developed and shown to be amenable to practical calculations for relatively simple configurations in terms of both material characteristics and beam geometry.[4–6]

In effect, the theory applies only if the following assumptions are verified: (a) The window is subjected to axially symmetric thermal loadings, (b) the elastic and photoelastic properties are isotropic in the plane of the window, and (c) the stresses obey either "thin-plate" or "long-rod" type distributions. Under those conditions, and in the absence of mechanical loadings, unconstrained laser windows are subjected to stresses characterized by cylindrically symmetric radial and azimuthal components σ_ρ and σ_θ, which relate to the temperature profile in a relatively simple manner. Furthermore, the distortion of the beam can be described by means of two principal phase shifts, $\delta\phi_\rho$ and $\delta\phi_\theta$, that is, the phase shifts experienced by a normally incident light ray polarized in either the radial or the azimuthal direction. These phase shifts arise from the change in path length as the heated portion of the window expands and bulges outward, from the temperature dependence of the refractive index, and from photoelastic effects associated with nonuniform heating patterns. Considering that σ_ρ and σ_θ differ everywhere except on-axis, it follows that for windows made of stress-birefringent material, the two principal phase shifts are also different, which leads to two optical distortion coefficients for describing the thermal lensing process. My purpose here is to present this approach in a more coherent manner compared to earlier treatments[4–9] but simple enough to allow the laser systems designer to predict the window-induced degradation in focal intensity and, thus, to assess the "performance" of a window-material candidate in the context of a design methodology, as outlined in Ref. 7.

Early investigations of window-induced thermal lensing made use of methods such as conventional geometric optics,[10] Kirchhoff's vector diffraction theory,[11] and Jones's aberration-matrix formalism.[12] Bendow and Gianino,[11] in particular, performed extensive analyses, but because of the large number of parameters involved, their results are difficult to exploit for relating

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key features of the transmitted beam to specific window-material characteristics. For this reason, I have adopted the aberration-function expansion procedure to derive optical distortion coefficients and thus characterize the degradation in beam intensity at the Gaussian focus of an initially diffraction-limited laser beam focused through a weakly absorbing medium. In Sec. 2, I briefly review how the degradation can be described by means of a symmetric and an antisymmetric combination of the phase shifts $\delta\phi_\rho$ and $\delta\phi_\theta$, which then yield proper expressions (the "chi approximation") for obtaining the two optical distortion coefficients that relate the spatial temperature distribution to the phase-aberration functions (Sec. 3). For the purpose of elucidating how the reduction in target irradiance relates to specific features of the laser window as well as the laser beam (Sec. 4), I consider truncated Gaussian beam amplitudes and focus attention on pulsed modes of operation (the "short-time approximation"), for which local temperature increases mirror the local heat deposition, thus ignoring thermal diffusion effects. I also address the problem of obtaining proper values for the relevant elastic and photoelastic properties (Sec. 5), to emphasize that using single-crystal constants, as is often done in the literature, is basically incorrect. Finally, conclusions are stated in Sec. 6.

2. WINDOW-INDUCED IRRADIANCE DEGRADATION

We assume that the thermal lensing process gives rise to radially dependent wavefront distortions but does not alter the incident beam intensity. In that case, the Strehl ratio, or ratio of focal irradiances with and without aberration, is simply[11]

$$s = \frac{\left|\int_0^1 (\tfrac{1}{2})[\exp(i\delta\phi_\rho) + \exp(i\delta\phi_\theta)]\sqrt{I(\rho)}\,\rho d\rho\right|^2}{\left|\int_0^1 \sqrt{I(\rho)}\,\rho d\rho\right|^2} , \tag{1}$$

where $\delta\phi_\rho$ and $\delta\phi_\theta$ are the phase aberrations experienced by normally incident light polarized along radial and azimuthal directions, respectively (the other symbols are defined in Table I). This expression provides a convenient starting point for analyzing thermal lensing caused by high-power laser windows. For weak distortions, in particular, we may proceed by expanding the two multiplicative phase factors to second order in the manner originally suggested by Born and Wolf,[13] i.e.,

$$\exp(i\delta\phi) \simeq 1 + i\delta\phi - \frac{\delta\phi^2}{2} , \tag{2}$$

which yields

$$s \simeq 1 - \frac{1}{2}\langle\delta\phi_\rho^2 + \delta\phi_\theta^2\rangle + \frac{1}{4}\langle\delta\phi_\rho + \delta\phi_\theta\rangle^2 , \tag{3}$$

where the symbol $\langle\,\rangle$ refers to amplitude-weighted averages over the entire aperture:

$$\langle Y\rangle = \frac{\int_0^1 Y(\rho)\sqrt{I(\rho)}\,\rho d\rho}{\int_0^1 \sqrt{I(\rho)}\,\rho d\rho} . \tag{4}$$

On introducing symmetric and antisymmetric combinations of the radial and azimuthal phase aberrations,

TABLE I. List of symbols.

C_p'	: Heat capacity per unit volume
D	: Window diameter
E	: Young's modulus
F(W)	: Form factor, Strehl ratio
G	: Shear modulus
G(W)	: Form factor, distortion coefficient
$I(\rho,t)$	: Beam intensity
I_o	: Peak intensity
j_3	: Hershey's factor
K	: Bulk modulus
K_d	: Beam-profile factor for distortion
k	: Propagation constant $(2\pi/\lambda)$
L	: Window thickness
n	: Refractive index
$q_{\parallel}$	: Stress-optic coefficient, parallel orientation
$q_{\perp}$	: Stress-optic coefficient, perpendicular orientation
q_{ij}	: Piezo-optic tensor element, single crystal
q_{ij}^*	: Piezo-optic tensor element, aggregate
s	: Strehl ratio
s_{ij}	: Compliance tensor element, single crystal
s_{ij}^*	: Compliance tensor element, aggregate
t	: Exposure time
t_d	: Thermal lensing time
W	: Truncation parameter
z	: Axial position
α	: Expansion coefficient
β_{app}	: Apparent absorption coefficient
β_V	: Bulk absorption coefficient
β_S	: Surface absorptance
δL	: Thickness variation
δn	: Index variation
$(\delta n)_{stress}$	: Stress-induced index variation
$(\delta n)_{temp}$	: Temperature-induced index variation
δT	: Beam-induced temperature increment
$\delta\phi_\theta$	: Phase aberration, azimuthal polarization
$\delta\phi_\rho$	: Phase aberration, radial polarization
$\delta\phi_+$	: Symmetric phase-aberration function
$\delta\phi_-$	: Anti-symmetric phase-aberration function
ε_z	: Axial strain component
λ	: Laser wavelength
ν	: Poisson's ratio
ρ	: Normalized radial distance
σ_z	: Axial stress component
σ_θ	: Azimuthal stress component
σ_ρ	: Radial stress component
ϕ	: Window-induced phaseshift
X_+	: Optical distortion coefficient, symmetric
X_-	: Optical distortion coefficient, anti-symmetric
X_{eff}	: Effective optical distortion coefficient
ω	: Beam radius $(1/e^2)$

$$\delta\phi_+ = \frac{1}{2}(\delta\phi_\rho + \delta\phi_\theta) \ , \tag{5}$$

$$\delta\phi_- = \frac{1}{2}(\delta\phi_\rho - \delta\phi_\theta) \ , \tag{6}$$

this procedure then leads to a remarkably compact expression for the Strehl ratio,

$$s \simeq 1 - \{\mathrm{var}[\delta\phi_+] + \langle\delta\phi_-^2\rangle\} \ , \tag{7}$$

keeping in mind that the variance is defined in accord with

$$\mathrm{var}[Y] = \langle Y^2\rangle - \langle Y\rangle^2 \ . \tag{8}$$

The advantage of this formulation is quite obvious: Since $\delta\phi_-$ exists only if $\delta\phi_\rho$ and $\delta\phi_\theta$ are different, Eq. (7) specifies the impact of birefringence with regard to focal irradiances compared to all other sources of distortion.

3. THE TWO "CHI" COEFFICIENTS

Prior to the onset of thermal loadings, the window is assumed to be at uniform temperature and birefringence free; any normally incident light ray that traverses the window without experiencing significant deviation or loss in intensity then emerges with its phase shifted by an amount

$$\phi = k(n - 1)L \ , \tag{9}$$

where k designates the propagation constant, n is the unperturbed index of refraction, and L refers to the window thickness (path length). The absorption of laser power gives rise to radially dependent aberrations as a result of the change in path length and the change in index caused by beam-induced temperature gradients:

$$\delta\phi = k[(n - 1)\delta L + L\delta n] \ . \tag{10}$$

The variation in path length reflects the magnitude of the tensor component of strain in the z direction, i.e.,

$$\delta L = \varepsilon_z L \tag{11}$$

in single-subscript notations, which relates to the temperature field and the stress tensor by means of Hooke's law,[14]

$$\varepsilon_z = \alpha\delta T + \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_\rho + \sigma_\theta) \ , \tag{12}$$

where δT refers to the beam-induced local temperature rise averaged over the pane thickness:

$$\delta T = \frac{1}{L}\int_0^L \delta T(\rho,z,t)\,dz \ . \tag{13}$$

The stresses occurring in a solid window that is not constrained by external forces, has an axially symmetric temperature distribution, and is made of elastically isotropic material can be described analytically for two simple model situations[15]:

(a) The *plane-stress model,* which applies to "thin-disk" geometries, that is, when the window thickness is much smaller than the window radius $[L/(D/2) \lesssim 0.5]$, yields

$$\sigma_\rho = \alpha E\left[\int_0^1 \delta T\rho'\,d\rho' - \frac{1}{\rho^2}\int_0^\rho \delta T\rho'\,d\rho'\right] \ , \tag{14}$$

$$\sigma_\theta = \alpha E\left[\int_0^1 \delta T\rho'\,d\rho' + \frac{1}{\rho^2}\int_0^\rho \delta T\rho'\,d\rho' - \delta T\right] \ , \tag{15}$$

$$\sigma_z = 0 \ , \tag{16}$$

for the principal stresses; note that for this approximation to be valid, the axial stress must vanish.

(b) The *plane-strain model,* which yields

$$\sigma_z = \sigma_\rho + \sigma_\theta \ , \tag{17}$$

with planar stresses as in Eqs. (14) and (15) but for the factor αE, which must be replaced by $\alpha E/(1-\nu)$; in principle, this "long-rod" approximation should be used when the window aspect ratio satisfies the condition $L/(D/2) \gtrsim 2$ and axial stresses no longer can be ignored. With regard to the change in index, we must consider not only the temperature dependence but also the stress-induced photoelastic effect:

$$\delta n = (\delta n)_{\mathrm{temp}} + (\delta n)_{\mathrm{stress}} \ . \tag{18}$$

The effect of temperature can be easily formulated if the index of refraction varies more or less linearly over the temperature range of interest; in a first approximation, we may write

$$(\delta n)_{\mathrm{temp}} = \left(\frac{\partial n}{\partial T}\right)_{\sigma=0} \delta T \tag{19}$$

and set $(\partial n/\partial T)_{\sigma=0}$ equal to the thermo-optic coefficient, dn/dT, as measured at the reference temperature. Index variations caused by thermal stresses involve considerations relating to the photoelastic effect,[16] which show that

$$(\delta n)_{\mathrm{stress}} = \frac{-n^3}{2}[q_\parallel\sigma_\rho + q_\perp(\sigma_\theta + \sigma_z)] \tag{20}$$

for plane waves polarized along the radial direction and

$$(\delta n)_{\mathrm{stress}} = \frac{-n^3}{2}[q_\parallel\sigma_\theta + q_\perp(\sigma_\rho + \sigma_z)] \tag{21}$$

for azimuthal polarizations, the symbols $q_\parallel$ and $q_\perp$ referring to the stress-optic coefficients for stresses applied parallel and perpendicular to the polarization axis.

At this point, it becomes a straightforward matter to establish that the two aberration functions $\delta\phi_+$ and $\delta\phi_-$ of Sec. 2 relate to the temperature field in a fairly simple manner*:

$$\delta\phi_+ = k\chi_+ L\delta T + \rho\text{-independent terms} \ , \tag{22}$$

$$\delta\phi_- = k\chi_- L(\overline{\delta T} - \delta T) \ , \tag{23}$$

*Note that ρ-independent terms do not contribute to the variance of $\delta\phi_+$ and hence do not contribute to the degradation in focal intensity [see Eq. (7)].

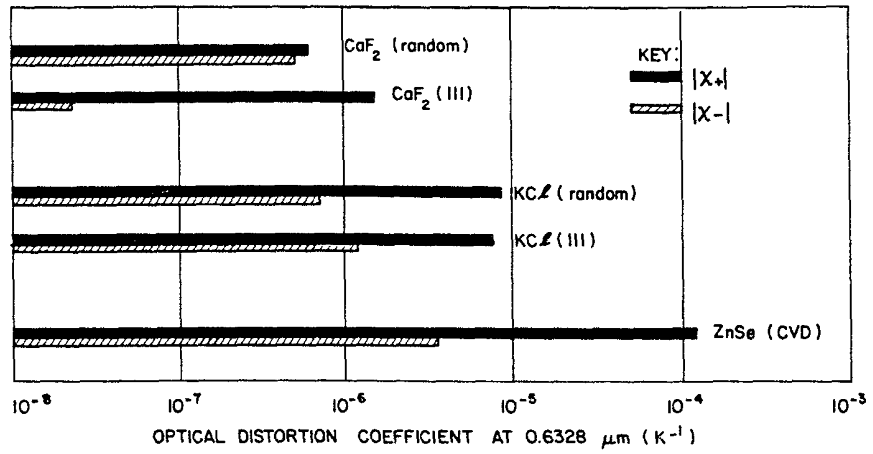

Fig. 1. Comparison of optical distortion coefficients for CaF_2, KCl, and ZnSe at the He-Ne laser wavelength; note that these coefficients are displayed on a log scale.

where $\overline{\delta T}$ represents the mean temperature increase from the window axis to the radial position ρ, i.e.,

$$\overline{\delta T} = \frac{2}{\rho^2}\int_0^{\rho} \delta T \rho' \, d\rho' \ . \tag{24}$$

Equations (22) and (23) define two optical distortion coefficients χ_+ and χ_-, which characterize the medium's sensitivity to thermal lensing and properly assemble the material parameters that control the thermal lensing process. In effect, they are the eigenvalues of the relevant Jones matrix, as discussed in Ref. 12. In a disk geometry, the calculation yields

$$\chi_+ = \frac{dn}{dT} + (n - 1)\alpha(1 + \nu) + \frac{n^3\alpha E}{4}(q_\parallel + q_\perp) \ , \tag{25}$$

$$\chi_- = \frac{n^3\alpha E}{4}(q_\parallel - q_\perp) \ , \tag{26}$$

which demonstrates that χ_+ combines the temperature-induced change in index, the contribution due to bulging, plus the average photoelastic effect for the two polarizations; the coefficient χ_- exists only if the medium is stress-birefringent. For "thick" windows we have

$$\chi_+ = \frac{dn}{dT} + \frac{n^3\alpha E/4}{1 - \nu}(q_\parallel + 3q_\perp) \ , \tag{27}$$

$$\chi_- = \frac{n^3\alpha E/4}{1 - \nu}(q_\parallel - q_\perp) \ , \tag{28}$$

which no longer includes a "bulging" term, thus reflecting the well-known observation[15] that the thickness variation δL is ρ-independent in that geometry. In both instances, birefringence-independent distortions replicate the temperature profile as created by the incident beam, whereas birefringence-dependent contributions are always minimal in the central window region but may become substantial at larger radial distances. With regard to the elastic and the stress-optic coefficients, I remind the reader that the theory holds only for macroscopically isotropic material or, in the case of windows made of highly oriented cubic material, if the beam propagates along the [111] direction, in which case the coefficients must be obtained from the compliance tensor and the piezo-optic tensor by means of the formulas given in Sec. 5.

At the He-Ne laser wavelength, for instance, the optical distortion coefficients of a "thin" laser window made of ZnSe, KCl, or CaF_2 are displayed in Fig. 1.[†]

On a macroscopic scale, polycrystalline, chemically vapor deposited ZnSe exhibits isotropic properties and has a large χ_+ coefficient ($\chi_+ = 1 \times 10^{-4}\ K^{-1}$) that originates primarily from the thermo-optic coefficient and, thus, ensures polarization-independent lensing in this material; there is, nevertheless, a substantial stress-birefringence contribution (see Fig. 1), but it does not affect the focal irradiance, as shown in the next section. At this point, we may note that in relation to the dissipated power, the wavefront deformation in ZnSe should be almost an order of magnitude greater than in KCl, or even two orders of magnitude greater than in CaF_2, which agrees with much experimental evidence.[17]

As shown in Fig. 1, randomly oriented KCl windows behave essentially in the same manner as ⟨111⟩-oriented windows, which suggests that KCl-induced thermal lensing does not depend much on crystalline order or preferred orientation. Isotropic contributions dominate in the sense that the "small-birefringence condition," $\chi_-^2/\chi_+^2 << 1$, appears to be satisfied, but it should be pointed out that χ_+ is actually negative, which substantiates the observation that KCl windows behave in the manner of a negative lens.[17]

For randomly oriented CaF_2, the two χ coefficients are very small but comparable in magnitude, which explains why depolarization can become highly apparent with polycrystalline CaF_2 windows.[17] With ⟨111⟩-oriented material, however, χ_- becomes practically insignificant (see Fig. 1) because the critical direction of Joiner, Marburger, and Steier[18] almost coincides with the [111] direction, which suggests that ⟨111⟩-oriented CaF_2 laser windows as recently developed at Harshaw Crystals and Electronics (Solon, Ohio)[19] should not exhibit any detectable birefringence, and this irrespective of the polarization of the incident beam.

[†] For a discussion of the wavelength dependence, the reader may consult Ref. 5.

4. THE SHORT-TIME APPROXIMATION

Returning now to Eq. (7) and making use of the two χ coefficients, it is seen that

$$\mathrm{var}[\delta\phi_+] = (k\chi_+L)^2\mathrm{var}[\delta T] \ , \tag{29}$$

$$\langle\delta\phi_-^2\rangle = (k\chi_-L)^2\langle(\delta T - \overline{\delta T})^2\rangle \ , \tag{30}$$

which clearly specifies how the temperature distribution impacts the focal point intensity. For our purposes, and since the birefringence is expected to play a relatively minor role (see Fig. 1), we may try to relate the intensity degradation to the variance of the temperature rise; in other words, we may rewrite the Strehl ratio expression as follows:

$$s = 1 - (k\chi_{\mathrm{eff}}L)^2\mathrm{var}[\delta T] \ . \tag{31}$$

This amounts to injecting the concept of an "effective optical distortion coefficient,"

$$\chi_{\mathrm{eff}} = |\chi_+|\left[1 + G\left(\frac{\chi_-}{\chi_+}\right)^2\right]^{1/2} \ , \tag{32}$$

which involves a temperature-profile-dependent factor G,

$$G = \frac{\langle(\delta T - \overline{\delta T})^2\rangle}{\langle\delta T^2\rangle - \langle\delta T\rangle^2} \ , \tag{33}$$

and indicates that birefringence now can be accounted for by means of the factor $[1 + G(\chi_-/\chi_+)^2]^{1/2}$; from the point of view of the reduction in target intensity, the ratio χ_-^2/χ_+^2 thus measures the relative weight of birefringence compared to all other sources of optical distortion.

In principle, the temperature distribution δT induced by the passage of a laser beam through a solid window must be obtained by solving the heat-diffusion equation in conjunction with appropriate boundary conditions. In a pulsed mode of operation, however, if one assumes that the time scales are such that planar thermal conduction as well as surface-cooling effects can be ignored, the local temperature rise is simply given by

$$\delta T = \int_0^t \frac{\partial T(\rho,t')dt'}{\partial t'} \ , \tag{34}$$

where $\partial T/\partial t$ relates linearly to the power per unit area absorbed by the window:

$$\frac{\partial T(\rho,t)}{\partial t} = \frac{\beta_{\mathrm{app}}I(\rho,t)}{C_p'} \ . \tag{35}$$

Here, it is understood that β_{app} refers to an "apparent absorption coefficient" defined in the same manner as δT in Eq. (13), which means[20]

$$\beta_{\mathrm{app}} = \beta_V + \frac{2\beta_S}{L} \ , \tag{36}$$

where β_V is the bulk absorption coefficient and β_S characterizes the localized surface absorption.

For the purpose of exercising this model, we now consider the case of an incident beam possessing a Gaussian power-density profile:

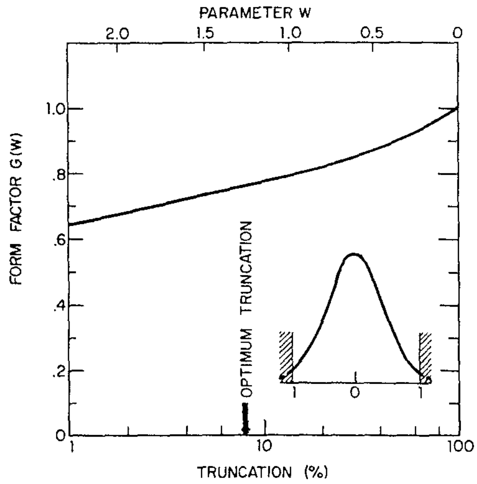

Fig. 2. Form factor G of Eq. (42); the parameter W characterizes the truncation of the Gaussian beam [the truncation is $\exp(-2W)$], as illustrated in the inset.

$$I(\rho,t) = I_o(t)\exp(-2W\rho^2) \tag{37}$$

with $I_o(t)$ representing the on-axis intensity at the window aperture and W referring to the truncation parameter,‡

$$W = \frac{(D/2)^2}{\omega^2} \ . \tag{38}$$

In the short-time approximation (STA), or prediffusion regime, local window temperatures reflect local beam intensities and δT can be expressed simply as a product of two single-variable functions,

$$\delta T = f(\rho)g(t) \ . \tag{39}$$

The function $f(\rho)$ has the shape of the incident beam,

$$f(\rho) = \exp(-2W\rho^2) \ , \tag{40}$$

while $g(t)$ is proportional to the beam fluence on-axis:

$$g(t) = \frac{\beta_{\mathrm{app}}}{C_p'}\int_0^t I_o(t')dt' \ . \tag{41}$$

On this basis, it is immediately seen that the form factor G defined in Eq. (33) becomes a function of truncation only,

$$G(W) = \frac{\langle[f(\rho) - \overline{f(\rho)}]^2\rangle}{\langle f^2(\rho)\rangle - \langle f(\rho)\rangle^2} \ , \tag{42}$$

which is amenable to analytic/numeric evaluation, keeping in mind that $\overline{f(\rho)}$ represents an average as in Eq. (24). The results are displayed in Fig. 2; we note that the dependence on beam shape is quite weak with $G(W)$ close to 0.75 for the type of truncations normally anticipated ($W \simeq 1$). In the context of the

‡Since the power passed through the aperture is $P = P_0[1 - \exp(-2W)]$, the term $\exp(-2W)$ measures the truncation in percent; in the absence of aberrations, optimum truncation in terms of focal irradiances at a fixed beam-power level P_0 corresponds to a truncation parameter W of 1.26.

STA model, we therefore take it that

$$\chi_{\text{eff}} = |\chi_+| \left[1 + 0.75 \left(\frac{\chi_-}{\chi_+} \right)^2 \right]^{1/2} \tag{43}$$

is a good approximation, which validates the concept of an "effective chi."

In this light, we may reconsider the problem of evaluating the reduction in focal intensity and, by the same token, derive a criterion for characterizing the optical performance of candidate window materials. For this purpose, we write [see Eq. (39)]

$$\text{var}[\delta T] = [g(t)]^2 \text{var}[f(\rho)] \tag{44}$$

and evaluate the variance of $f(\rho)$ for Gaussian shapes defined as in Eq. (40):

$$\text{var}[f(\rho)] = \frac{1 - \exp(-5W)}{5[1 - \exp(-W)]} - \left\{ \frac{1 - \exp(-3W)}{3[1 - \exp(-W)]} \right\}^2$$

$$= F^2(W) \ . \tag{45}$$

If the beam intensity remains steady throughout the exposure, this leads immediately to an explicit expression for the window-induced degradation in focal intensity,

$$s = 1 - \left[\frac{kF(W)\beta_{\text{app}} L \chi_{\text{eff}} I_o t}{C_p'} \right]^2 , \tag{46}$$

which is applicable to relatively weak distortions, or distortions that are acceptable in the sense of Maréchal.[13] Assume now that the apparent absorption coefficient is essentially equal to the bulk absorption coefficient, that is, assume that the coatings are "good enough" to satisfy the relation $2\beta_S << \beta_V L$. In that case, the "lensing time," or time required for thermal lensing to substantially degrade ($s = 0.8$) the performance of an initially diffraction-limited window, is given by the product of a beam-related term and a window-related term,

$$t_d = \frac{K_d}{kI_o} \frac{(\text{FOM})_d}{L} , \tag{47}$$

where K_d represents a beam-profile factor for distortion,

$$K_d = \frac{\sqrt{0.2}}{F(W)} , \tag{48}$$

and $(\text{FOM})_d$ is the figure of merit for distortion,

$$(\text{FOM})_d = \frac{C_p'}{\beta_V \chi_{\text{eff}}} , \tag{49}$$

which regroups all window-material features that affect thermal lensing. Since the function $F(W)$ shows little dependence on truncation for $W \gtrsim 1$ but drops rapidly to zero for large truncations, lensing proceeds more slowly for broader beams, smaller intensities, longer wavelengths, and thinner panes, and with window materials having a good figure of merit in terms of heat capacity (C_p'), light absorption (β_V), and sensitivity to distortion (χ_{eff}).

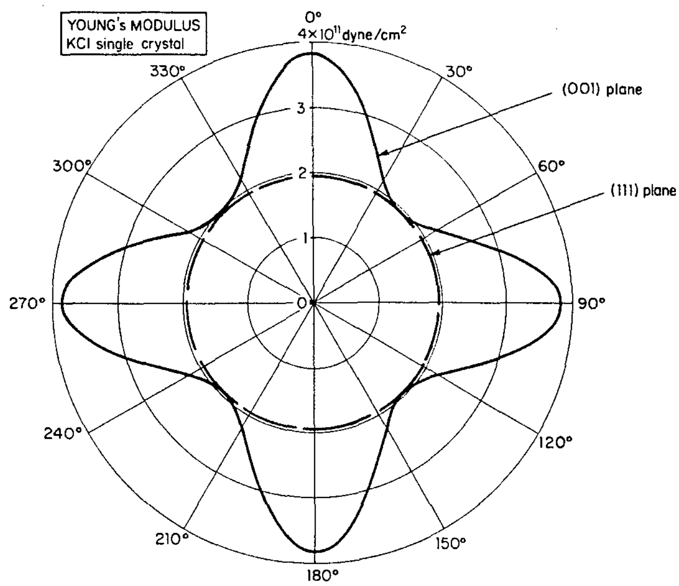

Fig. 3. Young's modulus of single-crystal KCl in the (001) plane and the (111) plane; the modulus may exhibit substantial enhancement along the principal crystallographic axes.

5. ELASTIC AND PHOTOELASTIC PROPERTIES

In cubic single crystals, many physical properties are anisotropic; directionality equations, therefore, must be used to describe the orientational dependence.[21] Take, for instance, the (001) plane in KCl: The elastic modulus varies with direction in the manner portrayed in Fig. 3 and exhibits protuberances along the crystallographic axes. Actually, of all the standard crystallographic configurations, only the (111) plane has isotropic elastic and photoelastic properties. Specifically, we have

$$E_{(111)} = \frac{2}{s_{11} + s_{12} + (1/2)s_{44}} , \tag{50}$$

$$\nu_{\{111\}}^{\{111\}} = -\frac{2}{3} \frac{s_{11} + 2s_{12} - (1/2)s_{44}}{s_{11} + s_{12} + (1/2)s_{44}} , \tag{51}$$

for Young's modulus and Poisson's ratio of relevance here. These two expressions reduce to the commonly used formulas ($E = 1/s_{11}$ and $\nu = -s_{12}/s_{11}$) only if the isotropy condition holds for the elastic compliances. Regarding the elasto-optic properties, Turley and Sines's approach[21] can be easily extended[4] to derive suitable expressions for the two stress-optic coefficients that enter our equations for the optical distortion [Eqs. (25) through (28)]:

$$q_{\parallel} = \frac{1}{2}(q_{11} + q_{12} + q_{44}) , \tag{52}$$

$$q_{\perp} = \frac{1}{6}(q_{11} + 5q_{12} - q_{44}) . \tag{53}$$

Here, the q_{ij}'s are the piezo-optic constants obtained from photoelastic measurements performed on single crystals and listed in the handbooks; as in the case of elastic properties, there is no angular dependence in the (111) plane.

Turning now to macroscopically isotropic solids, we know that the following holds[16]:

$$s_{44}^* = 2(s_{11}^* - s_{12}^*) \ , \tag{54}$$

$$q_{44}^* = q_{11}^* - q_{12}^* \ , \tag{55}$$

if the s_{ij}^*'s and the q_{ij}^*'s are the elastic compliances and the piezo-optic constants as measured for polycrystalline aggregates or for amorphous compositions. Therefore, the elastic properties of concern in a thermal lensing situation are simply

$$E = \frac{1}{s_{11}^*} \ , \tag{56}$$

$$\nu = -\frac{s_{12}^*}{s_{11}^*} \ . \tag{57}$$

Similarly, if the piezo-optic constants are available, as for chemically vapor deposited ZnSe ($q_{11}^* = -1.44\times10^{-12}$ Pa^{-1}, $q_{12}^* = 0.17\times10^{-12}$ Pa^{-1} at 0.6328 μm[22]), we have

$$q_{\parallel} = q_{11}^* \ , \tag{58}$$

$$q_{\perp} = q_{12}^* \ , \tag{59}$$

which indeed is consistent with Eqs. (52) and (53), considering the isotropy condition [Eq. (55)].

If the macroscopic property values of an isotropic aggregate of cubic material are not available, we may proceed as follows: Since the bulk modulus is an invariant, we know that the relation

$$K = \frac{1}{3(s_{11} + 2s_{12})} \tag{60}$$

always holds and yields an exact number for the aggregate *à partir de* single-crystal compliances. The shear modulus, however,

$$G = \frac{1}{2(s_{11}^* - s_{12}^*)} \ , \tag{61}$$

does not relate to single-crystal constants in an obvious manner but falls between narrow bounds, as tabulated in Ref. 23. Because these two moduli, K and G, suffice to completely characterize the elastic features of a solid, it is then a simple matter to obtain both Young's modulus and Poisson's ratio of the aggregate:

$$E = \frac{9KG}{G + 3K} \ , \tag{62}$$

$$\nu = \frac{3K - 2G}{2(G + 3K)} \ . \tag{63}$$

For a discussion of the photoelastic properties of such aggregates, I refer the reader to Ref. 5. Starting from the Flannery-Marburger equations[24] for the strain-optic coefficients, it is seen that the two stress-optic coefficients are best expressed as follows:

$$q_{\parallel} = q_{11} + \frac{2j_3}{5}\left[q_{44} - (q_{11} - q_{12})\right] \ , \tag{64}$$

$$q_{\perp} = q_{12} - \frac{j_3}{5}\left[q_{44} - (q_{11} - q_{12})\right] \tag{65}$$

if the factor j_3 is as given by Hershey,[25]

$$j_3 = \frac{5c_{44}(3K + 4G)}{G(9K + 8G) + 6c_{44}(K + 2G)} \ , \tag{66}$$

keeping in mind that the c_{44} elastic stiffness is the reciprocal of the s_{44} elastic compliance. Again, we note that these expressions are consistent with Eqs. (52) and (53) if the isotropy condition holds; they also have been shown to be compatible with some available experimental evidence.[5]

6. CONCLUSIONS

In its present form, the theory of thermal lensing applies only if the window is exposed to axially symmetric laser beams and has isotropic properties in the plane perpendicular to the axis; this requirement implies window panes made of either ⟨111⟩-oriented cubic crystals, randomly orientated polycrystals, or amorphous materials with elastic and elasto-optic coefficients as discussed in Sec. 5.

In a fixed-focus, long-focal-length configuration, the degradation in focal irradiance caused by thermal lensing is best described by means of the two phase-aberration functions $\delta\phi_+$ and $\delta\phi_-$ defined in Eqs. (5) and (6); an evaluation of the Strehl ratio as carried out in Sec. 2 then yields a convenient expression [Eq. (7)] for assessing the impact of stress birefringence on the far-field irradiance.

The two optical distortion coefficients introduced in Sec. 3, χ_+ and χ_-, relate the spatial temperature distribution to the aberration functions $\delta\phi_+$ and $\delta\phi_-$; Eqs. (25) through (28) then provide explicit expressions for obtaining these coefficients from intrinsic material-property values and apply to thin-disk or long-rod geometries.

In principle, the Strehl ratio relates directly to the variance of the temperature rise by means of an equation such as Eq. (31); this amounts to injecting the concept of an effective optical distortion coefficient [see Eq. (32)], which shows that birefringence gives rise to the correction factor $[1 + G(\chi_-/\chi_+)^2]^{1/2}$ and, thus, that the ratio χ_-^2/χ_+^2 measures the relative weight of stress birefringence compared to all other sources of optical distortion.

For the purpose of elucidating how the reduction in focal irradiance depends on specific features of both the laser window and the laser beam, it is of interest to consider the case of truncated Gaussians in the context of a pulsed mode of operation; it is then a straightforward task (see Sec. 4) to derive simple analytical expressions for the factor G as well as for the ponderated variance of the temperature distribution.

This procedure immediately leads to the definition of a figure of merit for distortion [Eq. (49)], which regroups all window-material-related features that affect thermal lensing; for weak distortions, the effective distortion coefficient χ_{eff} best characterizes the window's susceptibility to lensing but must be evaluated with care because of potentially significant cancellations among the three terms that contribute to the coefficient χ_+.

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